

1-2 Simplifying Expressions

Objective: To review the methods used to simplify numerical expressions and to evaluate algebraic expressions.

Vocabulary

Variable A symbol, usually a letter, used to represent one or more numbers.

Algebraic expression A numerical expression; a variable; or a sum, difference, product, or quotient that contains one or more variables.

Examples: $6 + 11$ x $x^2 - 5y + z$

Simplify To simplify an expression you replace it by the simplest or most common symbol having the same value.

Evaluate an expression To evaluate an algebraic expression, or find its value, replace each variable in the expression by a given value and simplify the result.

Power A product of equal factors. The repeated factor is the *base*. A positive *exponent* tells the number of times the base occurs as a factor. Example:

$5 \times 5 \times 5 = 5^3$ is a power in which 5 is the base and 3 is the exponent.

Absolute value If x is positive or zero, $|x| = x$.
If x is negative, $|x| = -x$ (reads "the opposite of x ").

Symbols Grouping: () (parentheses) [] (brackets) — (fraction bar)

CAUTION Remember to use the correct order of operations. Expressions inside grouping symbols are simplified first, and then powers. Next, multiplication and division are done *in order from left to right*. Finally, addition and subtraction are done in order from left to right.

Example 1 Use one of the symbols $<$, $=$, or $>$ to make a true statement.

a. $5^2 + 3^2$? $(5 + 3)^2$ b. $(7 + 2) + 8$? $7 + (2 + 8)$

Solution Find the value of each side. Then compare the results.

a. $5^2 + 3^2 = 25 + 9 = 34$ b. $(7 + 2) + 8 = 9 + 8 = 17$
and $(5 + 3)^2 = 8^2 = 64$ and $7 + (2 + 8) = 7 + 10 = 17$,
Since 34 is less than 64, so $(7 + 2) + 8 = 7 + (2 + 8)$.
 $5^2 + 3^2 < (5 + 3)^2$.

Use one of the symbols $<$, $=$, or $>$ to make a true statement.

1. $1 \cdot 4$? $1 \div 4$

2. $4 \cdot 1$? $4 \div 1$

3. $4^2 \cdot 5^2$? $(4 \cdot 5)^2$

4. $4^2 + 5^2$? $(4 + 5)^2$

5. $\frac{5 + 3}{5 - 3}$? $\frac{7 + 5}{7 - 5}$

6. $\frac{5 + 4}{5 - 4}$? $\frac{8 + 4}{8 - 4}$

7. $(8 + 5) + 1$? $8 + (5 + 1)$

8. $(8 - 5) - 1$? $8 - (5 - 1)$

9. $(8 \cdot 5) \cdot 2$? $8 \cdot (5 \cdot 2)$

10. $(16 \div 4) \div 2$? $16 \div (4 \div 2)$

1-2 Simplifying Expressions (continued)**Example 2** Simplify $9^2 - 6 \div 3 + 4$.

$$\begin{aligned} \text{Solution} \quad 9^2 - 6 \div 3 + 4 &= 81 - \underbrace{6 \div 3} + 4 \\ &= \underbrace{81 - 2} + 4 \\ &= 79 + 4 \\ &= 83 \end{aligned}$$

Simplify the power: $9^2 = 9 \times 9$.

Divide.

Subtract.

Add.

Example 3 Simplify $11 + 5[8 - 3(6 - 4)]$.

$$\begin{aligned} \text{Solution} \quad 11 + 5[8 - 3(6 - 4)] &= 11 + 5 \cdot [8 - 3 \cdot 2] \\ &= 11 + 5 \cdot [8 - 6] \\ &= 11 + 5 \cdot 2 \\ &= 11 + 10 \\ &= 21 \end{aligned}$$

Subtract inside parentheses.

Multiply inside brackets.

Subtract inside brackets.

Multiply.

Add.

Simplify.

11. a. $18 - 7 + 3 - 1$

b. $18 - (7 + 3) - 1$

c. $18 - (7 + 3 - 1)$

12. a. $6 \cdot 4 + 5 \cdot 2$

b. $6 \cdot (4 + 5) \cdot 2$

c. $6 \cdot (4 + 5 \cdot 2)$

13. a. $6^2 - 8 \div 2 + 5$

b. $(6^2 - 8) \div 2 + 5$

c. $(6^2 - 8) \div (2 + 5)$

14. $3 \cdot 2^3 - (7^2 - 5^2)$

15. $16 - 3[9 - 2(5 - 3)]$

16. $[4(5 - 2) + 2^3] \div 2$

17. $\frac{3^3}{4 - (4 - 1)}$

18. $\frac{1}{2} \left| \frac{1 + 9^2}{5^2} \right|$

19. $\frac{4^3 + 6}{4^2 - 6}$

Example 4 Evaluate each expression if $r = 2$, $s = 5$, $t = 6$, and $u = -9$.

a. $3r^2 + s - 6$

b. $\frac{t^2 - 3rs}{t + r^2}$

c. $3|u| - |s|$

Solution Substitute the given values for the variables. Then simplify.

a. $3r^2 + s - 6 = 3(2)^2 + 5 - 6 = 3 \cdot 4 + 5 - 6 = 12 + 5 - 6 = 11$

b. $\frac{t^2 - 3rs}{t + r^2} = \frac{6^2 - 3 \cdot 2 \cdot 5}{6 + 2^2} = \frac{36 - 30}{6 + 4} = \frac{6}{10} = \frac{3}{5}$

c. $3|u| - |s| = 3|-9| - |5| = 3 \cdot 9 - 5 = 27 - 5 = 22$

Evaluate each expression if $w = -8$, $x = 2$, $y = 3$, and $z = 7$.

20. $y^2 - y + 1$

21. $3x^2 + x - 8$

22. $(xy - x)^3$

23. $\frac{2xy}{z^2 - y^3}$

24. $\left(\frac{xyz}{x + y + z} \right)^2$

25. $|w| - |x|$

26. $|w| + 3|y|$

27. $|w|^2 - |z^2|$