

7-9 Negative Exponents

Objective: To use negative exponents.

Definitions

a^{-n} If a is a nonzero real number and n is a positive integer, $a^{-n} = \frac{1}{a^n}$.

For example, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$. (Notice that 2^{-3} is not a negative number.)

a^0 If a is a real number not equal to zero, $a^0 = 1$. For example, $2^0 = 1$, $3^0 = 1$, $25^0 = 1$, and so on. The expression 0^0 has no meaning.

Example 1	a. $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$	b. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$	c. $8^{-1} = \frac{1}{8^1} = \frac{1}{8}$
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Simplify.

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|--------------|-------------|---------------|--------------|
| 1. 3^{-1} | 2. 6^{-1} | 3. 4^{-3} | 4. 3^{-4} |
| 5. 5^{-2} | 6. 7^{-2} | 7. 2^{-5} | 8. 11^{-2} |
| 9. 9^0 | 10. -7^0 | 11. 1^{-3} | 12. 5^{-1} |
| 13. 6^{-2} | 14. 5^0 | 15. 10^{-4} | 16. 7^{-3} |

All of the rules for positive exponents also hold for zero and negative exponents:

Summary of Rules for Exponents	Examples
Let m and n be any integers. Let a and b be any nonzero integers.	
1. Products of Powers: $b^m b^n = b^{m+n}$	$2^2 \cdot 2^{-4} = 2^{2+(-4)} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
2. Quotients of Powers: $b^m \div b^n = b^{m-n}$	$3^2 \div 3^5 = 3^{2-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$
3. Power of a Power: $(b^m)^n = b^{mn}$	$(2^2)^{-3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$
4. Power of a Product: $(ab)^m = a^m b^m$	$(2x)^{-3} = 2^{-3} \cdot x^{-3} = \frac{1}{2^3} \cdot \frac{1}{x^3} = \frac{1}{8x^3}$
5. Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{\frac{1}{2^2}}{\frac{1}{3^2}} = \frac{1}{2^2} \cdot \frac{3^2}{1} = \frac{3^2}{2^2} = \frac{9}{4}$

CAUTION Remember that in $3x^2$, the exponent is applied to x but not to 3. However, in $(3x)^2$, the exponent is applied both to 3 and to x .

7-9 Negative Exponents (continued)**Example 2** Simplify. Give your answers using positive exponents.

a. $\frac{3}{3^{-2}}$ b. $(x^{-1})^{-2}$ c. $(2x^{-1})^3$

Solution a. $\frac{3}{3^{-2}} = 3^{1 - (-2)}$ Use the rule for quotients of powers.
 $= 3^3$
 $= 27$

b. $(x^{-1})^{-2} = x^{(-1)(-2)}$ Use the rule for a power of a power.
 $= x^2$

c. $(2x^{-1})^3 = 2^3 \cdot x^{(-1)(3)}$ { Use the rule for a power of a power and
 $= 8x^{-3}$ the rule for a power of a product.
 $= \frac{8}{x^3}$ Use the rule for negative exponents.

Simplify. Give your answers using positive exponents.

17. $\frac{2}{2^{-3}}$

18. $\frac{4^{-2}}{4^{-3}}$

19. $3^{-3} \cdot 3^5$

20. $(5^{-1})^2$

21. $\left(\frac{2^{-1}}{1}\right)^2$

22. $\left(\frac{4^3}{4^{-2}}\right)^0$

23. $(5^{-1})^{-2}$

24. $(3^{-2})^{-1}$

25. $\left(\frac{3}{2}\right)^{-2}$

26. $\left(\frac{3^4}{3^{-2}}\right)^0$

27. $\frac{5^{-2} \cdot 5}{5^{-1}}$

28. $\frac{3^{-4} \cdot 3^2}{3^{-2}}$

29. $2x^{-2}$

30. $3x^{-3}$

31. $(2x)^{-2}$

32. $(3x)^{-3}$

33. $x^{-2}y$

34. $a^{-2}b^3$

35. $a^5 \cdot a^{-3}$

36. $n^3 \cdot n^{-4}$

37. $(m^{-2})^3$

38. $(x^{-3})^2$

39. $(2x^{-2})^2$

40. $(3x^{-1})^2$

41. $\frac{y^2}{y^{-3}}$

42. $\frac{u^{-3}}{u^7}$

43. $\frac{c^{-5}}{c^3}$

44. $\frac{d^3}{d^{-3}}$

45. $\frac{x^{-4}}{x^{-2}}$

46. $\frac{m^{-6}}{m^{-8}}$

Mixed Review Exercises

Simplify. Give restrictions on the variables.

1. $\frac{24x^2y}{16xy^2}$

2. $\frac{y^2 - 7y + 10}{y^2 - 10y + 25}$

3. $\left(\frac{-2a}{b}\right)^3$

4. $\frac{6}{5mn} - \frac{2}{n}$

5. $2 - \frac{4a}{a-1}$

6. $\frac{x^2 - 4}{x^2 + 3x + 2}$

Divide. Write your answer as a polynomial or as a mixed expression.

7. $\frac{3x^2 + 10x + 3}{x + 3}$

8. $\frac{a^3 + 8}{a + 2}$

9. $\frac{32}{x^2 - 25} \div \frac{24}{x^2 + 10x + 25}$