

8 Introduction to Functions

8-1 Equations in Two Variables

Objective: To solve equations in two variables over given domains of the variables.

Vocabulary

Ordered pair A pair of numbers for which the order of the numbers is important.

Solution of an equation in two variables An ordered pair of numbers that makes the equation true.

To solve an equation To find the set of all solutions of the equation.

Symbols (a, b) (The ordered pair a, b .)

CAUTION 1 (x, y) is not the same as (y, x) ; the order is important.

CAUTION 2 The equation $2x + 1 = 5$ is a *one-variable equation* and has one number, $\{2\}$, for its solution. The equation $2x + y = 6$ is a *two-variable equation* and will have pairs of numbers for its solution. The numbers in a solution pair of an equation in two variables are written in the alphabetical order of the variables.

Example 1 State whether each ordered pair is a solution of $2x + y = 6$.

- a. $(1, 4)$ b. $(-1, 8)$ c. $(2, -2)$ d. $(\frac{5}{2}, 1)$

Solution Substitute each ordered pair in the equation $2x + y = 6$.

1. **yes, yes**
 a. $(1, 4)$ is a solution because $2(1) + 4 = 6$.
 2. **no, no**
 b. $(-1, 8)$ is a solution because $2(-1) + 8 = 6$.
 3-6. **yes, yes**
 7. **yes, no**
 c. $(2, -2)$ is *not* a solution because $2(2) + (-2) \neq 6$.
 8-9. **yes, yes**
 10. **yes, no**
 11-12. **yes, yes**
 13. **yes, no**
 d. $(\frac{5}{2}, 1)$ is a solution because $2(\frac{5}{2}) + 1 = 6$.

State whether each ordered pair is a solution of the given equation.

1. $x - y = 5$ 2. $2x + y = 8$ 3. $x + 3y = 6$ 4. $12 - y = 2x$
 $(6, 1), (3, -2)$ $(3, -2), (-3, -2)$ $(3, 1), (-3, 3)$ $(3, 6), (4, 4)$
5. $5x - 3y = 0$ 6. $2x - 4y = 0$ 7. $3a - 4b = 12$ 8. $2m - 3n = 6$
 $(3, 5), (-3, -5)$ $(2, 1), (1, \frac{1}{2})$ $(4, 0), (0, 3)$ $(6, 2), (9, 4)$
9. $2x + 5y = 18$ 10. $5m - 4n = 11$ 11. $xy = 8$ 12. $2xy = 4$
 $(4, 2), (\frac{3}{2}, 3)$ $(3, 1), (2, \frac{1}{4})$ $(16, \frac{1}{2}), (-4, -2)$ $(\frac{1}{4}, 8), (-2, -1)$
13. $x^2 + y^2 = 5$ 14. $x^2 - y^2 = 10$ 15. $x^2 - 2y^2 = 15$ 16. $2x^2 + 3y^2 = 30$
 $(2, -1), (3, -2)$ $(3, -1), (1, -3)$ $(5, 5), (4, 1)$ $(3, 2), (-3, 2)$

8-1 Equations in Two Variables (continued)

Example 2 Solve $2x + 3y = 6$ for y in terms of x .

Solution $2x + 3y = 6$
 $3y = 6 - 2x$ Subtract $2x$ from both sides of the equation.
 $y = \frac{6 - 2x}{3}$ Divide both sides of the equation by 3.

Solve each equation for y in terms of x . Answers may vary.

17. $3x + y = 6$ $y = 6 - 3x$ 18. $2x - y = 5$ $y = 2x - 5$ 19. $3x + 2y = 7$ $y = \frac{7 - 3x}{2}$
 20. $x + 3y = 9$ $y = \frac{9 - x}{3}$ 21. $4x + 2y = 0$ $y = -2x$ 22. $5x + 4y = 10$ $y = \frac{10 - 5x}{4}$

Example 3 Solve $xy + x = 4$ if x and y are whole numbers.

Solution 1. Solve the equation for y in terms of x .
 $y = \frac{4 - x}{x}$

2. Replace x with successive whole numbers and find the corresponding values of y . If y is a whole number, you have found a solution pair.

The solutions are $(1, 3)$, $(2, 1)$, and $(4, 0)$.

x	$y = \frac{4 - x}{x}$	Solution
0	denominator = 0	No
1	$\frac{4 - 1}{1} = 3$	$(1, 3)$
2	$\frac{4 - 2}{2} = 1$	$(2, 1)$
3	$\frac{4 - 3}{3} = \frac{1}{3}$	No
4	$\frac{4 - 4}{4} = 0$	$(4, 0)$

23. $(0, 4), (2, 0), (1, 2)$ 24. $(0, 7), (2, 1), (1, 4)$
 25. $(0, 2), (3, 1), (6, 0)$ 26. $(5, 0), (3, 1), (1, 2)$

Values of x greater than 4 give negative values of y .

Solve each equation if x and y are whole numbers.

23. $2x + y = 4$ 24. $3x + y = 7$ 25. $x + 3y = 6$ 26. $x + 2y = 5$
 27. $2x + 3y = 8$ 28. $3x + y = 9$ 29. $2x + 3y = 6$ 30. $xy = 3$
 31. $xy + 1 = 7$ 32. $xy + 2 = 9$ 33. $xy + y = 3$ 34. $xy - 2y = 4$
 27. $(1, 2), (4, 0)$ 28. $(3, 0), (0, 9), (2, 3), (1, 6)$ 29. $(0, 2), (3, 0)$ 30. $(1, 3), (3, 1)$
 31. $(1, 6), (2, 3), (3, 2), (6, 1)$ 32. $(1, 7), (7, 1)$

Mixed Review Exercises 33. $(0, 3), (2, 1)$ 34. $(3, 4), (4, 2), (6, 1)$

Write each number in scientific notation.

1. 28,000,000 2.8×10^7 2. 0.00461 4.61×10^{-3} 3. 104 million 1.04×10^8
 4. 0.0000325 3.25×10^{-5} 5. 37,000 3.7×10^4 6. 6,302,000 6.302×10^6

Simplify. Give answers in terms of positive exponents.

7. $\frac{4n^2}{2n}$ $2n$ 8. $(2x)^{-3} \frac{1}{8x^3}$ 9. $\frac{42x^3y^2}{14x^2y}$ $3xy$ 10. $\frac{a^{-5}}{a^2} \frac{1}{a^7}$

8-2 Points, Lines, and Their Graphs

Objective: To graph ordered pairs and linear equations in two variables.

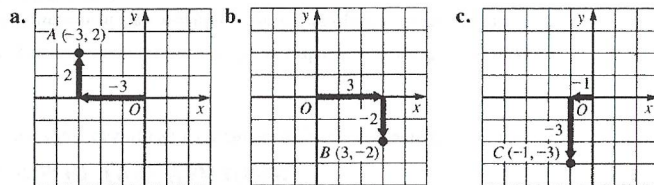
Vocabulary

- Plot a point** Locate the graph of an ordered pair in a number plane.
- Horizontal axis** The horizontal number line in a number plane; the *x*-axis.
- Origin** The intersection of the axes on a number plane. The zero point on each axis.
- Vertical axis** The vertical number line in a number plane; the *y*-axis.
- Graph of an ordered pair** The point in a number plane associated with an ordered pair.
- Abscissa** The first coordinate in an ordered pair of numbers; the *x*-coordinate.
- Ordinate** The second coordinate in an ordered pair of numbers; the *y*-coordinate.
- Coordinates of a point** The abscissa and ordinate of the point, written as an ordered pair.
- Coordinate axes** The *x*- and *y*-axes in a number plane.
- Coordinate plane** A number plane; a plane in which a coordinate system has been set up.
- Quadrant** One of the four regions into which the coordinate axes separate a number plane.
- Graph of an equation in two variables** All the points that are the graphs of the solutions of the equation.
- Linear equation** An equation whose graph is a line.
- Standard form of a linear equation** The form $ax + by = c$, where a , b , and c are integers and a and b are not both zero.

Example 1 Plot each point in a number plane.

- a. $A(-3, 2)$ b. $B(3, -2)$ c. $C(-1, -3)$

Solution



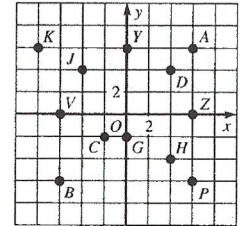
8-2 Points, Lines, and Their Graphs (continued)

Plot each point in a coordinate plane. Answers given at the back of this Answer Key.

1. $A(4, 2)$ 2. $B(6, 3)$ 3. $C(-4, -2)$ 4. $D(-5, -1)$
5. $E(-5, 0)$ 6. $F(0, -5)$ 7. $G(-3, 2)$ 8. $H(3, -2)$

Refer to the diagram at the right. Name the point(s) described.

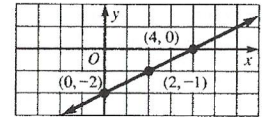
9. The point on the positive *x*-axis. **Z**
10. The point on the negative *y*-axis. **G**
11. The points on the vertical line through *Z*. **A, P**
12. The points on the horizontal line through *Y*. **A, K**
13. The *x*-coordinate is zero. **G, Y**
14. The *y*-coordinate is zero. **V, Z**
15. The points have equal *x*- and *y*-coordinates. **A, B, C, D**
16. The points have opposite *x*- and *y*-coordinates. **H, J, P**



Example 2 Graph $x - 2y = 4$ in a coordinate plane.

Solution

Let $y = 0$:	Let $x = 0$:
$x - 2(0) = 4$	$0 - 2y = 4$
$x = 4$	$-2y = 4$
Solution $(4, 0)$	$y = -2$
	Solution $(0, -2)$



A third solution, such as $(2, -1)$ can be used as a check.

Graph each equation. You may wish to verify your graphs on a computer or a graphing calculator. Graphs given at the back of this Answer Key.

17. $x - y = 4$ 18. $x + y = 5$ 19. $y = 2x + 6$ 20. $y = -2x + 2$
21. $2x + y = 4$ 22. $x - 3y = 6$ 23. $2x - 3y = 6$ 24. $2x + 3y = 6$

Mixed Review Exercises

State whether each ordered pair is a solution of the given equation.

- | | | | |
|--|---|--|--|
| 1. $2x + y = 7$
$(4, -1), (-1, 9)$
yes, yes | 2. $3a + 2b = 6$
$(2, -6), (2, 0)$
no, yes | 3. $x + 3y = 11$
$(2, 3), (-3, -2)$
yes, no | 4. $2m + 3n = 7$
$(2, 1), (-1, 3)$
yes, yes |
|--|---|--|--|

Solve.

5. $x^2 + 5x + 6 = 0$ $\{-2, -3\}$ 6. $-z + 9 = 3$ $\{6\}$ 7. $2b^2 - 6b - 8 = 0$ $\{-1, 4\}$
8. $\frac{10 - 5y}{3} = 5$ $\{-1\}$ 9. $5x + 9 = 3x - 11$ $\{-10\}$ 10. $10 = \frac{2}{5}n$ $\{25\}$

8-3 Slope of a Line

Objective: To find the slope of a line.

Vocabulary

Slope If (x_1, y_1) and (x_2, y_2) are any two different points on a line,

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{difference between } y\text{-coordinates}}{\text{difference between } x\text{-coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Positive slope The slope of a line that rises from left to right is positive.

Negative slope The slope of a line that falls from left to right is negative.

Zero slope A horizontal line has slope 0.

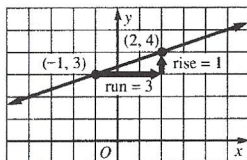
No slope A vertical line has no slope.

Collinear points Points that lie on the same line.

Example 1 Find the slope of the line through $(-1, 3)$ and $(2, 4)$.

Solution Let $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (2, 4)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{2 - (-1)} = \frac{1}{3}$$



Example 2 Find the slope of the line through $(1, -3)$ and $(4, -3)$.

Solution Slope = $\frac{-3 - (-3)}{4 - 1} = \frac{0}{3} = 0$ The line has slope 0.

Example 3 Find the slope of the line through $(2, -1)$ and $(2, 5)$.

Solution Slope = $\frac{5 - (-1)}{2 - 2} = \frac{6}{0}$ (undefined) The line has no slope.

Find the slope of the line through the given points.

- | | | |
|---------------------------------------|-------------------------------------|---------------------------------------|
| 1. $(5, -6), (2, -4)$ $-\frac{2}{3}$ | 2. $(-3, 6), (-5, 4)$ 1 | 3. $(0, 1), (2, -2)$ $-\frac{3}{2}$ |
| 4. $(1, 2), (4, 6)$ $\frac{4}{3}$ | 5. $(2, 1), (8, -2)$ $-\frac{1}{2}$ | 6. $(-1, 5), (0, 0)$ -5 |
| 7. $(4, 3), (2, 7)$ -2 | 8. $(5, 2), (-1, 2)$ 0 | 9. $(-3, -4), (1, 2)$ $\frac{3}{2}$ |
| 10. $(-5, 2), (7, -6)$ $-\frac{2}{3}$ | 11. $(1, 4), (-3, 0)$ 1 | 12. $(4, 4), (-4, 6)$ $-\frac{1}{4}$ |
| 13. $(8, -1), (6, 0)$ $-\frac{1}{2}$ | 14. $(3, -1), (-2, 4)$ -1 | 15. $(7, 4), (7, -4)$ no slope |

8-3 Slope of a Line (continued)

Example 4 Find the slope of the line with the equation $2x + 3y = 6$.

Solution 1. First find any two points on the line.

If $x = 0$:	$2(0) + 3y = 6$	If $y = 0$:	$2x + 3(0) = 6$
	$3y = 6$		$2x = 6$
	$y = 2$		$x = 3$

One point: $(0, 2)$ Another point: $(3, 0)$

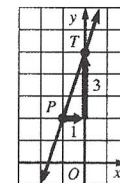
2. Now use the slope formula. Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{3 - 0} = -\frac{2}{3}$

Find the slope of each line. If the line has no slope, say so.

- | | | | |
|-------------------|--------------------|--------------------|------------------|
| 16. $y = 2x - 1$ | 17. $y = 3x + 2$ | 18. $y = 4 - 2x$ | 19. $y = 6 - 3x$ |
| 20. $6x + 2y = 3$ | 21. $2x - 5y = 10$ | 22. $3x + 6y = 12$ | 23. $x - 2y = 4$ |
| 24. $y = 5$ | 25. $y + 2 = 0$ | 26. $x = 1$ | 27. $2x - 3 = 0$ |
- no slope

Example 5 Draw a line through the point $P(-1, 2)$ with a slope of 3.

- Solution**
- Plot point P .
 - Write the slope as $\frac{3}{1}$. Rise = 3. Run = 1.
 - From P , measure 1 unit to the right and 3 units up to locate a second point, T .
 - Draw the line through P and T .



Graphs given at the back of

Through the given point, draw a line with the given slope. this Answer Key.

- | | | |
|---------------------------------------|---------------------------------------|--------------------------------------|
| 28. $A(2, 1)$; slope 2 | 29. $B(-2, 3)$; slope -3 | 30. $C(1, -4)$; slope 4 |
| 31. $D(-3, -2)$; slope $\frac{2}{3}$ | 32. $E(-4, 1)$; slope $-\frac{1}{2}$ | 33. $F(3, 0)$; slope $-\frac{3}{4}$ |
| 34. $G(-2, -1)$; slope $\frac{2}{5}$ | 35. $H(-5, 2)$; slope -2 | 36. $I(2, -3)$; slope -1 |

Mixed Review Exercises

Solve.

1. $\frac{x+2}{2} + \frac{x}{4} = 0$	2. $-3 = \frac{9b}{4} \left\{ -\frac{4}{3} \right\}$	3. $\frac{2+z}{3z} = \frac{4}{z} \{10\}$	4. $-3(y+2) = 9$
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Evaluate if $x = -2, y = 1, a = 3,$ and $b = -4$.

5. $\frac{a+2b}{2a-b} - \frac{1}{2}$	6. $3(x+3y) \cdot 3$	7. $\frac{1}{2}(3x+4y) - 1$	8. $(2a-3b) + 5 \cdot 23$
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8-4 The Slope-Intercept Form of a Linear Equation

Objective: To use the slope-intercept form of a linear equation.

Vocabulary

y-intercept The y-coordinate of a point where a graph intersects the y-axis. Since the point is on the y-axis, its x-coordinate is 0.

Slope-intercept form of an equation The equation of a line in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Parallel lines Lines in the same plane that do not intersect. Lines with the same slope and different y-intercepts are parallel.

Example 1 Find the slope and y-intercept of each line: a. $y = \frac{5}{2}x + 4$ b. $y = \frac{5}{2}x$ c. $y = 4$

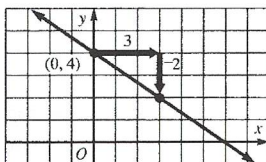
Solution Use the slope-intercept form, $y = mx + b$.

a. $y = \frac{5}{2}x + 4$	b. $y = \frac{5}{2}x$	c. $y = 4$
$y = \frac{5}{2}x + 4$	$y = \frac{5}{2}x + 0$	$y = 0x + 4$
$\begin{array}{c} \uparrow \\ m \end{array}$ $\begin{array}{c} \uparrow \\ b \end{array}$	$\begin{array}{c} \uparrow \\ m \end{array}$ $\begin{array}{c} \uparrow \\ b \end{array}$	$\begin{array}{c} \uparrow \\ m \end{array}$ $\begin{array}{c} \uparrow \\ b \end{array}$
The slope is $\frac{5}{2}$ and the y-intercept is 4.	The slope is $\frac{5}{2}$ and the y-intercept is 0.	The slope is 0 and the y-intercept is 4.

- Find the slope and the y-intercept.** 6. $-\frac{1}{3}; -3$
1. $y = x - 3$ 1; -3 2. $y = 2x + 3$ 2; 3 3. $y = -2$ 0; -2 4. $y = \frac{1}{3}x + 4$ $\frac{1}{3}$; 4
5. $y = -\frac{1}{2}x - \frac{1}{2}$ 0 6. $y = -\frac{1}{3}x - 3$ 7. $y = -2x + 6$ -2; 6 8. $y = -4x + 8$ -4; 8
9. $y = -x + 5$ -1; 5 10. $y = x - 9$ 1; -9 11. $y = 3x - 2$ 3; -2 12. $y = 3$ 0; 3

Example 2 Use only the slope and y-intercept to graph $y = -\frac{2}{3}x + 4$.

- Solution**
- Since the y-intercept is 4, plot (0, 4).
 - Since the slope $m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{rise}}{\text{run}}$, move 3 units to the right of (0, 4) and 2 units down to locate a second point.
 - Draw a line through the points.



Use only the slope and y-intercept to graph each equation. You may wish to verify your graphs on a computer or a graphing calculator.

13. $y = \frac{2}{3}x - 4$ 14. $y = \frac{3}{4}x - 3$ 15. $y = -\frac{1}{2}x$ 16. $y = -\frac{3}{4}x - 1$
17. $y = -x + 3$ 18. $y = 2x + 1$ 19. $y = -3$ 20. $y = 5$

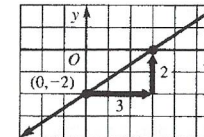
Graphs given at the back of this Answer Key.

8-4 The Slope-Intercept Form of a Linear Equation (continued)

Example 3 Use only the slope and y-intercept to graph $2x - 3y = 6$.

Solution $2x - 3y = 6$ { Solve for y to transform the equation
 $-3y = -2x + 6$ { into the form $y = mx + b$.
 $y = \frac{2}{3}x - 2$

- Since $b = -2$, plot (0, -2).
- Since $m = \frac{2}{3}$, move 3 units to the right and 2 units up to locate a second point.
- Draw a line through the points.



Use only the slope and y-intercept to graph each equation. You may wish to verify your graphs on a computer or a graphing calculator. Graphs given at the back of this Answer Key.

21. $2x + y = 4$ 22. $3x + y = 6$ 23. $2x - y = -6$ 24. $3x - y = 3$
25. $x + 2y = -2$ 26. $2x - 3y = 6$ 27. $4x - 3y = 12$ 28. $x + 4y = 4$

Example 4 Determine whether the lines with equations $4x + 5y = 20$ and $4x + 5y = 10$ are parallel.

Solution Write each equation in slope-intercept form:

$4x + 5y = 20$	$4x + 5y = 10$
$5y = -4x + 20$	$5y = -4x + 10$
$y = -\frac{4}{5}x + 4$	$y = -\frac{4}{5}x + 2$
slope = $-\frac{4}{5}$ y-intercept = 4	slope = $-\frac{4}{5}$ y-intercept = 2

Since both lines have the same slope and different y-intercepts, they are parallel.

Determine whether the lines whose equations are given are parallel.

29. $2x - y = 5$ 30. $x - 3y = 2$ 31. $2x - y = 6$
 $2x - y = 8$ **yes** $-2x + 6y = 12$ **yes** $2y - x = 6$ **no**
32. $3x - y = 2$ 33. $\frac{1}{2}x - \frac{1}{2}y = 4$ 34. $4x + \frac{1}{4}y = 2$
 $-6x + 2y = 8$ **yes** $2x - 2y = 3$ **yes** $4x + 4y = 2$ **no**

Mixed Review Exercises

- Find the slope of the line through each pair of given points. -3
1. (-2, 1), (-1, 2) 1 2. (1, 2), (3, -2) -2 3. (-3, 4), (-1, -2) 4. (1, 5), (2, 8) 3

- Factor. $(2x + 3)(x + 2)$ $2(x - 1)^2$ $(2y + 5z)(2y - 5z)$ $(m - 5n)(m + 2n)$
5. $2x^2 + 7x + 6$ 6. $2x^2 - 4x + 2$ 7. $4y^2 - 25z^2$ 8. $m^2 - 3mn - 10n^2$

8-5 Determining an Equation of a Line

Objective: To find an equation of a line given the slope and one point on the line, or given two points on the line.

Vocabulary

x-intercept The x-coordinate of the point where a line crosses the x-axis.

Example 1 Write an equation of a line that has slope 3 and y-intercept 2.

Solution Substitute 3 for m and 2 for b in $y = mx + b$.
The equation is $y = 3x + 2$.

Write an equation in slope-intercept form of each line described.

- slope 2; y-intercept 3 $y = 2x + 3$
- slope -4; y-intercept 2 $y = -4x + 2$
- slope $\frac{1}{2}$; y-intercept 5 $y = \frac{1}{2}x + 5$
- slope $\frac{1}{3}$; y-intercept 6 $y = \frac{1}{3}x + 6$
- slope $-\frac{1}{2}$; y-intercept 4 $y = -\frac{1}{2}x + 4$
- slope $-\frac{1}{4}$; y-intercept 4 $y = -\frac{1}{4}x + 4$
- slope $\frac{2}{3}$; y-intercept -6 $y = \frac{2}{3}x - 6$
- slope 3; y-intercept -7 $y = 3x - 7$
- slope -5; y-intercept 2 $y = -5x + 2$
- slope $-\frac{2}{5}$; y-intercept -1 $y = -\frac{2}{5}x - 1$

Example 2 Write an equation of a line that has slope -2 and passes through (5, 0).

Solution

- Substitute -2 for m in $y = mx + b$
 $y = -2x + b$
- To find b , substitute 5 for x and 0 for y in $y = -2x + b$.
 $0 = -2(5) + b$
 $0 = -10 + b$
 $10 = b$
The equation is $y = -2x + 10$.

$$13. y = -4x + 11$$

$$14. y = -2x - 5$$

Write an equation in slope-intercept form of each line described.

- slope 2; passes through (3, -1) $y = 2x - 7$
- slope 3; passes through (-1, 2) $y = 3x + 5$
- slope -4; passes through (2, 3)
- slope -2; passes through (-3, 1)
- slope $\frac{2}{3}$; passes through (0, 3) $y = \frac{2}{3}x + 3$
- slope $-\frac{4}{3}$; passes through (1, 0)
 $y = -\frac{4}{3}x + \frac{4}{3}$
- slope $-\frac{3}{5}$; passes through (-1, -4)
- slope -1; passes through (3, 1) $y = -x + 4$
- slope 0; passes through $(\frac{1}{4}, 2)$ $y = 2$
- slope 0; passes through $(-2, \frac{3}{8})$ $y = \frac{3}{8}$

8-5 Determining an Equation of a Line (continued)

Example 3 Write an equation of the line passing through the points (-3, 2) and (1, -2).

Solution

- Find the slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{1 - (-3)}$
 $= \frac{-4}{4} = -1$

Substitute -1 for m in $y = mx + b$.

$$y = -x + b$$

- Choose one of the points, say (-3, 2).

Substitute -3 for x and 2 for y .

$$y = -x + b$$

$$2 = -(-3) + b$$

$$2 = 3 + b$$

$$-1 = b$$

The equation is $y = -x - 1$.

Write an equation in slope-intercept form of the line passing through the given points.

- (4, 5), (2, 1) $y = 2x - 3$
- (-1, 2), (4, 7) $y = x + 3$
- (1, 2), (4, 4) $y = \frac{2}{3}x + \frac{4}{3}$
- (3, 4), (4, 6) $y = 2x - 2$
- (3, 1), (5, 2) $y = \frac{1}{2}x - \frac{1}{2}$
- (0, -2), (-3, 2) $y = -\frac{4}{3}x - 2$
- (0, -1), (-2, 3) $y = -2x - 1$
- (6, 4), (2, 1) $y = \frac{3}{4}x - \frac{1}{2}$
- (-2, 8), (1, 2) $y = -2x + 4$
- (0, 3), (-1, 0) $y = 3x + 3$
- (-1, 3), (2, 0) $y = -x + 2$
- (1, -7), (2, -1) $y = 6x - 13$

Write an equation in slope-intercept form for each line described.

- y-intercept -1; x-intercept 4 $y = \frac{1}{4}x - 1$
- y-intercept -4; x-intercept 1 $y = 4x - 4$
- x-intercept -4; y-intercept -3
- horizontal line through (-1, -2) $y = -2$
- horizontal line through (2, 4) $y = 4$
- vertical line through (-1, -2) $x = -1$
- $y = -\frac{3}{4}x - 3$

Mixed Review Exercises

Simplify.

- $(\frac{2}{5}t^2)(10t^3) 4t^5$
- $\frac{1}{3}(6s^2 - 9st) 2s^2 - 3st$
- $(6pq^2)^2 36p^2q^4$
- $(-2m^2n^3)^4 16m^8n^{12}$
- $2 \cdot 5 - 3^2 1$
- $(2a^2b^3)(-3ab^2) - 6a^3b^5$
- $2 \cdot (6 - 1)^2 50$
- $(6x + 2y) - (x + y) 5x + y$

NAME _____

DATE _____

8-6 Functions Defined by Tables and Graphs

Objective: To understand what a function is and to define a function by using tables and graphs.

Vocabulary

Function A correspondence between two sets, the *domain* and *range*, that assigns to each member of the domain exactly one member of the range.

Example 1 State the domain and range of the function shown by the table. Then give the correspondence as a set of ordered pairs.

High school	Northern	Central	Eastern	Western	Southern
Number of teachers	65	52	49	98	80

Solution Domain = {Northern, Central, Eastern, Western, Southern}
 Range = {49, 52, 65, 80, 98}
 (Northern, 65), (Central, 52), (Eastern, 49), (Western, 98), (Southern, 80)

State the domain and range of each function shown by each table. Answers given at the back of this Answer Key. Then give each correspondence as a set of ordered pairs.

1.

Animal	Antelope	Cheetah	Greyhound	Racehorse	Rabbit
Maximum speed (mi/h)	60	70	40	50	18

2. Inventory

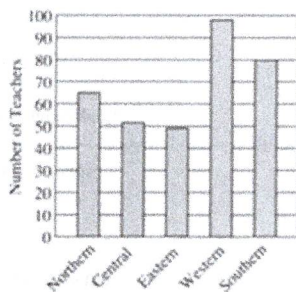
Item	Clock	Radio	Toaster	TV	Blender	Cookbook
Number	37	28	46	19	25	55

3. Electrical energy production

Year	1965	1970	1975	1980	1985
Billions of kilowatt-hours	1000	1500	2000	2250	2500

Example 2 Draw a bar graph for the function in the table in Example 1.

Solution Choose the horizontal axis for the members of the domain. List the members of the range along the vertical axis. For each member of the domain, draw a vertical bar to represent the corresponding value in the range of the function. Start the scale of the bars at zero, so that the relative lengths are correct.



NAME _____

DATE _____

8-6 Functions Defined by Tables and Graphs (continued)

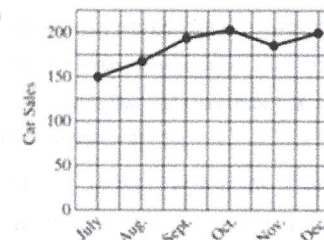
4-6. Draw a bar graph for the functions shown in each table in Exercises 1-3. Graphs given at the back of this Answer Key.

Example 3 Draw a broken-line graph for the function shown in the table.

Monthly car sales

Month	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number of sales	150	170	195	205	185	200

Solution List the members of the domain along the horizontal axis. For each member of the domain plot a point to represent the corresponding value in the range of the function. Then connect the points by line segments.



Graphs given at the back of this Answer Key.

Draw a broken-line graph for the function shown in each table. Answer Key.

7. Average monthly rainfall

Month	Apr.	May	June	July	Aug.	Sept.
Rainfall (mm)	60	50	85	78	40	52

9. Average monthly overtime

Month	July	Aug.	Sept.	Oct.	Nov.	Dec.
Hours of overtime	16	30	22	28	34	43

10. Average weekly pay

Year	1960	1965	1970	1975	1980	1985
Average weekly pay	\$ 88	\$122	\$190	\$289	\$371	\$386

8. Yearly profits

Year	Profit (in thousands)
1983	\$200
1984	\$215
1985	\$236
1986	\$270
1987	\$300
1988	\$350

Mixed Review Exercises

1. $y = -4x + 2$

Write an equation in slope-intercept form of each line described.

$y = -2x - 3$

1. passes through $(-1, 6)$ and $(-2, 10)$

2. slope -2 ; passes through $(-2, 1)$

3. slope $\frac{1}{2}$; y -intercept -4 $y = \frac{1}{2}x - 4$

4. passes through $(4, 5)$ and $(5, 0)$

$y = -5x + 25$

Graph each equation. Graphs given at the back of this Answer Key.

5. $y = -2x + 1$

6. $x + y = 5$

7. $y = -\frac{1}{2}x + 4$

8. $x = -2$

8-7 Functions Defined by Equations

Objective: To define a function by using equations.

Vocabulary

Arrow notation A notation involving an arrow used to define a function.

For example, $P: n \rightarrow 5n - 500$.

Functional notation A notation involving an equals sign used to define a function.

For example, $P(n) = 5n - 500$.

Values of a function Members of the range of the function.

Symbols $g(2) = 6$ (Read "g of 2 equals 6" or "the value of g at 2 is 6.")

CAUTION $g(2)$ is not the product of g and 2. It names the number that g assigns to 2.

Example 1 List the range of $g: x \rightarrow x^2 - x - 6$ if the domain $D = \{-2, -1, 0, 1, 2\}$.

Solution In $x^2 - x - 6$ replace x with each member of D to find the members of the range R .

x	$x^2 - x - 6$
-2	$(-2)^2 - (-2) - 6 = 0$
-1	$(-1)^2 - (-1) - 6 = -4$
0	$(0)^2 - (0) - 6 = -6$
1	$(1)^2 - (1) - 6 = -6$
2	$(2)^2 - (2) - 6 = -4$

$R = \{0, -4, -6\}$

Note: The function g assigns -4 to both -1 and 2 , and -6 to both 0 and 1 . In listing the range of g , you name -4 and -6 only once each.

Find the range of each function.

- | | |
|--|---|
| 1. $g: x \rightarrow 2x + 1, D = \{-1, 0, 1\}$
$R = \{-1, 1, 3\}$ | 2. $f: x \rightarrow 3x - 2, D = \{1, 2, 3\}$
$R = \{1, 4, 7\}$ |
| 3. $h: x \rightarrow 1 - 4x, D = \{-2, 0, 2\}$
$R = \{-7, 1, 9\}$ | 4. $h(y) = 3y + 1, D = \{-3, 0, 1\}$
$R = \{-8, 1, 4\}$ |
| 5. $G: a \rightarrow 3a - 2, D = \{-2, 0, 2\}$
$R = \{-8, -2, 4\}$ | 6. $F(x) = 2 - 4x, D = \{-1, 0, 1\}$
$R = \{-2, 2, 6\}$ |
| 7. $F(x) = 5x - 4, D = \{-1, 2, 3\}$
$R = \{-9, 6, 11\}$ | 8. $Q(n) = 4n - 3, D = \{0, 2, 3\}$
$R = \{-3, 5, 9\}$ |
| 9. $P(z) = z^2 - 2z, D = \{-1, 0, 1\}$
$R = \{-1, 0, 3\}$ | 10. $H: b \rightarrow b^2 - b - 2, D = \{-1, 0, 2\}$
$R = \{-2, 0\}$ |
| 11. $g: x \rightarrow x^2 + 3x - 4, D = \{-1, 2, 4\}$
$R = \{-6, 6, 24\}$ | 12. $f: x \rightarrow x^2 - x - 6, D = \{-2, 0, 3\}$
$R = \{-6, 0\}$ |
| 13. $F(x) = x^3 + x^2 + 2x, D = \{-1, 0, 1\}$
$R = \{-2, 0, 4\}$ | 14. $N(a) = a^3 - 2a^2 + 3a, D = \{0, 2, 3\}$
$R = \{0, 6, 18\}$ |

8-7 Functions Defined by Equations (continued)

Example 2 Given $f: x \rightarrow x^2 - x$ with the set of real numbers as the domain. Find:

- a. $f(2)$ b. $f(-3)$ c. $f(4)$

Solution First write the equation: $f(x) = x^2 - x$

Then substitute: a. $f(2) = 2^2 - 2 = 4 - 2 = 2$

 b. $f(-3) = (-3)^2 - (-3) = 9 + 3 = 12$

 c. $f(4) = 4^2 - 4 = 16 - 4 = 12$

Find the values for each given function with the set of real numbers as the domain.

- | | | | |
|----------------------------------|--------------|----------------|----------------|
| 15. $f(x) = 3x - 2$ | a. $f(2)$ 4 | b. $f(-2)$ -8 | c. $f(-4)$ -14 |
| 16. $p(x) = 4 - 2x$ | a. $p(1)$ 2 | b. $p(0)$ 4 | c. $p(-2)$ 8 |
| 17. $R: t \rightarrow t + 2$ | a. $R(2)$ 4 | b. $R(-1)$ 1 | c. $R(-3)$ -1 |
| 18. $G: n \rightarrow n - 3$ | a. $G(0)$ -3 | b. $G(2)$ -1 | c. $G(-3)$ -6 |
| 19. $h(a) = 2a^2 + 1$ | a. $h(3)$ 19 | b. $h(-2)$ 9 | c. $h(0)$ 1 |
| 20. $k(t) = 2t^2 - 3$ | a. $k(4)$ 29 | b. $k(-2)$ 5 | c. $k(-3)$ 15 |
| 21. $g(x) = x^2 - 1$ | a. $g(4)$ 15 | b. $g(-4)$ 15 | c. $g(0)$ -1 |
| 22. $h(y) = 3y^2 + 1$ | a. $h(2)$ 13 | b. $h(-2)$ 13 | c. $h(-1)$ 4 |
| 23. $R: y \rightarrow y^3 + 2$ | a. $R(0)$ 2 | b. $R(-2)$ -6 | c. $R(2)$ 10 |
| 24. $N: t \rightarrow t^3 - 8$ | a. $N(3)$ 19 | b. $N(-3)$ -35 | c. $N(0)$ -8 |
| 25. $f: x \rightarrow x^2 + 2x$ | a. $f(-2)$ 0 | b. $f(2)$ 8 | c. $f(-1)$ -1 |
| 26. $g: t \rightarrow 3t^2 - 2t$ | a. $g(3)$ 21 | b. $g(1)$ 1 | c. $g(-1)$ 5 |
| 27. $P(y) = y - y^2$ | a. $P(2)$ -2 | b. $P(0)$ 0 | c. $P(-2)$ -6 |

Mixed Review Exercises

Simplify.

- | | | |
|--|--|---|
| 1. $\frac{3n-1}{2n^2} + \frac{2}{n} - \frac{7n-1}{2n^2}$ | 2. $3\frac{1}{3} + 2\frac{3}{4} + 5\frac{2}{3} + 1\frac{1}{4}$ | 13. $(-12)\left(\frac{x}{4}\right) - 3x$ |
| 4. $(-2)(3a + 2b - c)$
$-6a - 4b + 2c$ | 5. $-[8 + (-3)]$ -5 | 6. $2(3m - 5) - 6m - 10$ |
| 7. $-80\left(\frac{1}{4}\right)\left(\frac{1}{5}\right) - 4$ | 8. $\frac{2e^2f}{3ef^2} \cdot \frac{6de^2}{8ef} \cdot \frac{de^2}{2f^2}$ | 9. $\frac{x^2 - 4}{x^2 + 4x + 4} \cdot \frac{x - 2}{x + 2}$ |

8-8 Linear and Quadratic Functions

Objective: To graph linear and quadratic functions.

Vocabulary

Graph of a function The graph of an equation that defines a function.

Linear function A function defined by $f(x) = mx + b$. For example, $f(x) = 2x + 3$.

Quadratic function A function defined by $f(x) = ax^2 + bx + c$ ($a \neq 0$). For example, $f(x) = 2x^2 - x - 1$.

Parabola The graph of $f(x) = ax^2 + bx + c$, where the domain of f is the set of real numbers and $a \neq 0$. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward.

Maximum point of a parabola The highest point on a parabola that opens downward; the point whose y -coordinate is the *greatest value* of the corresponding function.

Minimum point of a parabola The lowest point on a parabola that opens upward; the point whose y -coordinate is the *least value* of the corresponding function.

Axis of symmetry of a parabola The vertical line containing the maximum or minimum point of the parabola. The axis of symmetry of

$$y = ax^2 + bx + c \text{ (} a \neq 0 \text{) is } x = -\frac{b}{2a}.$$

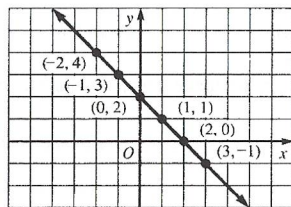
Vertex of a parabola The maximum or minimum point of the parabola. The

$$x\text{-coordinate of the vertex of } y = ax^2 + bx + c \text{ (} a \neq 0 \text{) is } x = -\frac{b}{2a}.$$

Example 1 Graph the function h defined by the equation $y = h(x) = -x + 2$.

Solution Find the coordinates of selected points as shown in the table below. Plot the points and connect them with a line.

x	$-x + 2 = y$
-2	$-(-2) + 2 = 4$
-1	$-(-1) + 2 = 3$
0	$-(0) + 2 = 2$
1	$-(1) + 2 = 1$
2	$-(2) + 2 = 0$
3	$-(-3) + 2 = -1$



Draw the graph of each linear function. You may wish to verify your graphs on a computer or a graphing calculator. Graphs given at the back of this Answer Key.

- $g: x - x - 2$
- $f: x - -x + 3$
- $g(x) = 2 - \frac{1}{2}x$
- $d(x) = -\frac{2}{3}x$
- $h(x) = -4$
- $n(x) = 5$

8-8 Linear and Quadratic Functions (continued)

Example 2 Find the coordinates of the vertex of the function $g(x) = x^2 - 2x - 3$. Then give the equation of the axis of symmetry. Use the vertex and four other points to graph the equation.

Solution

1. x -coordinate of vertex $= -\frac{b}{2a} = -\frac{-2}{2} = 1$

2. To find the y -coordinate of the vertex, substitute 1 for x .

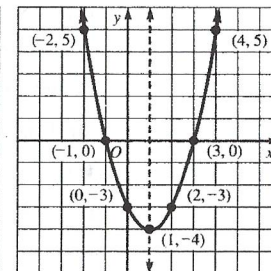
$$y = x^2 - 2x - 3 = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$$

The vertex is $(1, -4)$.

3. The axis of symmetry is the line $x = 1$.

4. For values of x , select three numbers greater than 1 and three numbers less than 1 to obtain paired points with the same y -coordinate.

x	$x^2 - 2x - 3 = y$
-2	$(-2)^2 - 2(-2) - 3 = 5$
-1	$(-1)^2 - 2(-1) - 3 = 0$
0	$(0)^2 - 2(0) - 3 = -3$
1	$(1)^2 - 2(1) - 3 = -4$
2	$(2)^2 - 2(2) - 3 = -3$
3	$(3)^2 - 2(3) - 3 = 0$
4	$(4)^2 - 2(4) - 3 = 5$



5. Plot the points. Connect them with a smooth curve.

Find the vertex and the axis of symmetry of the graph of each equation. Use the vertex and at least four other points to graph the equation. Graphs given at the back of this Answer Key.

- $y = 2 - x^2$ (0,2); $x = 0$
- $y = -2x^2$ (0,0); $x = 0$
- $y = x^2 - 3x$
- $y = -x^2 + x$ ($\frac{1}{2}, \frac{1}{4}$); $x = \frac{1}{2}$
- $y = -x^2 - x + 2$ ($-\frac{1}{2}, \frac{9}{4}$); $x = -\frac{1}{2}$
- $y = x^2 + 2x - 3$ ($\frac{3}{2}, -\frac{9}{4}$); $x = \frac{3}{2}$
- $(-1, -4)$; $x = -1$

Mixed Review Exercises

Find the range of each function. $R = \{3, 5, 11\}$

- $f(x) = 2x^2 + 3, D = \{0, 1, 2\}$
 - $m(b) = b^3 + 4, D = \{-1, 1, 2\}$
- $R = \{3, 5, 12\}$

Translate each phrase into a variable expression.

- 3 times the sum of a number and 2 $3(x + 2)$
- The difference between a number and 6 $n - 6$
- The product of a number and 7 $7n$
- 4 less than one half of a number $\frac{1}{2}x - 4$

8–9 Direct Variation

Objective: To use direct variation to solve problems.

Vocabulary

Direct variation A function defined by an equation of the form $y = kx$, where k is a nonzero constant. For example, $y = 5x$.

Constant of variation The nonzero constant k in a direct variation defined by $y = kx$. Also called the *constant of proportionality*.

Symbols $y = kx$ (y varies directly as x).

Example 1 Given that m varies directly as n and that $m = 75$ when $n = 25$, find the following:

- a. the constant of variation b. the value of m when $n = 15$

Solution Let $m = kn$.

- a. Substitute $m = 75$ and $n = 25$:

$$\begin{aligned} 75 &= k \cdot 25 \\ 3 &= k \end{aligned}$$

- b. Substitute $k = 3$ and $n = 15$: $m = 3 \cdot 15 = 45$

In Exercises 1–6, find the constant of variation.

- y varies directly as x , and $y = 18$ when $x = 3$. **6**
- y varies directly as x , and $y = 52$ when $x = 13$. **4**
- t varies directly as s , and $t = -36$ when $s = -4$. **9**
- h varies directly as m , and $h = 368$ when $m = 23$. **16**
- y varies directly as x , and $y = 252$ when $x = 18$. **14**
- t varies directly as s , and $t = 490$ when $s = 14$. **35**

Solve.

- y varies directly as x , and $y = 300$ when $x = 5$. Find y when $x = 15$. **900**
- y varies directly as x , and $y = 10$ when $x = 2$. Find y when $x = 9$. **45**
- h varies directly as a , and $a = 20$ when $h = 4$. Find a when $h = 3$. **15**
- h varies directly as a , and $a = 24$ when $h = 8$. Find a when $h = 4$. **12**
- y varies directly as x , and $y = 240$ when $x = 25$. Find y when $x = 40$. **384**
- h varies directly as a , and $a = 6$ when $h = 15$. Find a when $h = 5$. **2**

8–9 Direct Variation (continued)

Example 2 The amount of interest earned on savings is directly proportional to the amount of money saved. If \$26 interest is earned on \$325, how much interest will be earned on \$900 in the same period of time?

Solution 1

Step 1 The problem asks for the interest earned on \$900 if the interest on \$325 is \$26.

Step 2 Let i , in dollars, be the interest on d dollars. $i_1 = 26$ $i_2 = ?$
 $d_1 = 325$ $d_2 = 900$

Step 3 An equation can be written in the form $\frac{i_1}{d_1} = \frac{i_2}{d_2}$

$$\frac{26}{325} = \frac{i_2}{900}$$

Step 4 $26(900) = 325i_2$

$$23,400 = 325i_2$$

$$72 = i_2$$

Step 5 The check is left for you. The interest earned on \$900 will be \$72.

Solution 2 To solve Example 2 by the method shown in Example 1, first write the equation $i = kd$. Then solve for the constant of variation, k , by using the fact that $i = 26$ when $d = 325$. Use the value of k to find the value of i when $d = 900$. You may wish to complete the problem this way.

Solve.

- An employee's wages are directly proportional to the time worked. If an employee earns \$120 for 8 h, how much will the employee earn for 20 h? **\$300**
- A certain car used 21 gal of gasoline in 7 h. If the rate of gasoline used is constant, how much gasoline will the car use on a 6-hour trip? **18 gal**
- The distance traveled by a bus at a constant speed varies with the length of time it travels. If a bus travels 192 mi in 4 h, how far will it travel in 9 h? **432 mi**
- The number of words typed is directly proportional to the time spent typing. If a typist can type 325 words in 5 min, how long will it take the typist to type a 1040-word report? **16 min**

Mixed Review Exercises

Multiply. $6x^2 - 11x + 3$

$3x^3 + x^2 - 11x + 6$

$-6x + 10x^2$

1. $(2x - 3)(3x - 1)$

2. $(3x - 2)(x^2 + x - 3)$

3. $-2x(3 - 5x)$

4. $(2x + 5)(2x - 5)$
 $4x^2 - 25$

5. $(t - 2)(3t + 5)$
 $3t^2 - t - 10$

6. $(5y - 3)(2y + 3)$
 $10y^2 + 9y - 9$

8-10 Inverse Variation

Objective: To use inverse variation to solve problems.

Vocabulary

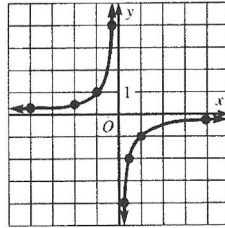
Inverse variation A function defined by an equation of the form $xy = k$, where k is a nonzero constant. For example, $xy = 6$.

Hyperbola The graph of $xy = k$ for any nonzero value of k .

Example 1 Graph the equation $xy = -1$.

Solution

x	y	x	y
-4	$\frac{1}{4}$	$\frac{1}{4}$	-4
-2	$\frac{1}{2}$	$\frac{1}{2}$	-2
-1	1	1	-1
$-\frac{1}{4}$	4	4	$-\frac{1}{4}$



Graphs given at the back of this Answer Key.

Graph each equation if the domain and the range are both the set of real numbers.

You may wish to verify your graphs on a computer or graphing calculator.

1. $xy = 8$ 2. $xy = 16$ 3. $xy = -4$ 4. $xy = -6$
 5. $x = \frac{4}{y}$ 6. $y = \frac{6}{x}$ 7. $\frac{x}{3} = \frac{-3}{y}$ 8. $\frac{x}{2} = \frac{6}{y}$

Example 2 (x_1, y_1) and (x_2, y_2) are ordered pairs of the same inverse variation.

Find the missing value: $x_1 = 2, y_1 = 28, x_2 = 4, y_2 = ?$.

Solution An inverse variation $xy = k$ can also be expressed as $x_1y_1 = x_2y_2$.

$2 \cdot 28 = 4 \cdot y_2$ Replace x_1 with 2, y_1 with 28, and x_2 with 4.

$56 = 4y_2$ Solve the equation.

$14 = y_2$, or $y_2 = 14$.

(x_1, y_1) and (x_2, y_2) are ordered pairs of the same inverse variation. Find the missing value.

9. $x_1 = 6, y_1 = 5, x_2 = 2, y_2 = ?$ 15 10. $x_1 = 8, y_1 = 24, x_2 = ?$, $y_2 = 48$ 4
 11. $x_1 = 5, y_1 = 8, x_2 = 10, y_2 = ?$ 4 12. $x_1 = 6, y_1 = ?$, $x_2 = 9, y_2 = 8$ 12
 13. $x_1 = ?$, $y_1 = 20, x_2 = 8, y_2 = 5$ 2 14. $x_1 = 8, y_1 = 9, x_2 = ?$, $y_2 = 18$ 4

8-10 Inverse Variation (continued)

Example 3 If a 12 g mass is 60 cm from the fulcrum of a lever, how far from the fulcrum is a 45 g mass that balances the 12 g mass?

Solution A lever is a bar pivoted at a point called the *fulcrum*. If masses m_1 and m_2 are placed at distances d_1 and d_2 from the fulcrum, and the bar is balanced, then $m_1d_1 = m_2d_2$.

Let $m_1 = 12, d_1 = 60$, and $m_2 = 45, d_2 = ?$.

Use $m_1d_1 = m_2d_2$.

$$12 \cdot 60 = 45 \cdot d_2$$

$$720 = 45d_2$$

$$16 = d_2$$

The distance of the 45 g mass from the fulcrum is 16 cm.

In Exercises 15-22, refer to the lever at balance in Example 3. Find the missing value.

15. $m_1 = 12, m_2 = 8, d_1 = 45, d_2 = ?$ 67.5 16. $m_1 = 60, m_2 = ?$, $d_1 = 8, d_2 = 12$ 40

17. $m_1 = 24, m_2 = 8, d_1 = ?$, $d_2 = 18$ 6 18. $m_1 = ?$, $m_2 = 40, d_1 = 5, d_2 = 7$ 56

19. $m_1 = 12, m_2 = 9, d_1 = ?$, $d_2 = 40$ 30 20. $m_1 = 108, m_2 = 60, d_1 = ?$, $d_2 = 9$ 5

Solve.

21. Sarah weighs 105 lb and Wyatt weighs 140 lb. If Sarah sits 8 ft from the seesaw support, how far from the support must Wyatt sit to balance the seesaw? 6 ft
 22. Yoko weighs 120 lb and Lars weighs 180 lb. If Yoko sits 6 ft from the seesaw support, how far from the support must Lars sit to balance the seesaw? 4 ft

Mixed Review Exercises

Show that the lines whose equations are given are parallel.

1. $x + 2y = 3$
 $x + 2y = 5$ $m = -\frac{1}{2}$; $b_1 = \frac{3}{2}, b_2 = \frac{5}{2}$ 2. $2x + 6y = 7$
 $x + 3y = 1$ $m = -\frac{1}{3}$; $b_1 = \frac{7}{6}, b_2 = \frac{1}{3}$
 3. $x - y = 3$
 $y - x = 3$ $m = 1$; $b_1 = -3, b_2 = 3$ 4. $-6x + 9y = 2$
 $2x - 3y = 6$ $m = \frac{2}{3}$; $b_1 = \frac{2}{9}, b_2 = -2$

Find the constant of variation.

5. t varies directly as s , and $t = 12$ when $s = -3$. -4
 6. y varies directly as x , and $y = 8$ when $x = 32$. $\frac{1}{4}$
 7. m varies directly as n , and $m = 27$ when $n = 3$. 9