NAME _____ DATE _____

10–6 Absolute Values of Products in Open Sentences

Objective: To extend your skill in solving open sentences that involve absolute value.

Property

The absolute value of a product of numbers equals the product of their absolute values.

$$|ab| = |a| \cdot |b|$$

Examples: $|-3 \cdot 5| = |-15| = 15 = 3 \cdot 5 = |-3| \cdot |5|$

$|-6 \cdot (-2)| = |12| = 12 = 6 \cdot 2 = |-6| \cdot |-2|$

Example 1 Solve $|2x + 1| = 5$.

Solution 1 $|2x + 1| = 5$ is equivalent to the disjunction:

$$\begin{array}{rcl} 2x + 1 = -5 & \text{or} & 2x + 1 = 5 \\ 2x + 1 - 1 = -5 - 1 & | & 2x + 1 - 1 = 5 - 1 \\ 2x = -6 & | & 2x = 4 \\ x = -3 & \text{or} & x = 2 \end{array}$$

The solution set is $\{-3, 2\}$.

Solution 2

$$|2x + 1| = 5$$

$$\left|2\left(x + \tfrac{1}{2}\right)\right| = 5$$

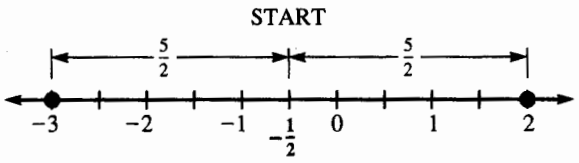
$$|2| \cdot \left|x + \tfrac{1}{2}\right| = 5$$

$$2\left|x + \tfrac{1}{2}\right| = 5$$

$$\left|x + \tfrac{1}{2}\right| = \tfrac{5}{2}$$

$$\left|x - \left(-\tfrac{1}{2}\right)\right| = \tfrac{5}{2}$$

Thus the distance between x and $-\frac{1}{2}$ is $\frac{5}{2}$.



Starting at $-\frac{1}{2}$ the numbers -3 and 2 are exactly $\frac{5}{2}$ units away in either direction.

The solution set is $\{-3, 2\}$.

Solve each open sentence and graph its solution set.

1. $|2y| = 6$
2. $|6y| = 24$
3. $|5x| = 10$
4. $\left|\frac{x}{3}\right| = 2$
5. $\left|\frac{x}{2}\right| = 4$
6. $|2a - 1| = 5$
7. $|2x + 1| = 7$
8. $|3x - 1| = 5$
9. $\left|\frac{x}{2} - 1\right| = 3$

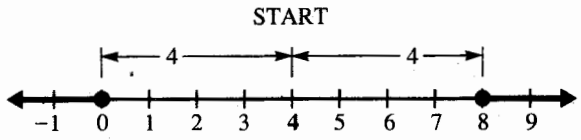
NAME _____ DATE _____

10–6 Absolute Values of Products in Open Sentences *(continued)*

Example 2 Solve $|8 - 2k| \geq 8$ and graph its solution set.

Solution 1

$$\begin{aligned} |8 - 2k| &\geq 8 \\ |-2k + 8| &\geq 8 \\ |(-2)(k - 4)| &\geq 8 \quad \text{Factor.} \\ |-2| \cdot |k - 4| &\geq 8 \quad \text{Use the property about the absolute value of a product.} \\ |k - 4| &\geq 4 \end{aligned}$$



The distance between k and 4 must be 4 or more, as shown above.

Thus the given inequality is equivalent to the disjunction

$$k \leq 0 \quad \text{or} \quad k \geq 8$$

The solution set is {0, 8, and the real numbers less than 0 or greater than 8}.

The graph is shown above.

Solution 2 $|8 - 2k| \geq 8$ is equivalent to the disjunction

$$\begin{array}{rcr} 8 - 2k \leq -8 & \text{or} & 8 - 2k \geq 8 \\ -2k \leq -16 & | & -2k \geq 0 \\ k \geq 8 & \text{or} & k \leq 0 \end{array}$$

The solution set and graph are as given in Solution 1.

Solve each open sentence and graph its solution set.

10. $|2y - 1| \leq 5$ **11.** $|2x + 1| \geq 1$ **12.** $|2x - 3| < 7$

13. $|2n - 1| \geq 3$ **14.** $|4x - 13| > 7$ **15.** $|6 - 3k| \geq 9$

16. $|4 - 2k| \leq 4$ **17.** $\left|\frac{x}{2} - 1\right| \geq 3$ **18.** $\left|\frac{x}{3} - 2\right| \leq 2$

Mixed Review Exercises

Give the slope and *y*-intercept of each line.

1. $y = 3x + 1$ **2.** $3y = 12x - 6$ **3.** $3y - 2x + 6 = 0$

4. $y = 6$ **5.** $2x - y = 5$ **6.** $x = -2y + 4$

Graph each equation.

7. $y = -x + 2$ **8.** $y = 2x - 3$ **9.** $x = -2$

10. $y = 3$ **11.** $y = \frac{2}{3}x + 1$ **12.** $y = -\frac{1}{2}x - 2$