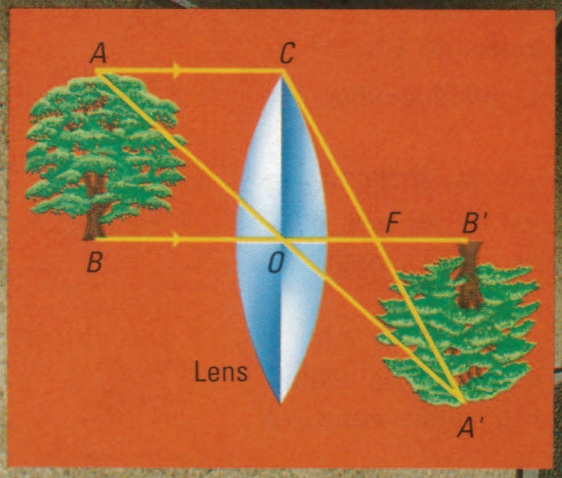
6 Fractions



A fractional equation describes the relationships among these distances: between object and lens (OB), lens and film (OB′), and lens and focal point (OF).

$$\frac{1}{OB} + \frac{1}{OB'} = \frac{1}{OF}$$

Algebraic Fractions

6-1 Simplifying Fractions

Objective To simplify algebraic fractions.

When the numerator and denominator of an algebraic fraction have no common factor other than 1 and -1, the fraction is said to be in *simplest form*. To simplify a fraction, first factor the numerator and the denominator.

Example 1 Simplify $\dfrac{3a+6}{3a+3b}$.

Solution
$$\frac{3a+6}{3a+3b} = \frac{3(a+2)}{3(a+b)}$$
Factor the numerator and denominator and look for common factors.
$$= \frac{a+2}{a+b} \quad (a \neq -b) \quad \textbf{\textit{Answer}}$$

Remember that you cannot divide by zero. You must *restrict* the variables in a denominator by excluding any values that make the denominator equal to zero. In Example 1, a cannot equal $-b$.

Example 2 Simplify $\dfrac{x^2-9}{(2x+1)(3+x)}$.

Solution
$$\frac{x^2-9}{(2x+1)(3+x)} = \frac{(x+3)(x-3)}{(2x+1)(3+x)}$$
$x+3$ and $3+x$ are equal.
$$= \frac{x-3}{2x+1} \quad \left(x \neq -\tfrac{1}{2},\ x \neq -3\right) \quad \textbf{\textit{Answer}}$$

Note: To see which values of the variable to exclude, look at the denominator of the *original* fraction. Neither $2x+1$ nor $3+x$ can equal zero. Since $2x+1 \neq 0$ and $3+x \neq 0$, $x \neq -\frac{1}{2}$ and $x \neq -3$.

Example 3 Simplify $\dfrac{2x^2+x-3}{2-x-x^2}$.

Solution First factor the numerator and the denominator. If you don't see any common factors, look for opposites.

(Solution continues on next page.)

$$\frac{2x^2 + x - 3}{2 - x - x^2} = \frac{(x - 1)(2x + 3)}{(1 - x)(2 + x)}$$

$(x - 1)$ and $(1 - x)$ are opposites.
$(1 - x) = -(x - 1)$

$$= \frac{(x - 1)(2x + 3)}{-(x - 1)(2 + x)}$$

$$= \frac{2x + 3}{-(2 + x)}, \text{ or } -\frac{2x + 3}{x + 2} \quad (x \neq 1, x \neq -2) \quad \textit{Answer}$$

Example 4 Solve $ax - a^2 = bx - b^2$ for x.

Solution

$ax - a^2 = bx - b^2$

$ax - bx = a^2 - b^2$ — Collect all terms with x on one side of the equation and all other terms on the other side.

$(a - b)x = (a + b)(a - b)$ — Factor both sides of the equation.

$$x = \frac{(a + b)(a - b)}{(a - b)}$$ — Divide both sides by the coefficient of x.

$x = a + b \quad (a \neq b)$ ***Answer***

Oral Exercises

Simplify. State the restrictions on the variable.

1. $\frac{3m + 9}{m + 3}$
2. $\frac{2n + 8}{3n + 12}$
3. $\frac{a^2 - 16}{a - 4}$
4. $\frac{b^2 - 9}{b + 3}$
5. $\frac{x^2 - 2x + 1}{x - 1}$
6. $\frac{x + 6}{36 - x^2}$
7. $\frac{14 - 2c}{7 - c}$
8. $\frac{2c - 2d}{2c + 2d}$
9. $\frac{2t - 1}{1 - 2t}$
10. $\frac{(2y - 8)^2}{(2y - 8)^3}$
11. $\frac{(x + 5)^2}{5 + x}$
12. $\frac{(4 - x)(x^2 - 9)}{(x - 4)(x - 3)}$

Which of the following fractions *cannot* be simplified?

13. $\frac{4x - 6y}{4x + 6y}$
14. $\frac{4x - 7y}{7y + 4x}$
15. $\frac{4x^2 - y^2}{2x - y}$
16. $\frac{4x^2 + y^2}{2x + y}$

Written Exercises

Simplify. Give any restrictions on the variables.

A

1. $\frac{2x + 2y}{4}$
2. $\frac{12m - 15n}{9}$
3. $\frac{5a - 10}{a - 2}$
4. $\frac{4n + 16}{n + 4}$
5. $\frac{3n + 1}{9n + 3}$
6. $\frac{8x - 8y}{8x + 8y}$
7. $\frac{2x - 4}{x^2 - 4}$
8. $\frac{6y + 30}{y^2 - 25}$

9. $\dfrac{2xy}{x^2y - y^2x}$

10. $\dfrac{4p^2 - 8p}{4p^3}$

11. $\dfrac{(x + 4)(2x + 1)}{(1 + 2x)(x - 3)}$

12. $\dfrac{(x - 5)(2 + 7x)}{(5 - x)(7x + 2)}$

13. $\dfrac{a^2 + 8a + 16}{16 - a^2}$

14. $\dfrac{25 - b^2}{b^2 + 12b + 35}$

15. $\dfrac{2 - y}{y^2 - 4y + 4}$

16. $\dfrac{(a - 5)^2}{25 - a^2}$

17. $\dfrac{4n^2 + 144}{6n + 36}$

18. $\dfrac{25c + 15d}{50c^2 + 30d^2}$

19. $\dfrac{2n^2 - 5n - 3}{4n^2 - 8n - 5}$

20. $\dfrac{2w^2 + w - 6}{2w + 4}$

21. $\dfrac{3x^2 + 6x}{6x^2 + 7x - 10}$

22. $\dfrac{4b^2 - 5b - 6}{8b^2 + 6b}$

23. $\dfrac{10 - 3a - a^2}{a^2 - 4}$

24. $\dfrac{2y^2 - 9y + 4}{8y - 2y^2}$

25. $\dfrac{x^2 + xy}{x^2 - xy}$

26. $\dfrac{2ab + 2ac + 4a^2}{4b + 4c + 8a}$

27. $\dfrac{r^2 - s^2}{4r^2 + rs - 5s^2}$

28. $\dfrac{3t^2 - 4tv - 7v^2}{t^2 - v^2}$

Solve for x.

B **29.** $cx + dx = c^2 - d^2$

30. $ax + bx = a^2 + 2ab + b^2$

31. $abx - b = ax - 1$

32. $3ax + 6 = a^2x + 2a$

33. $5kx - x = 25k^2 - 10k + 1$

34. $4x - 4 = b^2 + 5b - bx$

35. $2cx + 3dx = 4c^2 + 12cd + 9d^2$

36. $2x + 5k = 6k^2 - 3kx - 6$

37. $a(x - a) + 6(x + 6) = 0$

38. $2n(x - n) = x - 5n + 2$

39. Miguel wants to evaluate $\dfrac{x^2 - 4y^2}{x - 2y}$ when $x = 3$ and $y = 1$. First he simplifies the fraction to $x + 2y$. Then he substitutes $x = 3$ and $y = 1$, getting 5 for his answer. Miguel also uses the simplified form $x + 2y$ to evaluate the given fraction when $x = 4$ and $y = 2$, getting 8 for his answer. Tell which one of these two answers is incorrect and explain why.

40. Donna wants to simplify $\dfrac{x^2 - 4y^2}{x^2 - 2xy}$.

She gives this solution: $\dfrac{x^2 - 4y^2}{x^2 - 2xy} = \dfrac{x^2 - 4y^2}{x^2 - 2xy} = \dfrac{-4y^2}{-2xy} = \dfrac{2y}{x}$

Choose values of x and y to show that her solution is incorrect. Simplify the fraction correctly.

Simplify. Give any restrictions on the variables.

41. $\dfrac{25x^2 - 36y^2}{10x^2 + 3xy - 18y^2}$

42. $\dfrac{12x^2 - 5xy - 2y^2}{4y^2 - 4xy - 3x^2}$

43. $\dfrac{6a^3 + 10a^2}{36a^3 - 100a}$

44. $\dfrac{8a^2 + 6ab - 5b^2}{16a^2 - 25b^2}$

Simplify. Give any restrictions on the variables.

45. $\dfrac{x(a+b)-2(a+b)}{2-x}$

46. $\dfrac{w^4-1}{w^4(w+1)+(w+1)}$

C **47.** $\dfrac{2x^3-13x^2+15x}{15x-7x^2-2x^3}$

48. $\dfrac{x^4-10x^2+9}{3-2x-x^2}$

49. $\dfrac{4a^2-b^2-2a+b}{(2a+b)^2-1}$

50. $\dfrac{x^2+2x+1-y^2}{(x+1)^2+2y(x+1)+y^2}$

For which value(s) of x does each fraction equal zero?

51. $\dfrac{x^2-4}{x^2-4x+4}$

52. $\dfrac{3x^2+x}{2x-x^3}$

53. $\dfrac{x^2-2x-15}{x^2+3x-40}$

54. $\dfrac{x^4-x^2}{x^3+x^2-2x}$

Mixed Review Exercises

Simplify. Assume that no denominator equals zero.

1. $12\left(\frac{1}{3}u+\frac{1}{4}v\right)$

2. $(-42n+28p)\left(-\frac{1}{4}\right)$

3. $\dfrac{15a^7b^8}{30a^2b^3}$

4. $\dfrac{(-3y)^5}{(y^2)^5}$

5. $\dfrac{3x^5+6x^3-12x^2}{x^2}$

6. $(-20)(-7)(-4)(-5)$

Solve.

7. $-y+15=9$

8. $68=-\frac{n}{5}$

9. $5p+8=-47$

10. $5(x+2)+2=27$

11. $9y-(7y+5)=11$

12. $(3n-6)-(5-3n)=7$

Computer Exercises

For students with some programming experience.

1. a. Write a BASIC program to evaluate each algebraic fraction for $x = 10, 20, 30, 40, 50$.

(1) $\dfrac{x}{x^2+1}$ (2) $\dfrac{x^2}{x^2+1}$ (3) $\dfrac{x^2}{x+1}$

b. As the value of x increases, what happens to the value of each of these algebraic fractions? Explain why.

2. a. Write a BASIC program that uses READ . . . DATA statements to evaluate the algebraic fraction $\dfrac{x^5+1}{x^4-x^3+x^2-x+1}$ for $x = 2, 13, 22, 50, 99$.

b. On the basis of your results in part (a), suggest a general formula that evaluates the given fraction for any value of x.

6-2 Multiplying Fractions

Objective To multiply algebraic fractions.

The property of quotients given in Lesson 5-2 states that

$$\frac{ac}{bd} = \frac{a}{b} \cdot \frac{c}{d}.$$

You can rewrite this result to get the multiplication rule for fractions.

Multiplication Rule for Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example: $\frac{3}{8} \cdot \frac{5}{2} = \frac{3 \cdot 5}{8 \cdot 2} = \frac{15}{16}$

To multiply fractions, you multiply their numerators and multiply their denominators.

Example 1 Multiply: $\frac{8}{9} \cdot \frac{3}{10}$.

Solution 1

$$\frac{8}{9} \cdot \frac{3}{10} = \frac{8 \cdot 3}{9 \cdot 10} = \frac{\cancel{24}^{4}}{\cancel{90}_{15}} = \frac{4}{15}$$

You can multiply first and then simplify.

Solution 2

$$\frac{\cancel{8}^{4}}{\cancel{9}_{3}} \cdot \frac{\cancel{3}^{1}}{\cancel{10}_{5}} = \frac{4}{15}$$

You can simplify first and then multiply.

Example 2 Multiply: **a.** $\frac{6x}{y^3} \cdot \frac{y^2}{15}$ **b.** $\frac{x^2 - x - 12}{x^2 - 5x} \cdot \frac{x^2 - 25}{x + 3}$

Solution

a. $\frac{6x}{y^3} \cdot \frac{y^2}{15} = \frac{\cancel{3} \cdot 2x}{\cancel{y^2} \cdot y} \cdot \frac{\cancel{y^2} \cdot 1}{\cancel{3} \cdot 5} = \frac{2x}{5y} \quad (y \neq 0)$

b. $\frac{x^2 - x - 12}{x^2 - 5x} \cdot \frac{x^2 - 25}{x + 3} = \frac{(x - 4)\cancel{(x + 3)}}{x\cancel{(x - 5)}} \cdot \frac{(x + 5)\cancel{(x - 5)}}{\cancel{(x + 3)}}$

$$= \frac{(x - 4)(x + 5)}{x} \quad (x \neq 0,\ x \neq 5,\ x \neq -3)$$

Answer

Another way to write the answer to Example 2(b) is $\frac{x^2 + x - 20}{x}$. The factored form of the answer, as shown in Example 2, is the one we'll show in this book.

In Example 2(b), the denominators of $\frac{x^2 - x - 12}{x^2 - 5x} \cdot \frac{x^2 - 25}{x + 3}$ equal zero when x is 0, 5, or -3, so the product is restricted to values of x other than 0, 5, and -3.

From now on, assume that the domains of the variables do not include values for which any denominator is zero. Therefore, it will not be necessary to show the excluded values of the variables.

In Chapter 4, you learned the rule of exponents for a power of a product:

For every positive integer m, $(ab)^m = a^m b^m$.

The rule below is similar.

Rule of Exponents for a Power of a Quotient

For every positive integer m,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.$$

Example 3 Simplify $\left(\frac{x}{3}\right)^3$.

Solution $\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3}$

$= \frac{x^3}{27}$ *Answer*

Example 4 Simplify $\left(-\frac{c}{2}\right)^2 \cdot \frac{4}{3c}$.

Solution $\left(-\frac{c}{2}\right)^2 \cdot \frac{4}{3c} = \frac{c^2}{4} \cdot \frac{4}{3c}$

$= \frac{c \cdot c}{\cancel{4}} \cdot \frac{\cancel{4}}{3\cancel{c}}$

$= \frac{c}{3}$ *Answer*

Oral Exercises

Multiply. Express each product in simplest form.

1. $\frac{6}{5} \cdot \frac{10}{3}$
2. $\frac{9}{8} \cdot \frac{16}{3}$
3. $-\frac{3}{4} \cdot \frac{8}{9}$
4. $\frac{a}{b} \cdot \frac{b}{a}$
5. $\frac{2}{x} \cdot \frac{x}{14}$
6. $\frac{n}{6} \cdot \frac{16}{n}$
7. $\frac{5y^2}{3} \cdot \frac{6}{y^2}$
8. $\frac{2a}{3} \cdot \frac{a}{4}$
9. $(3c)^2 \cdot \frac{4}{c}$
10. $\frac{b}{(2a)^2} \cdot \frac{a^2}{b}$
11. $\frac{(x-1)^2}{8} \cdot \frac{4}{x-1}$
12. $\frac{3n-2}{n^2} \cdot \frac{n^4}{2-3n}$

Simplify.

13. $\left(\frac{5a}{b}\right)^2$
14. $\left(\frac{x}{2y}\right)^3$
15. $\left(-\frac{c^2}{3}\right)^2$
16. $\left(-\frac{4}{n^2}\right)^3$

Written Exercises

Multiply. Express each product in simplest form.

A

1. $\frac{4}{7} \cdot \frac{21}{8}$
2. $\frac{4}{9} \cdot \frac{3}{16}$
3. $\frac{15}{4} \cdot \frac{8}{9}$
4. $-\frac{7}{2} \cdot \frac{10}{28}$
5. $\frac{3}{5} \cdot \frac{5}{7} \cdot \frac{7}{9}$
6. $\frac{9}{5} \cdot \frac{2}{3} \cdot \frac{15}{18}$
7. $\left(-\frac{5}{2}\right)^2 \cdot \frac{8}{5}$
8. $(-2)^3 \cdot \frac{35}{16}$
9. $\frac{6}{x^2} \cdot \frac{x^3}{3}$
10. $\frac{5y}{2} \cdot \frac{4}{15y}$
11. $\frac{a}{b} \cdot \frac{b}{c} \cdot \frac{c}{d}$
12. $\frac{4}{x^2} \cdot \frac{7x}{8}$
13. $\frac{4w}{v} \cdot \frac{v^3}{2w^2}$
14. $\frac{6a}{11b^4} \cdot \frac{22b}{3a^3}$
15. $\frac{4d^2e}{9ef} \cdot \frac{f^2}{6d}$
16. $\frac{2rs^2}{3t} \cdot \frac{9t^2}{4rs}$

Simplify.

17. $\left(\frac{a}{6}\right)^2$
18. $\left(\frac{c}{5}\right)^3$
19. $\left(\frac{2n}{7}\right)^2$
20. $\left(\frac{4x}{3}\right)^2$
21. $\left(\frac{2a}{5b^3}\right)^2$
22. $\left(\frac{4m}{7n^2}\right)^2$
23. $\left(\frac{-x^2}{10}\right)^4$
24. $-\left(\frac{5b^4}{6}\right)^2$
25. $\left(\frac{a}{b}\right)^2 \cdot \frac{b}{a}$
26. $\left(\frac{3x}{y}\right)^3 \cdot \frac{y^2}{9}$
27. $\left(-\frac{x}{4y}\right)^2 \cdot \left(-\frac{4y}{x}\right)$
28. $\left(\frac{3z}{y}\right)^3 \cdot \frac{2yz}{15}$

29. Find the area of a square if each side has length $\frac{2x}{7}$ in.

30. Find the volume of a cube if each edge has length $\frac{4n}{5}$ in.

31. A triangle has base $\frac{3x}{4}$ cm and height $\frac{8}{9x}$ cm. What is its area?

32. If you travel for $\frac{7t}{60}$ hours at $\frac{80r}{9}$ mi/h, how far have you gone?

B

33. Find the total dollar cost of $\frac{4y}{3}$ eggs if they cost d dollars per dozen.

34. Find the total dollar cost of n dozen pencils if each pencil costs $\frac{2c}{3}$ cents.

Simplify.

35. $\frac{c+2}{c^2} \cdot \frac{3c}{c^2-4}$
36. $\frac{x^2-1}{16x} \cdot \frac{4x^2}{5x+5}$
37. $\frac{a^2-x^2}{a^2} \cdot \frac{a}{3x-3a}$
38. $\frac{3r-rt}{6r^2t} \cdot \frac{3}{9-t^2}$
39. $(4b^2-3b) \cdot \frac{5b^2}{40b^3-30b^2}$
40. $\frac{2y}{3y^2+15y} \cdot (3y^3-75y)$
41. $\frac{10x^4}{6x-12} \cdot \frac{x^2-x-2}{4x}$
42. $\frac{2x^2+5x-3}{x+2} \cdot \frac{9x+18}{1-2x}$
43. $\frac{n^2-3n+2}{n^2+3n+2} \cdot \frac{8n+8}{4n+8}$
44. $\frac{x^2+4x-21}{x^2-6x-16} \cdot \frac{x^2-8x+15}{x^2+9x+14}$

Simplify.

45. $\dfrac{t^2 - 2t - 8}{4 - t^2} \cdot \dfrac{t^2 - 5t + 6}{t^2 - t - 12}$

46. $\dfrac{a^2 - 7a + 6}{2a^3 - 3a^2} \cdot \dfrac{4a^3 - 9a}{a^2 - 10a + 24}$

47. $\dfrac{3x^2 - 14x - 5}{8x^2 - 12x} \cdot \dfrac{8x^2 - 18}{3x^2 + 4x + 1}$

48. $\dfrac{25 - 16y^2}{6y^3 - 36y^2} \cdot \dfrac{10y + 8y^2}{10 - 3y - 4y^2}$

49. $\dfrac{3d^2 - 9d + 6}{2d^2 - 10d + 12} \cdot \dfrac{6 - 2d}{3 - 3d}$

50. $\dfrac{4c^2 - 8c - 5}{12c^2 + 46c + 40} \cdot \dfrac{20 + 8c}{5 - 2c}$

51. $\left(\dfrac{x - 2}{2}\right)^2 \cdot \left(\dfrac{2}{x - 2}\right)^3$

52. $\left(\dfrac{2n - 1}{3}\right)^4 \cdot \left(\dfrac{9}{2n - 1}\right)^2$

53. $\left(\dfrac{x - y}{x + y}\right)^2 \cdot \dfrac{x^2 + y^2}{x^2 - y^2}$

54. $\dfrac{4a^2 - b^2}{4c^2 - d^2} \cdot \left(\dfrac{2c - d}{2a + b}\right)^2$

C **55.** $\dfrac{9x^2 - 1}{6x^2} \cdot \dfrac{12x^2 + 12x}{(1 - 3x)^2} \cdot \dfrac{6 - 18x}{3x^2 + 4x + 1}$

56. $\dfrac{4n^2 - 4}{1 + n^2} \cdot \dfrac{1 - n}{2n} \cdot \dfrac{1 - 2n^2 + n^4}{2 + 2n}$

57. $\dfrac{x^3 + 3x^2 - 4x - 12}{2x^2 - 18} \cdot \dfrac{x^3 - 3x^2 + 3x - 9}{3x^3 - 12x}$

58. $\dfrac{a^2 - (b - c)^2}{2a - 2b + 2c} \cdot \dfrac{6a - 6b + 6c}{b^2 - (a - c)^2}$

59. Find all values of x for which $\dfrac{2x^3 - 2x}{3x^3} \cdot \dfrac{9}{x^2 + 3x + 2}$ is equal to zero.

Mixed Review Exercises

Factor completely.

1. $a^2 + 14a + 45$

2. $x^2 - 7x + 10$

3. $16x^4 - 81$

4. $2x^2 + 5x + 1$

5. $625y^2 - 4z^2$

6. $64 + 16c + c^2$

7. $xy + 3y - 4xz - 12z$

8. $9x^2 - 12x + 4$

9. $3x^2 + 14x - 5$

10. $x^4 + 7x^2 - 8x^3$

11. $n^2 + 5n - 14$

12. $y^2 - 5y - 24$

Challenge

What is wrong with this "proof" that $2 = 1$?

$$\begin{aligned} r &= s \\ r^2 &= rs \\ r^2 - s^2 &= rs - s^2 \\ (r + s)(r - s) &= s(r - s) \\ r + s &= s \\ s + s &= s \\ 2s &= s \\ 2 &= 1 \end{aligned}$$

6-3 Dividing Fractions

Objective To divide algebraic fractions.

To divide by a real number, you multiply by its reciprocal. You use the same rule to divide algebraic fractions.

Division Rule for Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example: $\frac{5}{8} \div \frac{2}{9} = \frac{5}{8} \cdot \frac{9}{2} = \frac{45}{16}$

To divide by a fraction, you multiply by its reciprocal.

Example 1 Divide: $\frac{x}{2y} \div \frac{xy}{4}$

Solution

$$\frac{x}{2y} \div \frac{xy}{4} = \frac{x}{2y} \cdot \frac{4}{xy}$$ {Multiply by the reciprocal.

$$= \frac{\cancel{x}}{\cancel{2} \cdot y} \cdot \frac{\cancel{2} \cdot 2}{\cancel{x} \cdot y}$$ {Factor and simplify.

$$= \frac{2}{y^2}$$ ***Answer***

Example 2 Divide: **a.** $\frac{18}{x^2 - 25} \div \frac{24}{x + 5}$ **b.** $\frac{x^2 + 3x - 10}{2x + 6} \div \frac{x^2 - 4}{x^2 - x - 12}$

Solution

a. $\frac{18}{x^2 - 25} \div \frac{24}{x + 5} = \frac{18}{x^2 - 25} \cdot \frac{x + 5}{24}$

$$= \frac{\cancel{6} \cdot 3}{\cancel{(x + 5)}(x - 5)} \cdot \frac{\cancel{x + 5}}{\cancel{6} \cdot 4}$$

$$= \frac{3}{4(x - 5)}$$ ***Answer***

b. $\frac{x^2 + 3x - 10}{2x + 6} \div \frac{x^2 - 4}{x^2 - x - 12} = \frac{x^2 + 3x - 10}{2x + 6} \cdot \frac{x^2 - x - 12}{x^2 - 4}$

$$= \frac{(x + 5)\cancel{(x - 2)}}{2\cancel{(x + 3)}} \cdot \frac{\cancel{(x + 3)}(x - 4)}{(x + 2)\cancel{(x - 2)}}$$

$$= \frac{(x + 5)(x - 4)}{2(x + 2)}$$ ***Answer***

To simplify an expression that involves more than one operation, follow the order of operations on page 142, as shown in Example 3.

Example 3 Simplify $\left(\frac{2x}{y}\right)^3 \div \frac{x}{y^2} \cdot \frac{x}{4}$.

Solution

$$\left(\frac{2x}{y}\right)^3 \div \frac{x}{y^2} \cdot \frac{x}{4} = \frac{8x^3}{y^3} \cdot \frac{y^2}{x} \cdot \frac{x}{4}$$

$$= \frac{4 \cdot 2 \cdot x^3}{y \cdot y^2} \cdot \frac{y^2}{x} \cdot \frac{x}{4}$$

$$= \frac{2x^3}{y}$$ *Answer*

Oral Exercises

Simplify.

1. $\frac{5}{7} \div \frac{4}{7}$
2. $\frac{7}{2} \div \frac{4}{5}$
3. $-\frac{1}{4} \div \left(-\frac{2}{3}\right)$
4. $-\frac{4}{7} \div \frac{2}{5}$
5. $\frac{a}{b} \div \frac{c}{d}$
6. $\frac{x}{y} \div \frac{y}{x}$
7. $6a \div \frac{2}{a}$
8. $\frac{c}{2} \div 2c$
9. $\frac{y^2}{4} \div \frac{y^3}{16}$
10. $y \div \frac{1}{3y^2}$
11. $\frac{1}{2} \cdot \frac{1}{3} \div \frac{1}{4}$
12. $\frac{a}{b} \div \frac{c}{d} \cdot \frac{e}{f}$

Written Exercises

Divide. Give your answers in simplest form.

A

1. $\frac{6}{5} \div \frac{9}{10}$
2. $\frac{2}{3} \div \frac{5}{9}$
3. $\frac{a}{6} \div \frac{a}{3}$
4. $\frac{3x}{5} \div \frac{x}{15}$
5. $\frac{x}{y^2} \div \frac{x^2}{y}$
6. $\frac{3n^2}{5} \div \frac{9n}{20}$
7. $\frac{ab}{6} \div \frac{b}{a}$
8. $\frac{c}{2d} \div \frac{d}{8c}$
9. $\frac{3x^2}{4y} \div \frac{xy}{18}$
10. $\frac{2n}{3m^2} \div \frac{1}{12mn}$
11. $\frac{xy^2}{3} \div xy$
12. $\frac{9a^2}{2b} \div 6ab$
13. $1 \div \left(\frac{3x}{5}\right)^2$
14. $4 \div \left(\frac{2}{n}\right)^3$
15. $\frac{2 + 2b}{6} \div \frac{1 + b}{9}$
16. $\frac{6n - 2}{6n} \div \frac{3n - 1}{27}$
17. $\frac{x^2 - 1}{2} \div \frac{x + 1}{16}$
18. $\frac{m^2 - 9}{2m} \div \frac{m - 3}{10}$
19. $\frac{1}{x - y} \div \frac{1}{y - x}$
20. $\frac{1}{8 - 2a} \div \frac{5}{3a - 12}$

Divide. Give your answers in simplest form.

21. $\dfrac{2x+2y}{x^2} \div \dfrac{x^2-y^2}{4x}$

22. $\dfrac{3}{n^2-9} \div \dfrac{3n-9}{n+3}$

23. $\dfrac{w^2-4}{w^2-1} \div \dfrac{w^2-w-2}{w^2+w-2}$

24. $\dfrac{x^2-x-20}{5x-25} \div \dfrac{x^2+4x-5}{x^2-25}$

B

25. $\dfrac{4x^2-25}{x^2-16} \div \dfrac{12x+30}{2x^2+8x}$

26. $\dfrac{2x-y}{2y-x} \div \dfrac{4x^2-y^2}{4y^2-x^2}$

27. $\dfrac{c^4-d^4}{5c-5d} \div \dfrac{c^2+d^2}{5}$

28. $\dfrac{8-2p^4}{3p^4} \div \dfrac{2+p^2}{6p^2}$

29. $\dfrac{r^2-t^2}{r^2+t^2} \div (r+t)$

30. $\dfrac{3-3n}{n^2+2n-3} \div (2n-2)$

31. $\dfrac{6+x-x^2}{x^2-13x+42} \div \dfrac{2x^2-5x-3}{2x^2-13x-7}$

32. $\dfrac{2x^2-11x+12}{27-18x} \div \dfrac{x^3+4x^2}{6x^3-96x}$

Simplify.

33. a. $\dfrac{3}{8} \cdot \left(\dfrac{2}{3} \div \dfrac{1}{4}\right)$

b. $\dfrac{3}{8} \cdot \dfrac{2}{3} \div \dfrac{1}{4}$

34. a. $\dfrac{1}{2} \div \dfrac{1}{5} \cdot \dfrac{3}{4}$

b. $\dfrac{1}{2} \div \left(\dfrac{1}{5} \cdot \dfrac{3}{4}\right)$

35. a. $\dfrac{r}{5} \div \dfrac{t}{r} \cdot \dfrac{5}{t^2}$

b. $\dfrac{x}{y^2} \cdot \dfrac{2}{x^2} \div \dfrac{x}{y}$

36. a. $\left(\dfrac{n}{2}\right)^2 \div \dfrac{n}{4} \cdot \dfrac{n}{3}$

b. $(2c)^3 \cdot \dfrac{3c}{d} \div \dfrac{6c^3}{d}$

37. $\dfrac{c-d}{c+2d} \cdot \dfrac{2d+c}{d+c} \div \dfrac{d-c}{d+c}$

38. $\dfrac{r^2}{r^2-s^2} \cdot \dfrac{r-s}{r+s} \div \left(\dfrac{r}{r+s}\right)^2$

39. $\dfrac{2q-6}{3} \div \left(\dfrac{3-q}{6}\right)^2$

40. $\left(\dfrac{2p-7}{9}\right)^3 \div \left(\dfrac{7-2p}{3}\right)^4$

41. $\dfrac{1}{ab} \div \dfrac{1}{ab^2} \div \dfrac{1}{a^2b}$

42. $\dfrac{6y}{6y-14} \div \dfrac{21}{9y-21} \div \dfrac{y^2}{35}$

C

43. $\dfrac{x^2-2x}{x^2-3x-4} \cdot \dfrac{x^2-25}{x^2-4x-5} \div \dfrac{x^2+5x}{5x^2+10x+5}$

44. $\dfrac{b^2+6b-7}{6b^2-7b-20} \cdot \dfrac{2b^2+b-15}{b^2+2b-3} \div \dfrac{b^2+5b-14}{3b^2-2b-8}$

45. $\dfrac{2d+2c-cd-c^2}{2+d} \div \dfrac{d^2-c^2}{2+c} \cdot \dfrac{c-d}{c^2-4}$

46. $\dfrac{x^2+2xy+y^2-16}{16x^4-16y^4} \div \dfrac{x+y-4}{4x^2+4y^2} \cdot \dfrac{x+y}{x+y+4}$

Mixed Review Exercises

Solve.

1. $4k = 5k - 13$
2. $4p + 20 = 48$
3. $(5b - 2) - (3 - 2b) = 9$
4. $\frac{1}{2}(6x - 2) = 5$
5. $2n^3 - 8n = 0$
6. $3x^2 + x = 4$

Give the prime factorization of each number.

7. 256
8. 156
9. 120
10. 1350

Self-Test 1

Simplify. Give any restrictions on the variable.

1. $\frac{15c - 5c^2}{3c - c^2}$
2. $\frac{4a^2 - 9}{6a^2 + 13a + 6}$

Obj. 6-1, p. 247

Simplify.

3. $\frac{42a}{6c^3} \cdot \frac{-4c}{3a^2b}$
4. $\left(\frac{2x}{3}\right)^3 \cdot \frac{27}{48x}$

Obj. 6-2, p. 251

5. $\frac{4x^2}{y} \div \frac{2xy}{7}$
6. $\frac{x^2 - 4}{x^2 - x - 6} \div \frac{x - 2}{2x}$

Obj. 6-3, p. 255

Check your answers with those at the back of the book.

Calculator Key-In

You can use a calculator to evaluate algebraic fractions for given values of their variables. First evaluate the denominator and store its value in the calculator's memory. (You may want to review the method for evaluating a polynomial given on page 203.) Then evaluate the numerator and divide by what is stored in memory (that is, the value of the denominator).

Evaluate each fraction for the given value of the variable.

1. $\frac{5n - 16}{2n}$; $n = 5$
2. $\frac{7a + 20}{3a}$; $a = 4$
3. $\frac{7x^2 + 4x + 12}{x}$; $x = 9$
4. $\frac{4m^2 + 11m - 60}{2m + 8}$; $m = -6$
5. $\frac{a^2 + 8a - 10}{2a}$; $a = 0.5$
6. $\frac{5y^2 - 22y + 30}{6y - 10}$; $y = -2$

Adding and Subtracting Fractions

6-4 Least Common Denominators

Objective To express two or more fractions with their least common denominator.

You have learned that you can write a fraction in simpler form by *dividing* its numerator and denominator by the same nonzero number.

$$\frac{bc}{bd} = \frac{c}{d} \quad (b \neq 0)$$

You can rewrite this rule as

$$\frac{c}{d} = \frac{bc}{bd} \quad (b \neq 0)$$

Using this form of the rule, you can write a fraction in a different form by *multiplying* its numerator and denominator by the same nonzero number.

Example 1 Complete: $\frac{3}{7} = \frac{?}{35}$

Solution

$\frac{3}{7} = \frac{?}{35}$ ← 7 is multiplied by 5 to get 35.

$\frac{3}{7} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{15}{35}$ ← Therefore, multiply 3 by 5 to get 15.

Example 2 Complete: $\frac{8}{3a} = \frac{?}{18a^2}$

Solution

$\frac{8}{3a} = \frac{?}{18a^2}$ ← $3a$ is multiplied by $6a$ to get $18a^2$.

$\frac{8}{3a} = \frac{8 \cdot 6a}{3a \cdot 6a} = \frac{48a}{18a^2}$ ← Therefore, multiply 8 by $6a$ to get $48a$.

Example 3 Complete: $\frac{2}{x-5} = \frac{?}{(x-5)(x+1)}$

Solution

$\frac{2}{x-5} = \frac{?}{(x-5)(x+1)}$ ← $x - 5$ is multiplied by $x + 1$.

$\frac{2}{x-5} = \frac{2(x+1)}{(x-5)(x+1)} = \frac{2(x+1)}{(x-5)(x+1)}$ ← Therefore, multiply 2 by $x + 1$.

You can use the method shown in Examples 1, 2, and 3 to rewrite two or more fractions so that they have equal denominators. When you add and subtract fractions in the next lesson, you'll find that it may simplify your work if you use the *least common denominator* (LCD) of the fractions.

Finding the Least Common Denominator

1. Factor each denominator completely. Write any integral factor as a product of primes.
2. Find the product of the greatest power of each factor occurring in the denominators.

Example 4 Find the LCD of the fractions $\frac{3}{4}$, $\frac{11}{30}$, and $\frac{7}{45}$.

Solution

1. Factor each denominator into prime numbers.

   $4 = 2^2 \qquad 30 = 2 \cdot 3 \cdot 5 \qquad 45 = 3^2 \cdot 5$

2. Greatest power of 2: 2^2
   Greatest power of 3: 3^2
   Greatest power of 5: 5

   $2^2 \cdot 3^2 \cdot 5 = 180$

   $\therefore$ the LCD is 180. ***Answer***

Example 5 Find the LCD of $\frac{3}{6x - 30}$ and $\frac{8}{9x - 45}$.

Solution

1. Factor each denominator completely. Factor integers into primes.

   $6x - 30 = 6(x - 5) = 2 \cdot 3(x - 5)$
   $9x - 45 = 9(x - 5) = 3^2(x - 5)$

2. Form the product of the greatest power of each factor.

   $2 \cdot 3^2(x - 5)$, or $18(x - 5)$

   $\therefore$ the LCD is $18(x - 5)$ ***Answer***

Example 6 Rewrite $\frac{9}{x^2 - 8x + 16}$ and $\frac{5}{x^2 - 7x + 12}$ with their LCD.

Solution

$x^2 - 8x + 16 = (x - 4)^2$
$x^2 - 7x + 12 = (x - 3)(x - 4)$

{First find the LCD of the fractions.

The LCD is $(x - 3)(x - 4)^2$.

(Solution continues on next page.)

Then rewrite each fraction using the LCD.

$$\frac{9}{x^2-8x+16}=\frac{9}{(x-4)^2}=\frac{9(x-3)}{(x-4)^2(x-3)}=\frac{9(x-3)}{(x-3)(x-4)^2}$$

$$\frac{5}{x^2-7x+12}=\frac{5}{(x-3)(x-4)}=\frac{5(x-4)}{(x-3)(x-4)(x-4)}=\frac{5(x-4)}{(x-3)(x-4)^2}$$

Oral Exercises

Complete.

1. $\frac{2}{5}=\frac{?}{10}$
2. $\frac{3}{8}=\frac{?}{40}$
3. $\frac{x}{6}=\frac{?}{30}$
4. $\frac{a}{3}=\frac{?}{21}$
5. $\frac{2x}{3}=\frac{?}{9}$
6. $\frac{a}{b}=\frac{?}{b^2}$
7. $\frac{5}{8n}=\frac{?}{16n^2}$
8. $\frac{5}{2-x}=\frac{?}{x-2}$

Find the LCD for each group of fractions.

9. $\frac{3}{2}, \frac{7}{10}$
10. $\frac{2}{5}, \frac{1}{9}$
11. $\frac{5}{18}, \frac{5}{12}$
12. $\frac{7}{90}, \frac{11}{60}$
13. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
14. $\frac{2}{5}, \frac{5}{6}, \frac{3}{10}$
15. $\frac{6}{5a}, \frac{11}{20a^2}$
16. $\frac{8}{x}, \frac{1}{xy}$
17. $\frac{12}{x^2}, \frac{10}{xy^2}$
18. $\frac{2}{3a-b}, \frac{5}{6a-2b}$
19. $\frac{3}{x^2+x}, \frac{9}{x+1}, \frac{4}{x}$
20. $\frac{4}{y^2-1}, \frac{3}{y+1}, \frac{7}{y-1}$

Written Exercises

Complete.

A

1. $\frac{3}{4}=\frac{?}{28}$
2. $\frac{4}{9}=\frac{?}{27}$
3. $\frac{5x}{3}=\frac{?}{18}$
4. $\frac{2a}{17}=\frac{?}{51}$
5. $\frac{x-3}{4}=\frac{?}{12}$
6. $\frac{2n-5}{5}=\frac{?}{25}$
7. $\frac{6}{15x}=\frac{?}{30x^2}$
8. $\frac{3}{4a}=\frac{?}{16a^3}$
9. $\frac{x}{2y}=\frac{?}{10xy}$
10. $\frac{5m}{2n}=\frac{?}{8mn^2}$
11. $\frac{7}{n-3}=\frac{?}{(n-3)(n+2)}$
12. $\frac{4}{x+1}=\frac{?}{(x+1)(x-1)}$
13. $\frac{5}{2n-3}=\frac{?}{(2n-3)^2}$
14. $\frac{2y}{3y-2}=\frac{?}{(3y-2)^2}$
15. $\frac{4}{a-1}=\frac{?}{2a-2}$
16. $\frac{2}{3x-1}=\frac{?}{9x-3}$
17. $\frac{2}{x+1}=\frac{?}{x^2-1}$
18. $\frac{5}{n-3}=\frac{?}{n^2-9}$
19. $\frac{3}{4-y}=\frac{?}{4y-y^2}$
20. $\frac{2x}{3-x}=\frac{?}{3x-x^2}$

Find the LCD for each group of fractions.

21. $\frac{1}{6}, \frac{5}{9}$ **22.** $\frac{3}{8}, \frac{2}{5}$ **23.** $\frac{5}{2}, \frac{1}{6}, \frac{3}{5}$

24. $\frac{1}{3}, \frac{2}{9}, \frac{3}{4}$ **25.** $\frac{a+3b}{8}, \frac{2a-b}{12}$ **26.** $\frac{n-3}{20}, \frac{n+4}{15}$

27. $\frac{1}{4t}, \frac{7}{16rt^2}$ **28.** $\frac{9}{x^2}, \frac{2}{xy}$ **29.** $\frac{6}{x+1}, \frac{x}{x-2}$

30. $\frac{b}{b+5}, \frac{2b}{b-5}$ **31.** $\frac{7}{m+2}, \frac{m-1}{m^2-4}$ **32.** $\frac{3a}{a-1}, \frac{5}{a^2-3a+2}$

Rewrite each group of fractions with their LCD.

B **33.** $\frac{1}{2xy}, \frac{3}{x^2}$ **34.** $\frac{1}{3mn^2}, \frac{2}{m^2n}$ **35.** $\frac{11}{6x^2y^2}, \frac{4}{5xy^3}$

36. $\frac{1}{12a^2b}, \frac{5}{18b^2}$ **37.** $\frac{5}{x-3}, \frac{7}{4x-12}$ **38.** $\frac{4}{3x-6y}, \frac{1}{5x-10y}$

39. $\frac{6}{x-3}, \frac{4x}{(x-3)^2}$ **40.** $\frac{9}{(2n+1)^2}, \frac{-2n}{2n+1}$

41. $\frac{3y}{2y-4}, \frac{1}{y^2-4}$ **42.** $\frac{3}{c^2-6c}, \frac{5}{c^2+6c}$

43. $\frac{x}{x^2-x-6}, \frac{9}{x^2-9}$ **44.** $\frac{2}{a^2+3a-10}, \frac{4a}{a^2+10a+25}$

C **45.** The product of the first n positive integers, denoted by $n!$, is called ***n* factorial.**

a. Find 4!, 5!, and 6!. (*Hint:* $3! = 1 \cdot 2 \cdot 3 = 6$)

b. What is the LCD of the fractions $\frac{1}{4!}, \frac{1}{5!}$, and $\frac{1}{6!}$?

c. What is the LCD of the fractions $\frac{1}{n!}$ and $\frac{1}{(n+1)!}$?

Mixed Review Exercises

Factor completely.

1. $4n - 8q + 16$ **2.** $3x^2 - 3$ **3.** $x^2 - 11x + 18$

4. $x^2 - 5x - 24$ **5.** $2x^2 - x - 3$ **6.** $x^2 + 16x + 39$

7. $x^2 + 3x - 28$ **8.** $x^2 + 22x + 121$ **9.** $n^2 - 9n$

Write an equation for each sentence.

10. Seven is 4 less than twice the number p.

11. The number n decreased by $\frac{1}{2}$ is $5\frac{1}{4}$.

12. Two thirds of the number k is 16.

Computer Key-In

The following program will find the LCD for two integral denominators.

```
10 PRINT "TO FIND THE LEAST"
20 PRINT "COMMON DENOMINATOR."
30 INPUT "ENTER TWO DENOMINATORS: "; D1, D2
40 LET M=1
50 LET Q=(D2*M)/D1
60 IF Q=INT(Q) THEN 90
70 LET M=M+1
80 GOTO 50
90 PRINT "LCD ("; D1; "; "; D2 ;") = ";
100 PRINT D2 ; " X "; M; " = "; D2*M
110 END
```

Exercises

Run this program for each pair of denominators.

1. D1 = 7, D2 = 21

2. D1 = 21, D2 = 7

3. D1 = 24, D2 = 36

4. D1 = 36, D2 = 24

5. D1 = 13, D2 = 15

6. D1 = 15, D2 = 13

7–12. In order to see how the program works, insert

```
55 PRINT D2*M; "/" ; D1; "=" ; (D2*M)/D1
```

and run the program again for the data in Exercises 1–6.

13. How does each RUN where you enter the smaller denominator first (Exercises 1, 3, and 5) compare with each RUN where you enter the smaller denominator last (Exercises 2, 4, and 6)? That is, which RUN requires fewer steps of computation?

14. Explain how to use the program above to find the least common denominator for the three denominators 27, 36, and 30.

Challenge

Two horses approach each other along the same country road, one walking at 5.5 km/h and the other at 4.5 km/h. When the horses are 10 km apart, a horsefly leaves one horse and flies at 30 km/h to the other. No sooner does the fly reach that horse than it turns around (losing no time on the turn) and returns to the first horse. If the fly continues to fly back and forth between the approaching horses, how far has the fly flown when the horses meet?

6-5 Adding and Subtracting Fractions

Objective To add and subtract algebraic fractions.

In Lesson 2-9 you learned that

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}.$$

You can rewrite these results to get the following rules.

Addition Rule for Fractions

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Example: $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$

Subtraction Rule for Fractions

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example: $\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$

To add or subtract fractions with the same denominator, you add or subtract their numerators and write the result over the common denominator.

To simplify an expression involving fractions, you write it as a single fraction in simplest form.

Example 1 Simplify: **a.** $\frac{3c}{16} + \frac{5c}{16}$ **b.** $\frac{5x+4}{10} - \frac{3x-8}{10}$

Solution

a. $\frac{3c}{16} + \frac{5c}{16} = \frac{3c+5c}{16} = \frac{8c}{16}$

$$= \frac{8 \cdot c}{8 \cdot 2}$$

$$= \frac{c}{2} \quad \textit{Answer}$$

b. $\frac{5x+4}{10} - \frac{3x-8}{10} = \frac{5x+4-(3x-8)}{10}$

$$= \frac{5x+4-3x+8}{10}$$

$$= \frac{2x+12}{10}$$

$$= \frac{2(x+6)}{2 \cdot 5}$$

$$= \frac{x+6}{5} \quad \textit{Answer}$$

Example 2 Simplify: **a.** $\frac{3}{x+4} + \frac{1}{x+4}$ **b.** $\frac{2}{x-3} + \frac{7}{3-x}$

Solution

a. $\frac{3}{x+4} + \frac{1}{x+4} = \frac{3+1}{x+4} = \frac{4}{x+4}$ ***Answer***

b. $\frac{2}{x-3} + \frac{7}{3-x} = \frac{2}{x-3} + \frac{7}{-(x-3)}$ Since $3 - x = -(x - 3)$, the LCD is $x - 3$.

$= \frac{2}{x-3} - \frac{7}{x-3}$

$= \frac{2-7}{x-3}$

$= \frac{-5}{x-3}$, or $-\frac{5}{x-3}$ ***Answer***

Example 3 Simplify $\frac{a}{4} - \frac{5+12a}{18}$.

Solution Since the denominators are different, rewrite the fractions using their least common denominator, 36.

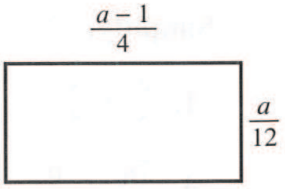
$\frac{a}{4} - \frac{5+12a}{18} = \frac{a \cdot 9}{4 \cdot 9} - \frac{(5+12a) \cdot 2}{18 \cdot 2}$

$= \frac{9a - 2(5+12a)}{36}$

$= \frac{9a - 10 - 24a}{36}$

$= \frac{-15a - 10}{36}$, or $-\frac{5(3a+2)}{36}$ ***Answer***

Example 4 Find the perimeter of the rectangle shown at the right.



Solution

Perimeter of a rectangle $= (2 \times \text{length}) + (2 \times \text{width})$

$= 2 \cdot \frac{a-1}{4} + 2 \cdot \frac{a}{12}$

$= \frac{a-1}{2} + \frac{a}{6}$

$= \frac{3(a-1)}{6} + \frac{a}{6}$

$= \frac{3a - 3 + a}{6}$

$= \frac{4a - 3}{6}$ ***Answer***

Example 5 Simplify $\frac{3}{2x} - \frac{1}{8x^2}$.

Solution
$$\frac{3}{2x} - \frac{1}{8x^2} = \frac{3 \cdot 4x}{2x \cdot 4x} - \frac{1}{8x^2}$$
$$= \frac{12x}{8x^2} - \frac{1}{8x^2} = \frac{12x - 1}{8x^2} \quad \textit{Answer}$$

Example 6 Simplify $\frac{a-3}{a^2-2a} - \frac{a-4}{a^2-4}$.

Solution
$$\frac{a-3}{a^2-2a} - \frac{a-4}{a^2-4} = \frac{a-3}{a(a-2)} - \frac{a-4}{(a-2)(a+2)}$$
$$= \frac{(a-3)(a+2)}{a(a-2)(a+2)} - \frac{a(a-4)}{a(a-2)(a+2)}$$
$$= \frac{a^2 - a - 6 - (a^2 - 4a)}{a(a-2)(a+2)}$$
$$= \frac{a^2 - a - 6 - a^2 + 4a}{a(a-2)(a+2)}$$
$$= \frac{3a - 6}{a(a-2)(a+2)}$$
$$= \frac{3(a-2)}{a(a-2)(a+2)}$$
$$= \frac{3}{a(a+2)} \quad \textit{Answer}$$

Oral Exercises

Simplify.

1. $\frac{4}{5} + \frac{3}{5}$ **2.** $\frac{7}{9} - \frac{4}{9}$ **3.** $\frac{x}{8} + \frac{3x}{8}$

4. $\frac{7a}{12} - \frac{a}{12}$ **5.** $\frac{4}{x} - \frac{1}{x}$ **6.** $\frac{7}{2x} + \frac{3}{2x}$

7. $\frac{3}{x+2} + \frac{x}{x+2}$ **8.** $\frac{n}{n+4} - \frac{5}{n+4}$ **9.** $\frac{x}{x+7} + \frac{7}{x+7}$

10. $\frac{2x}{2x-1} - \frac{1}{2x-1}$ **11.** $\frac{4}{a-b} + \frac{1}{b-a}$ **12.** $\frac{3}{x-5} - \frac{1}{5-x}$

13. $\frac{x}{2} + \frac{x}{8}$ **14.** $\frac{y}{4} - \frac{y}{6}$ **15.** $\frac{6}{a} + \frac{3}{2a}$ **16.** $\frac{5}{n} - \frac{4}{n^2}$

17. $\frac{6y}{5} - \frac{y+3}{5}$ **18.** $\frac{n^2}{4} - \frac{n^2-4}{4}$ **19.** $\frac{3c}{2} + \frac{c+1}{4}$ **20.** $\frac{x+1}{4} - \frac{x}{6}$

Written Exercises

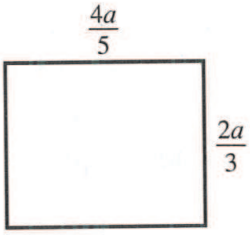
Simplify.

A

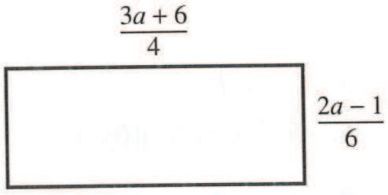
1. $\frac{x}{12} + \frac{3x}{12}$
2. $\frac{7x}{15} - \frac{4x}{15}$
3. $\frac{2}{n} - \frac{6}{n}$
4. $\frac{5}{3x} + \frac{9}{3x}$
5. $\frac{2x}{3} - \frac{x-1}{3}$
6. $\frac{3a}{10} - \frac{5a+1}{10}$
7. $\frac{y-3}{4} - \frac{3y-5}{4}$
8. $\frac{x+3}{4} - \frac{9x+11}{4}$
9. $\frac{2}{x+3} + \frac{5}{x+3}$
10. $\frac{9}{y-6} - \frac{3}{y-6}$
11. $\frac{x}{x-2} + \frac{1}{x-2}$
12. $\frac{n}{n-2} - \frac{3n-1}{n-2}$
13. $\frac{3}{x-5} + \frac{2}{5-x}$
14. $\frac{5}{4y-1} - \frac{3}{1-4y}$
15. $\frac{5c}{c-d} + \frac{5d}{d-c}$
16. $\frac{6}{3-2x} - \frac{4x}{2x-3}$
17. $\frac{2}{x} + \frac{3}{x^2}$
18. $\frac{8}{ab} - \frac{6}{ab^2}$
19. $\frac{1}{2x^2} - \frac{5}{4x}$
20. $\frac{4}{3xy} + \frac{3}{2x^2}$
21. $\frac{4}{5(x+1)} + \frac{x}{x+1}$
22. $\frac{2x}{x-3} - \frac{3}{2(x-3)}$
23. $\frac{1+2x}{4} + \frac{1+2x}{6}$
24. $\frac{5b+2}{8} + \frac{3b-4}{10}$
25. $\frac{3y-2}{6} - \frac{y-3}{9}$
26. $\frac{a-6}{15} + \frac{4-a}{10}$
27. $\frac{5n-2}{12} - \frac{3(n-3)}{8}$
28. $\frac{3(a-b)}{20} - \frac{5(a+b)}{12}$
29. $\frac{4x+3}{3} - \frac{7x}{4} + \frac{x-3}{6}$
30. $\frac{2m}{5} - \frac{m+4}{4} + \frac{2-m}{10}$

Find the perimeter of each figure.

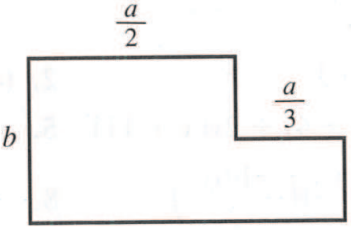
B 31.



32.

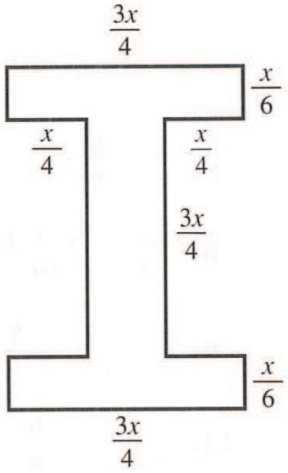


33.



Find the perimeter of the figure.

34.



Simplify.

35. $\frac{1}{x-1}+\frac{1}{x}$

36. $\frac{3}{y-6}-\frac{1}{y}$

37. $\frac{2}{x-3}+\frac{4}{x+3}$

38. $\frac{3}{x+4}-\frac{4}{x-2}$

39. $\frac{a+1}{a}-\frac{a}{a+1}$

40. $\frac{x}{x+y}+\frac{y}{x-y}$

41. $\frac{x}{x^2-1}+\frac{4}{x+1}$

42. $\frac{2y}{y^2-25}-\frac{y}{y-5}$

43. $\frac{2m}{2m-1}+\frac{1}{1-2m}$

44. $\frac{3a}{a-2b}+\frac{6b}{2b-a}$

45. $\frac{d+2}{d^2-1}-\frac{3}{2d+2}$

46. $\frac{2n}{n^3-5n^2}+\frac{2}{n^2+5n}$

47. $\frac{a}{ab-b^2}+\frac{b}{ab-a^2}$

48. $\frac{x}{x-x^2}-\frac{1}{x-x^3}$

49. $\frac{n}{n^2+4n}-\frac{n}{(n+4)^2}$

50. $\frac{1}{a^2+4a+4}+\frac{1}{a^2-4}$

51. $\frac{x-11}{x^2-9}-\frac{x-7}{x^2-3x}$

52. $\frac{c-2}{c^2+c-2}-\frac{c+2}{c^2-3c+2}$

C

53. $\frac{x^2+1}{x^2-1}+\frac{1}{x+1}+\frac{1}{x-1}$

54. $\frac{x}{2x-1}+\frac{x-1}{2x+1}-\frac{2x}{4x^2-1}$

55. $\frac{a+2}{a^2+5a+6}-\frac{2+a}{4-a^2}+\frac{2-a}{a^2+a-6}$

56. $\frac{x-3}{2x+6}-\frac{x+3}{3x-9}-\frac{5x^2+27}{6x^2-54}$

57. $\frac{b+1}{(b-1)^2}+\frac{2-2b}{(b-1)^3}+\frac{1}{b-1}$

58. $\frac{4}{c^2-4cd}-\frac{1}{cd-4d^2}-\frac{2}{cd}$

Mixed Review Exercises

Simplify.

1. $-8^2\cdot 3$

2. $(4\cdot 6-13)^2$

3. $(-4x^2)^3$

4. $5y(y-4)+2y(y+11)$

5. $n^2(n+8)-(2n^2-6)n$

6. $(5x^2y)(3x^3y^2)(6y^3)$

7. $\left(\frac{12x^2y^2}{5}\right)\left(\frac{-10xy^2}{3}\right)$

8. $-\frac{1}{6}(-42x+12y)$

9. $(3n-5p+2)-(-n+6p+1)$

Self-Test 2

Vocabulary least common denominator (LCD) (p. 260)

Find the missing numerators.

1. $\frac{3n}{20c} = \frac{?}{50c^3}$

2. $\frac{5}{1-a} = \frac{?}{a-a^2}$

Obj. 6-4, p. 259

Find the LCD for each group of fractions.

3. $\frac{3}{4x-8}, \frac{-12}{x^2-4x+4}$

4. $\frac{5}{a}, \frac{4}{b}, \frac{3}{a-b}$

Simplify.

5. $\frac{32c+3}{6c} - \frac{2c+9}{6c}$

6. $\frac{4}{3ab} + \frac{8}{7ab^2}$

Obj. 6-5, p. 264

7. $\frac{x}{x-y} + \frac{y}{y-x}$

8. $\frac{1}{n-1} - \frac{n}{5(n-1)}$

Check your answers with those at the back of the book.

Biographical Note / *Srinivasa Ramanujan*

Srinivasa Ramanujan (1887–1920) was a self-taught mathematician. At 16 he was awarded a scholarship to Government College in India for his proficiency in mathematics. However, he became so absorbed in his mathematical studies that he neglected to study English and lost his scholarship. He continued to study mathematics on his own, discovering over 100 theorems.

Friends convinced Ramanujan to write to G. H. Hardy, one of the leading number theorists at Cambridge University in England. Ramanujan sent about 120 of the theorems he had discovered. Convinced of Ramanujan's exceptional ability, Hardy brought him to England, where he was admitted to Trinity College. Hardy was not always sure how to teach a student with such profound mathematical insight but so little formal training; nevertheless Ramanujan progressed rapidly. While in England he did a great deal of work in number theory.

In 1918 Ramanujan was elected a fellow of the Royal Society and of Trinity College.

6-6 Mixed Expressions

Objective To write mixed expressions as fractions in simplest form.

A mixed number like $2\frac{3}{4}$ represents the sum of an integer and a fraction. You can write a mixed number as a single fraction in simplest form.

Example 1 Write $2\frac{3}{4}$ as a fraction in simplest form.

Solution

$$2\frac{3}{4} = 2 + \frac{3}{4}$$

$$= \frac{2}{1} + \frac{3}{4} \quad \{\text{Write 2 as } \frac{2}{1}.$$

$$= \frac{8}{4} + \frac{3}{4} \quad \{\text{LCD} = 4$$

$$= \frac{11}{4} \quad \textit{Answer}$$

The sum or difference of a polynomial and a fraction is called a **mixed expression.**

Example 2 Write each expression as a fraction in simplest form.

a. $c + \frac{5}{c}$ **b.** $5 - \frac{x-3}{x+2}$

Solution

a. $c + \frac{5}{c} = \frac{c}{1} + \frac{5}{c} \quad \{\text{Write } c \text{ as } \frac{c}{1}.$

$$= \frac{c^2}{c} + \frac{5}{c} \quad \{\text{LCD} = c$$

$$= \frac{c^2 + 5}{c} \quad \textit{Answer}$$

b. $5 - \frac{x-3}{x+2} = \frac{5}{1} - \frac{x-3}{x+2} \quad \{\text{Write 5 as } \frac{5}{1}.$

$$= \frac{5(x+2)}{x+2} - \frac{x-3}{x+2} \quad \{\text{LCD} = x + 2$$

$$= \frac{5x + 10 - x + 3}{x+2}$$

$$= \frac{4x + 13}{x+2} \quad \textit{Answer}$$

Example 3 Write as a fraction in simplest form: $x + \frac{5x+2}{x-1} - \frac{7}{x-1}$

Solution

$$x + \frac{5x+2}{x-1} - \frac{7}{x-1} = \frac{x(x-1)}{x-1} + \frac{5x+2}{x-1} - \frac{7}{x-1}$$

$$= \frac{x^2 - x + 5x + 2 - 7}{x-1}$$

$$= \frac{x^2 + 4x - 5}{x-1}$$

$$= \frac{(x-1)(x+5)}{x-1}$$

$$= \frac{x+5}{1} = x + 5 \quad \textit{Answer}$$

Oral Exercises

State each expression as a fraction in simplest form.

1. $2\frac{1}{8}$
2. $5\frac{2}{3}$
3. $-5\frac{2}{9}$
4. $-4\frac{4}{7}$
5. $1 + \frac{1}{x}$
6. $2 + \frac{4}{a}$
7. $n - \frac{3}{n}$
8. $\frac{2}{y} + y$
9. $2 - \frac{a}{b}$
10. $3 - \frac{1}{x+1}$
11. $\frac{5}{x-1} - 2$
12. $\frac{b}{b+1} + 1$
13. $2 + \frac{3}{x+1}$
14. $4 - \frac{1}{y+3}$
15. $\frac{3n}{3n+2} + 2$
16. $1 - \frac{2a}{a+1}$

Written Exercises

Write each expression as a fraction in simplest form.

A

1. $4\frac{1}{5}$
2. $2\frac{3}{8}$
3. $8 + \frac{1}{x}$
4. $3 + \frac{7}{a}$
5. $3a - \frac{2}{a}$
6. $5x - \frac{3}{x}$
7. $\frac{a}{b} - 3$
8. $2 - \frac{c}{d}$
9. $5 - \frac{4}{x+2}$
10. $8 + \frac{y}{y-1}$
11. $\frac{x}{x+2} + 6$
12. $\frac{a+3}{a} - 3$
13. $4 + \frac{2n+1}{n+1}$
14. $2 - \frac{k-3}{k-4}$
15. $6x - \frac{x}{x+1}$
16. $4y + \frac{y}{y-5}$
17. $8n - \frac{2}{n+1}$
18. $5a + \frac{a-5}{2a+3}$
19. $2a^2 - \frac{a-1}{2a+5}$
20. $y^2 - \frac{3y+1}{y+2}$

Write each expression as a fraction in simplest form.

B **21.** $x - \frac{8}{x+1} - \frac{6x-2}{x+1}$

22. $y - \frac{4(y+1)}{y+2} - \frac{4}{y+2}$

23. $\frac{b-1}{b} - \frac{3}{b-2} + 1$

24. $\frac{x}{x+1} - \frac{x+1}{x} + 2$

25. $\frac{3a}{a+1} + \frac{2}{a-1} - 1$

26. $2 - \frac{x}{x-3} - \frac{1}{x+3}$

27. $a - 1 - \frac{a^2+a-5}{a+2}$

28. $2a + 3b - \frac{2a^2-b^2}{2a-3b}$

29. $3x - \frac{x^2}{2x+3} - 2$

30. $(x+4)\left(\frac{4}{x} - 1\right)$

31. $\left(a + \frac{2}{a}\right)\left(a - \frac{2}{a}\right)$

32. $\left(2x + \frac{3}{x}\right)\left(x - \frac{2}{x}\right)$

33. $\left(\frac{a+b}{a} - 1\right)\left(\frac{a}{b} + 1\right)$

34. $\left(y - \frac{2}{y+1}\right)\left(1 - \frac{1}{y+2}\right)$

35. $\left(\frac{m}{n} - \frac{n}{m}\right) \div \left(\frac{1}{m} + \frac{1}{n}\right)$

36. $\left(9 - \frac{1}{x^2}\right) \div (3x - 1)$

37. $\left(1 - \frac{2}{a}\right) \div \left(1 - \frac{4}{a^2}\right)$

38. $1 + \frac{2x}{2x-1} - \frac{8x^2}{4x^2-1}$

39. As an algebra exercise, Amy, Don, and Julie were asked to simplify

$$1 - \frac{8x}{x^2-x} - \frac{2x}{1-x}.$$

Amy used a common denominator of $x(x-1)(1-x)$. Don used $x(x-1)$, and Julie used $x-1$. Explain why any of these three denominators could be used.

40. It took Jan y hours to drive 200 km. If she had increased her speed by 10 km/h and driven for 2 h less, how far could she have gone? (*Hint:* Make a chart. Answer in terms of y.)

41. Ted bought n rolls of film for a total of \$40. He then sold all but 2 of them for \$1 more per roll than he paid. How much did he receive for the rolls of film that he sold?

Write each expression as a fraction in simplest form.

C **42.** $\left(1 - \frac{b^2+c^2-a^2}{2bc}\right) \div \left(1 - \frac{a^2+b^2-c^2}{2ab}\right)$

43. $\left(2 - \frac{n}{n+1} + \frac{n}{1-n}\right) \div \left(\frac{1}{n-1} - \frac{1}{n+1}\right)$

44. Find the values of A and B if $\frac{A}{x+2} + \frac{B}{x-2} = \frac{4}{x^2-4}$.

45. Find the values of C and D if $\frac{C}{x-2} + \frac{D}{x+1} = \frac{6x}{x^2-x-2.}$.

46. Simplify: $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \ldots \left(1 - \frac{1}{n}\right)$

Mixed Review Exercises

Simplify.

1. $\frac{5a + 5b}{5a + 10}$
2. $\frac{a^2 - 9a + 14}{49 - a^2}$
3. $\frac{5x^2}{3y^3} \div \frac{10xy}{9}$
4. $\frac{n^2 - 4}{2} \div \frac{n + 2}{8}$
5. $\frac{8}{y^5} \cdot \frac{y^7}{4}$
6. $(-4b^2)^2$

Find the least common denominator.

7. $\frac{1}{2xy}, \frac{3}{x^2}$
8. $\frac{5}{b^2}, \frac{2}{ab}$
9. $\frac{2}{x - 1}, \frac{3}{1 + x}, \frac{1}{1 - x^2}$

Computer Exercises

For students with some programming experience.

In the Computer Exercises on page 145, you wrote a BASIC program to evaluate $n!$ for a value of n entered with an INPUT statement.

1. Write a BASIC program to find the value of each of the following.

$1 + \frac{1}{1!} + \frac{1}{2!}$

$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!}$

$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$

$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!}$

$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!}$

$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!}$

2. Based upon your results from Exercise 1, what do you think happens to the sum $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$ as n becomes larger?

Challenge

Find at least three numbers that satisfy all three conditions:

(1) there is a remainder of 1 when the number is divided by 2;
(2) there is a remainder of 2 when the number is divided by 3;
(3) there is a remainder of 3 when the number is divided by 4.

6-7 Polynomial Long Division

Objective To divide polynomials.

Dividing polynomials is very much like dividing real numbers. Compare the polynomial division to the numerical long division shown below.

Long Division

Step 1
$$\begin{array}{r} 4 \\ 23\overline{)949} \\ \underline{-92} \\ 29 \end{array}$$

Step 2
$$\begin{array}{r} 41 \\ 23\overline{)949} \\ \underline{-92} \\ 29 \\ \underline{-23} \\ 6 \end{array}$$

Check: $949 \stackrel{?}{=} 41 \cdot 23 + 6$

$949 \stackrel{?}{=} 943 + 6$

$949 = 949$ ✓

$\therefore \dfrac{949}{23} = 41\dfrac{6}{23}$

Polynomial Division

Step 1
$$\begin{array}{r} 2x \\ 4x + 1\overline{)8x^2 + 6x + 3} \\ \underline{8x^2 + 2x} \\ 4x + 3 \end{array} \quad \leftarrow \text{Subtract.}$$

Step 2
$$\begin{array}{r} 2x + 1 \\ 4x + 1\overline{)8x^2 + 6x + 3} \\ \underline{8x^2 + 2x} \\ 4x + 3 \\ \underline{4x + 1} \\ 2 \end{array} \quad \leftarrow \text{Subtract.}$$

Check: $8x^2 + 6x + 3 \stackrel{?}{=} (2x + 1)(4x + 1) + 2$

$8x^2 + 6x + 3 \stackrel{?}{=} (8x^2 + 6x + 1) + 2$

$8x^2 + 6x + 3 = 8x^2 + 6x + 3$ ✓

$\therefore \dfrac{8x^2 + 6x + 3}{4x + 1} = 2x + 1 + \dfrac{2}{4x + 1}$

In both divisions above, the answer was written in the following form:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

The following formula was used to check both divisions:

$$\text{Dividend} = \text{Quotient} \times \text{Divisor} + \text{Remainder}$$

When you divide polynomials, always arrange the terms in each polynomial in order of decreasing degree of the variable.

Example 1 Divide: $\dfrac{34x - 16 + 15x^2}{5x - 2}$

Solution First rewrite $34x - 16 + 15x^2$ in order of decreasing degree of x as $15x^2 + 34x - 16$.

$$
\begin{array}{r}
3x + 8 \\
5x - 2\overline{)15x^2 + 34x - 16} \\
\underline{15x^2 - 6x} \\
40x - 16 \\
\underline{40x - 16} \\
0
\end{array}
$$

Check: $15x^2 + 34x - 16 \stackrel{?}{=} (3x + 8)(5x - 2) + 0$

$15x^2 + 34x - 16 = 15x^2 + 34x - 16$ ✓

$\therefore \dfrac{15x^2 + 34x - 16}{5x - 2} = 3x + 8$ ***Answer***

In Example 1 the remainder is 0. Thus, both $3x + 8$ and $5x - 2$ are *factors* of $15x^2 + 34x - 16$.

Example 2 Divide $\dfrac{2a^3 + 5}{a - 3}$. Write the answer as a mixed expression.

Solution Using zero coefficients, insert missing terms in decreasing degree of a in $2a^3 - 5$. Then divide.

$$
\begin{array}{r}
2a^2 + 6a + 18 \\
a - 3\overline{)2a^3 + 0a^2 + 0a + 5} \\
\underline{2a^3 - 6a^2} \\
6a^2 + 0a \\
\underline{6a^2 - 18a} \\
18a + 5 \\
\underline{18a - 54} \\
59
\end{array}
$$

Division ends when the remainder is either 0 or of lesser degree than the divisor.

Check: $2a^3 + 5 \stackrel{?}{=} (2a^2 + 6a + 18)(a - 3) + 59$

$2a^3 + 5 \stackrel{?}{=} 2a^3 + 6a^2 + 18a - 6a^2 - 18a - 54 + 59$

$2a^3 + 5 \stackrel{?}{=} 2a^3 + (6a^2 - 6a^2) + (18a - 18a) - 54 + 59$

$2a^3 + 5 = 2a^3 + 5$ ✓

$\therefore \dfrac{2a^3 + 5}{a - 3} = 2a^2 + 6a + 18 + \dfrac{59}{a - 3}$ ***Answer***

Oral Exercises

How would you rewrite the terms of the divisor and the dividend before doing the long division? Do not divide.

1. $\dfrac{x^2 + 3x^3 + 5x - 2}{x + 1}$
2. $\dfrac{x^3 + 8}{x + 2}$
3. $\dfrac{2 - 2x^3 + x^2}{3 + 2x}$

Use the given information to find the dividend.

4. divisor = 5
   quotient = 11
   remainder = 2
5. divisor = $x + 1$
   quotient = $x - 1$
   remainder = 3
6. divisor = $x^2 + 1$
   quotient = $2x$
   remainder = $7x + 5$
7. When $x^3 - x - 6$ is divided by $x - 2$, the quotient is $x^2 + 2x + 3$ and the remainder is 0. This means that __?__ and __?__ are factors of __?__.

Written Exercises

Divide. Write the answer as a polynomial or a mixed expression.

A

1. $\dfrac{x^2 + 5x + 6}{x + 2}$
2. $\dfrac{x^2 - x - 12}{x - 4}$
3. $\dfrac{n^2 - 3n - 54}{n - 9}$
4. $\dfrac{n^2 - 11n - 26}{n + 2}$
5. $\dfrac{y^2 - 2y + 5}{y + 1}$
6. $\dfrac{a^2 - 3a + 7}{a - 2}$
7. $\dfrac{x^2 - 3x - 9}{x - 3}$
8. $\dfrac{z^2 - 8z - 12}{z + 4}$
9. $\dfrac{4 + n^2 - 2n}{n - 6}$
10. $\dfrac{s + s^2 - 8}{s + 4}$
11. $\dfrac{x^2 + 4}{x + 2}$
12. $\dfrac{y^2 - 9}{y + 9}$
13. $\dfrac{3x^2 + 10x - 9}{3x - 2}$
14. $\dfrac{2a^2 - 5a - 10}{2a + 1}$
15. $\dfrac{8 + 4x^2}{2x + 1}$
16. $\dfrac{9y^2 + 6}{3y - 1}$
17. $\dfrac{n^3 - 2n^2 + n + 2}{n + 2}$
18. $\dfrac{a^3 + 2a^2 + 3a + 4}{a - 1}$
19. $\dfrac{a^3 + 8}{a - 2}$
20. $\dfrac{n^3 - 1}{n - 1}$
21. $\dfrac{12x^3 - 2x^2 + x - 9}{3x + 1}$
22. $\dfrac{8n^3 - 6n^2 + 10n + 15}{4n + 1}$

B

23. $\dfrac{3x^4 - 4x^3 - x^2 - 16x - 12}{3x + 2}$
24. $\dfrac{6x^4 - x^3 - 11x^2 + 9x - 2}{2x - 1}$
25. $\dfrac{2n^4 - n^3 - 2n + 1}{2n - 1}$
26. $\dfrac{8y^4 + 10y^3 + 12y + 15}{4y + 5}$
27. $\dfrac{z^4 + 16}{z + 2}$
28. $\dfrac{b^4 + 84}{b - 3}$
29. $\dfrac{n^4 - n^3 + 3n^2 - 2n + 2}{n^2 + 2}$
30. $\dfrac{x^3 + 3x^2 - 4x - 12}{x^2 + 5x + 6}$

31. The volume of a rectangular solid is $12n^3 + 8n^2 - 3n - 2$. The length of the solid is $2n + 1$ and the width is $2n - 1$. Find the height.

32. Divide $a^4 + a^2 - 20$ by $a - 2$.

a. Use long division.

b. Factor $a^4 + a^2 - 20$ first. Then divide by $a - 2$.

c. Show that your answers to parts (a) and (b) are the same.

33. Factor $2n^3 - 14n + 12$ completely given that $n + 3$ is a factor.

34. Factor $4x^3 - 12x^2 - 37x - 15$ completely given that $2x + 1$ is a factor.

C **35.** Find the value of k if $x - 3$ is a factor of $4x^2 - 15x + k$.

36. Find the value of k if $2x - 1$ is a factor of $4x^3 - 6x^2 - 4x + k$.

37. Find the value of k if $y + 3$ is a factor of $y^3 + y^2 + ky + 3$.

38. When $4x^4 + x^3 - 7x^2 + 3x + k$ is divided by $x - 1$, the remainder is 5. Find the value of k.

Mixed Review Exercises

Simplify.

1. $\frac{x+4}{2} + \frac{2x-1}{2}$

2. $\frac{a^2}{a+3} - \frac{9}{a+3}$

3. $\frac{2}{y} + \frac{1}{3}$

4. $\frac{5c+1}{6c} + \frac{3}{2c}$

5. $\frac{2z+1}{6} - \frac{3z-5}{9}$

6. $\frac{x}{x^2-25} - \frac{1}{2x+10}$

7. $x + \frac{2}{x}$

8. $4 + \frac{n}{n+1}$

9. $y + 3 + \frac{2y-1}{y-2}$

Self-Test 3

Vocabulary mixed expression (p. 270)

Write each expression as a fraction in simplest form.

1. $3 - \frac{8}{y}$

2. $5n + \frac{1}{n}$

3. $a - \frac{a-1}{a+1}$

Obj. 6-6, p. 270

Divide. Write the answer as a polynomial or a mixed expression.

4. $\frac{-2 + 3y^2 + y}{y+1}$

5. $\frac{5b^3 + b^2 + b + 16}{b+2}$

Obj. 6-7, p. 274

Check your answers with those at the back of the book.

Extra / Complex Fractions

A complex fraction is a fraction whose numerator or denominator contains one or more fractions. To express a complex fraction as a simple fraction, use one of the methods below.

Method 1: Simplify the numerator and denominator. Express the fraction as a quotient using the $\div$ sign. Multiply by the reciprocal of the divisor.

Method 2: Find the LCD of all the simple fractions. Multiply the numerator and the denominator of the complex fraction by the LCD.

Example Simplify $\dfrac{\frac{1}{a}+\frac{1}{b}}{\frac{b}{2a}-\frac{a}{2b}}$.

Solution

Method 1:

$$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{b}{2a}-\frac{a}{2b}} = \frac{\frac{b+a}{ab}}{\frac{b^2-a^2}{2ab}}$$

$$= \frac{b+a}{ab} \div \frac{b^2-a^2}{2ab}$$

$$= \frac{\cancel{b+a}}{\cancel{ab}} \cdot \frac{2\cancel{ab}}{\cancel{(b+a)}(b-a)}$$

$$= \frac{2}{b-a}$$

Method 2:

The LCD of all the simple fractions is $2ab$.

$$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{b}{2a}-\frac{a}{2b}} = \frac{\left(\frac{1}{a}+\frac{1}{b}\right)2ab}{\left(\frac{b}{2a}-\frac{a}{2b}\right)2ab}$$

$$= \frac{2b+2a}{b^2-a^2}$$

$$= \frac{2\cancel{(b+a)}}{\cancel{(b+a)}(b-a)}$$

$$= \frac{2}{b-a}$$

Exercises

Simplify. Use either Method 1 or Method 2.

A

1. $\dfrac{\frac{m}{8}}{\frac{5m}{8}}$

2. $\dfrac{\frac{3a}{4}}{\frac{15a}{12}}$

3. $\dfrac{\frac{u}{v^2}}{\frac{u}{v}}$

4. $\dfrac{\frac{6e}{3}}{7e}$

5. $\dfrac{\frac{1}{4}+\frac{1}{8}}{\frac{1}{4}-\frac{1}{8}}$

6. $\dfrac{\frac{1}{6}+\frac{1}{3}}{\frac{1}{2}+\frac{1}{5}}$

7. $\dfrac{\frac{5}{6}-\frac{5}{7}}{\frac{5}{6}+\frac{5}{7}}$

8. $\dfrac{\frac{3}{x}}{\frac{1}{x}-\frac{1}{3x}}$

9. $\dfrac{\frac{9c^2}{5d^4}}{\frac{3c^2}{10d^3}}$

10. $\dfrac{n^2-25}{\frac{n}{n-5}}$

11. $\dfrac{\frac{r}{s}+2}{1-\frac{r}{s}}$

12. $\dfrac{\frac{2w}{3}+2}{\frac{5w}{3}-\frac{15}{w}}$

Simplify.

B **13.** $\dfrac{z - \dfrac{5z}{z+5}}{z + \dfrac{5z}{z-5}}$

14. $\dfrac{1 - \dfrac{6}{s^2+2}}{\dfrac{4s+2}{s^2+2} + 1}$

15. $\dfrac{k + \dfrac{k-3}{k+1}}{k - \dfrac{2}{k+1}}$

16. $\dfrac{\dfrac{1}{c} - \dfrac{1}{3-3c}}{\dfrac{1}{1-c} - \dfrac{3}{c}}$

17. $\dfrac{\dfrac{e}{e-f} - \dfrac{f}{e+f}}{\dfrac{f}{e-f} + \dfrac{e}{e+f}}$

18. $\dfrac{1-2u}{1+\dfrac{1}{2u}} \div \dfrac{1+2u}{1-\dfrac{1}{2u}}$

19. $\dfrac{\dfrac{2}{z+1} - 2}{\dfrac{2-z}{z^2-1} - 2}$

20. $\dfrac{\dfrac{m-n}{m+n} + \dfrac{n}{m}}{\dfrac{m}{n} - \dfrac{m-n}{m+n}}$

21. If $x = \dfrac{y-1}{y+1}$ and $y = \dfrac{1}{1-z}$, express x in terms of z.

22. If $a = \dfrac{b-c}{1+bc}$ and $b = \dfrac{1}{z-1}$ and $c = \dfrac{1}{z+1}$, find a in terms of z.

23. Sam drives d km at 50 km/h and returns the same distance at 30 km/h. Show that the average speed is 37.5 km/h. (*Hint:* Average speed = total distance divided by total time.)

24. A cyclist travels 12 km on a level road at x km/h and then goes 9 km on a downhill road at $2x$ km/h. Find her average speed in terms of x. (See *Hint* for Exercise 23.)

25. If n items can be purchased for 50 cents, how many items can be purchased for 50 cents after the price per item is decreased by 10 cents?

26. If $a = \dfrac{1-c}{1+c}$ and $c = \dfrac{1+b}{1-b}$, show that $a + b = 0$.

Simplify.

27. $\left(\dfrac{w}{6} - \dfrac{6}{w}\right) \div \left(\dfrac{6}{w} - 4 + \dfrac{w}{2}\right)$

28. $\left(\dfrac{x}{4-x^2} + \dfrac{1}{x-2}\right) \div \left(1 - \dfrac{2}{2+x}\right)$

29. $\left(\dfrac{1}{n-n^2} - \dfrac{1}{n^2+n}\right) \div \left(\dfrac{1}{n^2+1} - \dfrac{1}{n^2-1}\right)$

30. $\left(\dfrac{c^2+d^2}{cd} - 2\right) \div \left(\dfrac{4c^2-4d^2}{2cd}\right)$

31. $\left(\dfrac{9e^2-5}{e-1} - \dfrac{1}{6}\right) \div \left(1 - \dfrac{e-9e^2}{e-1}\right)$

32. $\left(\dfrac{u^2+v^2}{u^2-v^2}\right) \div \left(\dfrac{u-v}{u+v} - \dfrac{u+v}{u-v}\right)$

C **33.** $\dfrac{r}{2}\left(\dfrac{r^2-s^2}{r^2s+rs^2}\right)\left(\dfrac{1}{r-s}\right)\left(\dfrac{1}{r+s}\right) \div \dfrac{1}{r+s}$

34. $\left(\dfrac{4x^2-y^2}{3xy}\right) \div \left(\dfrac{2x^2-y^2}{xy} + 1\right)$

35. $1 - \dfrac{1}{1 - \dfrac{1}{c-2}}$

36. $2 + \dfrac{1}{1 + \dfrac{2}{a + \dfrac{1}{a}}}$

Chapter Summary

1. A fraction can be simplified by factoring its numerator and its denominator and dividing each by their common factors.
2. The *rule of exponents for a power of a quotient* (page 252) is sometimes used when simplifying fractions.
3. The following rules are used with fractions.

Multiplication Rule	Division Rule
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
Addition Rule	**Subtraction Rule**
$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

4. When adding or subtracting fractions with different denominators, rewrite the fractions using their *least common denominator (LCD)*. Then apply the appropriate rule. (See page 264.)
5. The sum or difference of a polynomial and a fraction is called a *mixed expression*. A mixed expression can be expressed as a fraction in simplest form.
6. When dividing polynomials, arrange the terms of the *divisor and dividend* in order of decreasing degree of a variable. Wherever the dividend is missing a term, insert one with a zero coefficient.

Chapter Review

Give the letter of the correct answer.

1. Express $\frac{9x^2 - 9}{1 - x^2}$ in simplest form. **6-1**

a. 0 **b.** -9 **c.** $\frac{9(x-1)}{x+1}$ **d.** $\frac{9(x+1)(x-1)}{x^2+1}$

2. Express $\frac{15xy}{10x^2y - y^2x}$ in simplest form.

a. $\frac{3}{2x-y}$ **b.** $\frac{15}{10x-y}$ **c.** $\frac{5}{3x}$ **d.** $\frac{5}{2x-y}$

3. Express $\left(-\frac{2}{5}\right)^3\left(-\frac{25}{4}\right)$ in simplest form. **6-2**

a. $\frac{2}{5}$ **b.** $-\frac{5}{2}$ **c.** $-\frac{2}{5}$ **d.** $\frac{5}{2}$

4. Express $\frac{2ab}{15} \cdot \frac{25}{3ab^2}$ in simplest form.

a. $10b$ **b.** $\frac{10}{9b}$ **c.** $\frac{10}{9b^2}$ **d.** $\frac{10}{3b}$

6-3

5. Express $5ab^2 \div \frac{10a}{b}$ in simplest form.

a. $\frac{b}{2}$ **b.** $\frac{b^3}{2}$ **c.** $\frac{50a^2}{b}$ **d.** $2b^3$

6. Express $\frac{x^2 - 36}{6x + 36} \div (6 - x)$ in simplest form.

a. $-\frac{1}{6}$ **b.** 6 **c.** $\frac{1}{6}$ **d.** -6

6-4

7. Complete: $\frac{2}{x - 3} = \frac{?}{x^2 - 6x + 9}$

a. 2 **b.** $2(x + 3)$ **c.** $2(9)$ **d.** $2(x - 3)$

8. Find the LCD for $\frac{4n}{9n - 6}$ and $\frac{2n}{15(3n - 2)^2}$.

a. $15(3n - 2)$ **b.** $45(3n - 2)^2$ **c.** $15(3n - 2)^2$ **d.** $5(3n - 2)$

6-5

9. Simplify $\frac{3x}{x - 2} + \frac{6}{2 - x}$.

a. $\frac{3x + 6}{x - 2}$ **b.** -3 **c.** 3 **d.** $\frac{3x - 6}{2 - x}$

10. Simplify $\frac{n - 9}{36} - \frac{n - 35}{108}$.

a. $\frac{n - 2}{27}$ **b.** $\frac{n + 4}{54}$ **c.** $2n + 8$ **d.** $\frac{n - 31}{54}$

6-6

11. Write $7 + \frac{x + 2}{x - 2}$ as a fraction in simplest form.

a. $\frac{6x}{x - 2}$ **b.** $\frac{4(x - 3)}{x - 2}$ **c.** $\frac{8x - 12}{x - 2}$ **d.** $\frac{-6x + 16}{x - 2}$

12. Simplify $y + 3 + \frac{1}{y - 3}$.

a. $\frac{y^2 - 8}{y - 3}$ **b.** $\frac{y + 4}{y - 3}$ **c.** $\frac{y^2 - 8}{y^2 - 9}$ **d.** $\frac{y^2 - 10}{y - 3}$

6-7

13. When $x^3 - 3x^2 + 3x + 4$ is divided by $x - 2$, what is the remainder?

a. 2 **b.** 4 **c.** 6 **d.** 8

14. Divide $\frac{27x^3 + 8}{3x - 2}$. Write the answer as a polynomial or a mixed expression.

a. $9x^2 + 4$ **b.** $9x^2 + 6x + 4 + \frac{16}{3x - 2}$

c. $9x^2 + 6x + 4$ **d.** $9x^2 - 6x + 4$

Chapter Test

Simplify. Give the restrictions on the variable.

1. $\frac{5x + 35}{x^2 - 49}$

2. $\frac{64 - n^2}{n^2 - 4n - 32}$

3. $\frac{3x^2 - 6x - 24}{3x^2 + 2x - 8}$

4. $\frac{15y^2 - 30y - 45}{5y^2 + 10y - 15}$

6-1

Express in simplest form.

5. $\left(-\frac{n^3}{7}\right)^2$

6. $\frac{(3b)^2}{5} \cdot \frac{b^3}{5}$

7. $\left(\frac{3a}{b}\right)^2 \cdot \frac{7ab}{54}$

6-2

8. $\frac{9}{11} \div \frac{11}{9}$

9. $\frac{4}{7} \div \frac{4}{7}$

10. $\frac{5x^2}{4y^2} \div 20xy$

6-3

11. $18 \div \left(\frac{3n}{2}\right)^3$

12. $\frac{6a + 36}{6a} \div \frac{a^2 - 36}{a^2}$

13. $\frac{y}{2x^3} \div \left(\frac{y}{2x}\right)^2$

Complete.

14. $\frac{7n}{16m} = \frac{?}{32m^2n}$

15. $\frac{3}{x + 5} = \frac{?}{x^2 - 25}$

6-4

Rewrite each group of fractions with their LCD.

16. $\frac{3}{8x}, \frac{5}{12y^2}, \frac{5}{6x^2y}$

17. $\frac{x - 4}{15}, \frac{x + 2}{10}$

Simplify.

18. $\frac{x}{x - 9} + \frac{1}{x - 9} - \frac{19 - x}{x - 9}$

19. $\frac{x - 1}{3} + \frac{3 - 2x}{6}$

6-5

20. $\frac{6n + 3}{n - 5} - \frac{4n + 9}{5 - n}$

21. $\frac{2}{y^2 - 2y} - \frac{3}{y^2 - y - 2}$

Write each expression as a fraction in simplest form.

22. $12 - \frac{n}{5}$

23. $2 + \frac{6}{y - 7}$

6-6

24. $4x - \frac{x + 1}{x - 1}$

25. $\frac{x}{x + 2} + \frac{2}{x - 2} + 1$

Divide. Write the answer as a polynomial or a mixed expression.

26. $\frac{45 - 13n + n^2}{n - 5}$

27. $\frac{2x^3 - x^2 - 5x - 2}{2x + 1}$

6-7

Cumulative Review *(Chapters 1–6)*

Perform the indicated operations. Express the answers in simplest form. Assume that no denominator is zero.

1. $0.3(-0.5)^2 + 0.7(-0.5)^2$

2. $\dfrac{45x^3y^2z^5}{-30(xyz)^2}$

3. $(4x^3yz^2)^3$

4. $(8a - 9b) + (5a + 6b)$

5. $(5x - 3)(4x + 3)$

6. $-5y(2y^3 - 7y^2 + 1)$

7. $(-13t + 6s) - (4t + 9s)$

8. $(-2b + 7c)^2$

9. $(4t^2s - 9)(4t^2s + 9)$

Evaluate if $a = -1$, $b = 1$, $c = -2$, and $d = 3$.

10. $\dfrac{a+b}{c} - cd$

11. $(a - b)^2 \div (c + d)^2$

12. $(a^2 + b^2) \div c^2 + d$

Factor completely. If the polynomial cannot be factored, write *prime*.

13. $6a^3b + 5a^3b^2 - 3a^2b^2$

14. $49x^2 + 14x + 1$

15. $8t^3 - 56t^2 + 98t$

16. $n^4 + 8n^3 + 15n^2$

17. $m^2 + 12m + 30$

18. $t^2 - 13t + 22$

19. $y^3 + 4y^2 - 32y$

20. $5c^2 + 8c - 4$

21. $4x^2 - 1 + 2x - 1$

Solve. If the equation is an identity or if it has no solution, write *identity* or *no solution*.

22. $6y - 3 = 27$

23. $\frac{1}{4}a - 3 = 9$

24. $6 - \frac{3}{4}d = -3$

25. $(n + 2)^2 = (n + 4)(n - 2)$

26. $10m^2 - m^3 = 25m$

27. $16c^2 - 9 = 0$

28. $t^2 + 12t + 15 = -5$

29. $x^2 - 6x = 7$

30. $6y^3 + 13y^2 - 5y = 0$

Perform the indicated operations. Express the answers in simplest form.

31. $\dfrac{x^2 + 5x + 4}{2x^2 + 3x + 1} \cdot \dfrac{1 + 2x}{x^2 - 16}$

32. $\dfrac{1}{4x - 8} + \dfrac{x}{x^2 - 3x + 2}$

33. $\dfrac{a^2 - b^2}{(cd)^3} \div \dfrac{a + b}{c^2d}$

34. $\dfrac{3}{a^2 + 6a + 9} - \dfrac{a + 1}{a^2 - 9}$

35. $\dfrac{10y^3 + 7y^2 - 52y + 44}{5y - 4}$

36. $x - \dfrac{x + 1}{x - 5}$

37. Jan worked 38 h last week. She worked four times as many 8-hour shifts as 6-hour shifts. How many 8-hour shifts did she work?

38. Jeremy has 24 quarters and half dollars. If he had twice as many half dollars and half as many quarters, he would have $2 more. How much money does he have?

39. It took Emily 25 min to ride her bicycle to the repair shop and 1 h 15 min to walk back home. If Emily can ride her bicycle 8 km/h faster than she can walk, how far is the repair shop from her house?

40. Find two numbers whose difference is 3 and whose squares total 65.

Maintaining Skills

Express each fraction as a decimal to the nearest hundredth.

Sample 1 $\frac{11}{13}$ **Solution** $13\overline{)11.000}$ (0.846) $\frac{11}{13} \approx 0.85$

1. $\frac{23}{25}$ **2.** $\frac{35}{20}$ **3.** $\frac{10}{4}$ **4.** $\frac{49}{50}$ **5.** $\frac{81}{25}$ **6.** $\frac{5}{3}$

7. $\frac{51}{30}$ **8.** $\frac{7}{8}$ **9.** $\frac{14}{6}$ **10.** $\frac{41}{24}$ **11.** $\frac{15}{22}$ **12.** $\frac{21}{17}$

Express each percent as a fraction in simplest form.

Sample 2 4.8% **Solution** $\frac{4.8}{100} = \frac{48}{1000} = \frac{6}{125}$

13. 62% **14.** 12% **15.** 85% **16.** 0.5% **17.** 0.03% **18.** 9.2%

Express each decimal as a percent.

Sample 3 0.73 **Solution** **(1)** $0.73 = \frac{73}{100} = 73\%$ **(2)** $0.73 = 73\%$

19. 0.91 **20.** 0.07 **21.** 0.8 **22.** 12 **23.** 0.032 **24.** 1.23

Find each number.

Sample 4 24% of 35 **Solution** $0.24 \times 35 = 8.4$

25. 32% of 85 **26.** 12% of 80 **27.** 0.2% of 40

28. 15.6% of 50 **29.** 130% of 40 **30.** 312% of 20

Find the value of each variable.

Sample 5 35% of $x = 7$ **Solution** $0.35x = 7$; $x = \frac{7}{0.35} = \frac{700}{35} = 20$

Sample 6 $n\%$ of $75 = 33$ **Solution** $\frac{n}{100} \cdot 75 = 33$; $\frac{n}{100} = \frac{33}{75}$; $n = 44$

31. 30% of $z = 21$ **32.** 15% of $m = 6$ **33.** 5% of $y = 0.6$

34. 24% of $t = 108$ **35.** $p\%$ of $50 = 30$ **36.** $a\%$ of $45 = 18$

37. $t\%$ of $105 = 21$ **38.** $28 = n\%$ of 112 **39.** $51 = x\%$ of 150