

6 Fractions

6-1 Simplifying Fractions

Objective: To simplify algebraic fractions.

Vocabulary

Simplest form of an algebraic fraction A form of the fraction in which the numerator and denominator have no common factor other than 1 and -1 .

CAUTION In a fraction, you cannot cancel terms. You must factor to

find common factors to cancel. For example, $\frac{x+y}{x+2} \neq \frac{y}{2}$.

Example 1 Simplify: a. $\frac{21x - 14y}{7}$ b. $\frac{3c - 24}{c - 8}$ c. $\frac{2a + 6}{4a - 12}$

Solution Factor. Then look for common factors to cancel.

$$\text{a. } \frac{21x - 14y}{7} = \frac{\cancel{7}(3x - 2y)}{\cancel{7}} = 3x - 2y$$

$$\text{b. } \frac{3c - 24}{c - 8} = \frac{\cancel{3}(c - \cancel{8})}{\cancel{c} - 8} = 3 \quad (c \neq 8) \quad \left\{ \begin{array}{l} \text{The denominator can't equal 0.} \\ \text{So } c - 8 \neq 0, \text{ or } c \neq 8. \end{array} \right.$$

$$\begin{aligned} \text{c. } \frac{2a + 6}{4a - 12} &= \frac{\cancel{2}(a + 3)}{\cancel{4}(a - 3)} \\ &= \frac{1(a + 3)}{2(a - 3)} \\ &= \frac{a + 3}{2(a - 3)}, \quad (a \neq 3) \quad \left\{ \begin{array}{l} \text{If } a = 3, a - 3 = 0. \\ \text{You must restrict the variable} \\ \text{in the denominator.} \end{array} \right. \end{aligned}$$

Simplify. Give any restrictions on the variables.

1. $\frac{3x - 3y}{9}$

2. $\frac{10m - 15n}{5}$

3. $\frac{4a - 20}{a - 5}$

4. $\frac{3n + 12}{n + 4}$

5. $\frac{4n + 24}{n + 6}$

6. $\frac{2n - 18}{n - 9}$

7. $\frac{2m + 3}{6m + 9}$

8. $\frac{6x + 6y}{6x - 6y}$

9. $\frac{3w + 5}{9w + 15}$

10. $\frac{4m - 4n}{4m + 4n}$

Example 2 Simplify $\frac{x^2 - 4}{2x^2 + 3x - 2}$.

Solution $\frac{x^2 - 4}{2x^2 + 3x - 2} = \frac{(x - 2)\cancel{(x + 2)}}{(2x - 1)\cancel{(x + 2)}}$ Factor. $x + 2$ is a common factor.
 $= \frac{x - 2}{2x - 1}, (x \neq -2, x \neq \frac{1}{2})$

To see which values of x to exclude, look at the denominator of the original fraction.

Since $2x - 1 \neq 0$ and $x + 2 \neq 0$, $x \neq \frac{1}{2}$ and $x \neq -2$.

6-1 Simplifying Fractions (continued)

Simplify. Give any restrictions on the variables.

11. $\frac{3x - 9}{x^2 - 9}$

12. $\frac{5y + 30}{y^2 - 36}$

13. $\frac{b^2 - 4}{b + 2}$

14. $\frac{x^2 - 49}{x + 7}$

15. $\frac{8n^2 - 72}{4n - 12}$

16. $\frac{15c + 25d}{90c^2 - 250d^2}$

17. $\frac{4xy}{x^2y - xy^2}$

18. $\frac{3x^2 - 6x}{3x^3}$

19. $\frac{a^2 - 3a - 10}{a^2 - 4}$

20. $\frac{a^2 - 5a - 36}{a^2 - 81}$

21. $\frac{2w^2 - w - 6}{2w - 4}$

22. $\frac{2x^2 + 5x - 3}{x^2 + 2x - 3}$

Example 3 Simplify: $\frac{2x^2 - 3x - 2}{4 - x^2}$.

Solution $\frac{2x^2 - 3x - 2}{4 - x^2} = \frac{(2x + 1)(x - 2)}{(2 + x)(2 - x)}$ { Factor. Since $(x - 2)$ and $(2 - x)$ are opposites, $(2 - x) = -(x - 2)$.

$$= \frac{(2x + 1)\cancel{(x - 2)}}{-(2 + x)\cancel{(x - 2)}}$$

$$= \frac{2x + 1}{-(2 + x)}, \text{ or } -\frac{2x + 1}{x + 2}, (x \neq 2, x \neq -2)$$

Simplify. Give any restrictions on the variables.

23. $\frac{(3n + 2)(n - 3)}{(3 + n)(3 - n)}$

24. $\frac{(x - 4)(3x + 4)}{(4 - x)(5x + 2)}$

25. $\frac{(x - 5)(2x - 7)}{(5 - x)(3x + 2)}$

26. $\frac{(x - 7)(x - 4)}{(7 - x)(x + 2)}$

27. $\frac{x^2 - 10x + 25}{25 - x^2}$

28. $\frac{6 - x}{x^2 - 2x - 24}$

29. $\frac{(a - 3)^2}{9 - a^2}$

30. $\frac{2n^2 - 72}{6n + 36}$

31. $\frac{6 + x - x^2}{x^2 - 9}$

32. $\frac{10 + 3x - x^2}{x^2 - 4}$

33. $\frac{2w^2 - w - 6}{2w - 4}$

34. $\frac{3x^2 - 6x}{6x^2 - 7x - 10}$

35. $\frac{2n^2 + 5n - 3}{4n^2 + 8n - 5}$

36. $\frac{2y^2 - 7y + 3}{6y - 2y^2}$

37. $\frac{3y^2 - 5y + 2}{6y^2 - y - 2}$

38. $\frac{3x^2 - 15x}{3x^2 - 16x + 5}$

Mixed Review Exercises

Simplify. Assume that no denominator equals zero.

1. $10\left(\frac{1}{2}u + \frac{1}{5}v\right)$

2. $(-36m + 24n)\left(-\frac{1}{6}\right)$

3. $\frac{20a^6b^5}{35a^2b^3}$

4. $\frac{(-2y)^4}{(y^2)^4}$

5. $\frac{2x^4 + 6x^3 + 10x^2}{2x^2}$

6. $(-10)(-6)(-2)(-5)$

Solve.

7. $3(x + 1) + 1 = 25$

8. $8y - (5y + 4) = 11$

9. $(2n - 3) - (5 - 2n) = 16$