

12 Quadratic Functions

12-1 Quadratic Equations with Perfect Squares

Objective: To solve quadratic equations involving perfect squares.

Vocabulary

Perfect square An expression such as x^2 , $(x - 1)^2$, or $(2x + 5)^2$.

Roots of $x^2 = k$ An equation in the form “perfect square = k ” ($k \geq 0$) can be solved by the method shown in Examples 1 and 2.

If $k > 0$, then $x^2 = k$ has 2 real roots: $x = \pm\sqrt{k}$.

If $k = 0$, then $x^2 = k$ has 1 real root: $x = 0$.

If $k < 0$, then $x^2 = k$ has no real roots.

Example 1 Solve: a. $m^2 = 36$ b. $3r^2 = 48$ c. $x^2 - 11 = 0$ d. $m^2 = -25$

Solution

a. $m^2 = 36$

$$m = \pm\sqrt{36}$$

$$m = \pm 6$$

The solution set is $\{6, -6\}$.

b. $3r^2 = 48$

$$r^2 = 16$$

$$r = \pm\sqrt{16} = \pm 4$$

The solution set is $\{4, -4\}$.

c. $x^2 - 11 = 0$

$$x^2 = 11$$

$$x = \pm\sqrt{11}$$

The solution set is $\{\sqrt{11}, -\sqrt{11}\}$.

d. $m^2 = -25$

Since the square of any real number is always a nonnegative number, there is *no real solution*.

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution*.

1. $x^2 = 49$

2. $2x^2 = 18$

3. $x^2 = \frac{25}{36}$

4. $a^2 = -16$

5. $2x^2 = 128$

6. $5x^2 = 125$

7. $9x^2 = 81$

8. $x^2 - 81 = 0$

9. $x^2 + 25 = 0$

10. $m^2 - 10 = 0$

11. $0 = 6x^2 - 24$

12. $0 = 3m^2 - 75$

Example 2 Solve $(x + 3)^2 = 49$

Solution

$$(x + 3)^2 = 49$$

$$x + 3 = \pm\sqrt{49}$$

$$x = -3 \pm 7$$

$$x = 4 \text{ or } x = -10$$

Check: $(4 + 3)^2 \stackrel{?}{=} 49$

$$7^2 \stackrel{?}{=} 49$$

$$49 = 49 \checkmark$$

Note that $(x + 3)^2$ is a perfect square.

Find the *positive or negative* square root of each side.

Subtract 3 from each side.

$$(-10 + 3)^2 \stackrel{?}{=} 49$$

$$(-7)^2 \stackrel{?}{=} 49$$

$$49 = 49 \checkmark$$

The solution set is $\{4, -10\}$.

12-1 Quadratic Equations with Perfect Squares (continued)

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution*.

13. $(x - 3)^2 = 0$

14. $(z - 1)^2 = 16$

15. $(r - 5)^2 = 100$

16. $(x - 1)^2 = 25$

17. $(2x + 9)^2 = 225$

18. $(t - 4)^2 = 9$

Example 3Solve: a. $3(x - 2)^2 = 21$ b. $y^2 + 10y + 25 = 36$ **Solution**a. $3(x - 2)^2 = 21$

$(x - 2)^2 = 7$

$x - 2 = \pm\sqrt{7}$

$x = 2 \pm \sqrt{7}$

$x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

The check is left to you.

The solution set is

$\{2 + \sqrt{7}, 2 - \sqrt{7}\}$.

b. $y^2 + 10y + 25 = 36$

$(y + 5)^2 = 36$

$y + 5 = \pm\sqrt{36}$

$y + 5 = \pm 6$

$y = -5 \pm 6$

$y = 1$ or $y = -11$

The check is left to you.

The solution set is $\{1, -11\}$.

Note: Example 3(b) could also have been solved by factoring.

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution*.

19. $9m^2 - 1 = 35$

20. $27 = 2r^2 - 5$

21. $3x^2 - 9 = 33$

22. $64 = 2t^2 - 8$

23. $2n^2 + 6 = 38$

24. $7x^2 + 1 = 64$

25. $3(m - 2)^2 = 15$

26. $400 = 4(z - 2)^2$

27. $2(x - 5)^2 = 98$

28. $25 = (2x + 1)^2$

29. $5(m - 3)^2 = 80$

30. $6(z + 5)^2 = 216$

31. $3(x - 1)^2 = -24$

32. $(3x - 1)^2 + 12 = 4$

33. $6(x + 5)^2 = 24$

34. $7(x + 2)^2 = 112$

35. $(x - 2)^2 - 1 = 35$

36. $2(3n - 1)^2 = 8$

37. $3(2x - 1)^2 = 27$

38. $2(x + 3)^2 - 4 = 68$

39. $5(x - 1)^2 + 3 = 23$

40. $x^2 - 2x + 1 = 9$

41. $x^2 + 18x + 81 = 98$

42. $x^2 - 12x + 36 = 64$

43. $x^2 - 4x + 4 = 16$

44. $x^2 + 10x + 25 = 81$

45. $n^2 - 8n + 16 = 36$

Mixed Review Exercises

Express each square as a trinomial.

1. $(x - 8)^2$

2. $(2x + 1)^2$

3. $(4x - 3)^2$

4. $(-2c + 3)^2$

5. $\left(x + \frac{1}{4}\right)^2$

6. $\left(x - \frac{1}{5}\right)^2$

7. $\left(\frac{1}{2}x + \frac{1}{3}\right)^2$

8. $\left(\frac{1}{4}x - \frac{2}{3}\right)^2$

9. $(x + 11)^2$