

1-6 Quotients

Objective: To review rules for dividing real numbers.

Vocabulary

Division To divide by any *nonzero* real number, multiply by its reciprocal.

Examples: $15 \div 3 = 15 \cdot \frac{1}{3} = 5$ $12 \div \left(-\frac{4}{3}\right) = 12 \cdot \left(-\frac{3}{4}\right) = -9$

Since zero has no reciprocal, *division by zero is undefined*.

Rules for division

1. The quotient of two real numbers with *like signs* is a *positive* real number.

Examples: $18 \div 3 = 6$ $(-22) \div (-2) = 11$

2. The quotient of two real numbers with *opposite signs* is a *negative* real number.

Example: $6 \div (-3) = -2$

3. For all *nonzero* real numbers a , b , and c :

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$$

Example 1 Simplify $-6 \div 9 \div \frac{1}{3}$.

Solution Work from left to right.

$$\begin{aligned} -6 \div 9 \div \frac{1}{3} &= \left(-6 \cdot \frac{1}{9}\right) \div \frac{1}{3} \\ &= -\frac{2}{3} \div \frac{1}{3} \\ &= -\frac{2}{3} \cdot 3 = -2 \end{aligned}$$

Simplify.

1. $-63 \div (-9)$

2. $-4 \div 16$

3. $-48 \div 8 \div (-2)$

4. $-18 \div \frac{2}{3}$

5. $-\frac{1}{3} \div \left(-\frac{1}{6}\right) \div (-4)$

6. $(-2)^3 \div [4(-6)]$

Example 2 Simplify:

a. $\frac{(-6)(-8) \div (-2)}{4(-3)}$

b. $\frac{3\left(\frac{3}{4} - \frac{1}{4}\right)}{-\frac{1}{2} \div \frac{3}{2}}$

Solution

Simplify the numerator and denominator separately. Then divide.

a. $\frac{(-6)(-8) \div (-2)}{4(-3)} = \frac{48 \div (-2)}{-12} = \frac{-24}{-12} = 2$

b. $\frac{3\left(\frac{3}{4} - \frac{1}{4}\right)}{-\frac{1}{2} \div \frac{3}{2}} = \frac{3\left(\frac{2}{4}\right)}{-\frac{1}{2} \cdot \frac{2}{3}} = \frac{\frac{3}{2}}{-\frac{1}{3}} = \frac{3}{2} \cdot (-3) = -\frac{9}{2}$

1-6 Quotients (continued)

Simplify.

7. $\frac{(-2)(-8)(-9)}{(-12)(-3)}$

8. $\frac{(-4)(-12) \div (-3)}{8(-2)}$

9. $\frac{7(-12) \div 14}{4(-3)}$

10. $\frac{4^2 - 3^2}{4 - (-3)}$

11. $\frac{-8\left(-\frac{1}{2} - \frac{1}{4}\right)}{-\frac{3}{4} \div 3}$

12. $\frac{-27\left[12 \div \left(-\frac{3}{4}\right)\right]}{12\left(-\frac{3}{4}\right)}$

Example 3 Simplify $\frac{72 - 8x^2}{-4}$.**Solution**

Apply the third rule for division on page 11. Then simplify.

$$\begin{aligned}\frac{72 - 8x^2}{-4} &= \frac{72}{-4} - \frac{8x^2}{-4} \\ &= -18 - (-2x^2) \\ &= -18 + 2x^2\end{aligned}$$

Simplify.

13. $\frac{6x^2 - 21}{-3}$

14. $\frac{56 - 4x^2}{-4}$

15. $\frac{2 - (-x)^2}{-1}$

16. $\frac{-3x^2 - 3^2}{-3}$

17. $\frac{12 + 4x - 8x^2}{4}$

18. $\frac{-20x^3 - 15x^2 - 10x}{-5}$

Example 4 Evaluate $\frac{(x^2 + 2)(x - 3)}{x - 1}$ for $x = -2$.**Solution**Substitute -2 for x . Then simplify using the order of operations.

$$\begin{aligned}\frac{(x^2 + 2)(x - 3)}{x - 1} &= \frac{[(-2)^2 + 2](-2 - 3)}{-2 - 1} \\ &= \frac{(4 + 2)(-5)}{-3} \\ &= \frac{6(-5)}{-3} \\ &= \frac{-30}{-3} = 10\end{aligned}$$

Evaluate each expression for the given values of the variables.

19. $\frac{x(x + 5)}{x - 2}$

a. $x = 1$

b. $x = 0$

c. $x = 4$

20. $\frac{(y^2 - 2)(y + 3)}{y + 1}$

a. $y = -2$

b. $y = 1$

c. $y = 3$

21. $\frac{(n - 3)(n + 2)(n - 1)}{\frac{1}{4}n - 4}$

a. $n = -4$

b. $n = 0$

c. $n = 4$

22. $\frac{a(a + 5)(a - 2)}{(a + 1)(a - 3)}$

a. $a = 1$

b. $a = -3$

c. $a = \frac{1}{2}$