

11 Rational Numbers

11-1 Properties of Rational Numbers

Objective: To learn and apply some properties of rational numbers.

Vocabulary

Rational number A real number that can be expressed as the quotient of two integers.

Examples: $\frac{2}{3}$, $6 = \frac{6}{1}$, $0 = \frac{0}{1}$, $4\frac{2}{3} = \frac{14}{3}$, $0.57 = \frac{57}{100}$, $-\frac{3}{5} = \frac{-3}{5}$

Average of two numbers The number halfway between two numbers.

Properties

Density Property of Rational Numbers Between every pair of different rational numbers there is another rational number.

Order Property of Rational Numbers For all integers a and b and all positive integers c and d :

1. $\frac{a}{c} > \frac{b}{d}$ if and only if $ad > bc$. 2. $\frac{a}{c} < \frac{b}{d}$ if and only if $ad < bc$.

Example 1 Which rational number is greater, $\frac{7}{3}$ or $\frac{9}{4}$?

Solution 1 The LCD is 12.

$$\frac{7}{3} = \frac{28}{12} \text{ and } \frac{9}{4} = \frac{27}{12}$$

Compare $\frac{28}{12}$ and $\frac{27}{12}$.

Since $28 > 27$, $\frac{28}{12} > \frac{27}{12}$. So $\frac{7}{3} > \frac{9}{4}$.

Solution 2

$$\frac{7}{3} \stackrel{?}{>} \frac{9}{4}$$

$$(7)(4) \stackrel{?}{>} (3)(9)$$

$$28 > 27$$

$$\text{So } \frac{7}{3} > \frac{9}{4}.$$

Arrange each group of numbers in order from least to greatest.

1. $\frac{3}{4}, \frac{5}{8}, \frac{4}{5}$

2. $\frac{2}{3}, \frac{11}{15}, \frac{5}{7}$

3. $4.6, \frac{105}{22}, -4$

4. $-\frac{31}{8}, -3.9, -\frac{41}{10}$

5. $-\frac{4}{9}, -\frac{6}{11}, -\frac{5}{7}, -\frac{3}{5}$

6. $\frac{5}{24}, \frac{4}{15}, \frac{5}{12}, \frac{1}{4}$

Example 2 Replace the $?$ with $<$, $=$, or $>$ to make a true statement.

a. $-\frac{3}{8} \stackrel{?}{>} -\frac{4}{11}$

b. $5\frac{3}{7} \stackrel{?}{>} \frac{49}{9}$

Solution

a. $\frac{-3}{8} \stackrel{?}{>} \frac{-4}{11}$

$$(-3)(11) \stackrel{?}{>} (8)(-4)$$

$$-33 < -32$$

$$\text{So } -\frac{3}{8} < -\frac{4}{11}.$$

{ Use the order property
of rational numbers.

b. $\frac{38}{7} \stackrel{?}{>} \frac{49}{9}$

$$(38)(9) \stackrel{?}{>} (7)(49)$$

$$342 < 343$$

$$\text{So } 5\frac{3}{7} < \frac{49}{9}.$$

11-1 Properties of Rational Numbers (continued)Replace the $?$ with $<$, $=$, or $>$ to make a true statement.

7. $\frac{5}{6} ? \frac{13}{18}$ 8. $\frac{3}{4} ? \frac{11}{16}$ 9. $\frac{7}{8} ? \frac{28}{32}$ 10. $\frac{2}{9} ? \frac{13}{54}$
11. $\frac{1}{6} ? \frac{5}{32}$ 12. $\frac{3}{4} ? \frac{11}{15}$ 13. $-\frac{3}{5} ? -\frac{7}{12}$ 14. $-\frac{7}{8} ? -\frac{5}{6}$
15. $\frac{2}{5} ? \frac{21}{50}$ 16. $-\frac{3}{4} ? -\frac{8}{10}$ 17. $-12\frac{1}{5} ? -\frac{86}{7}$ 18. $-\frac{107}{7} ? -15\frac{5}{8}$

Example 3 Find the number halfway between $\frac{3}{8}$ and $\frac{2}{5}$.

Solution
$$\begin{aligned} \frac{3}{8} + \frac{1}{2}\left(\frac{2}{5} - \frac{3}{8}\right) &= \frac{3}{8} + \frac{1}{2}\left(\frac{16}{40} - \frac{15}{40}\right) \\ &= \frac{3}{8} + \frac{1}{2}\left(\frac{1}{40}\right) \\ &= \frac{3}{8} + \frac{1}{80} \\ &= \frac{30}{80} + \frac{1}{80} \\ &= \frac{31}{80} \end{aligned}$$

Check: Is $\frac{3}{8} < \frac{31}{80}$?

Is $(3)(80) < (8)(31)$?
 $240 < 248 \checkmark$

Is $\frac{31}{80} < \frac{2}{5}$?

Is $(31)(5) < (80)(2)$?
 $155 < 160 \checkmark$

$\frac{31}{80}$ is a rational number halfway between $\frac{3}{8}$ and $\frac{2}{5}$.

Find the number halfway between the given numbers.

19. $\frac{3}{8}, \frac{5}{9}$ 20. $\frac{7}{12}, \frac{5}{6}$ 21. $\frac{6}{11}, \frac{2}{3}$
22. $-\frac{11}{15}, -\frac{7}{12}$ 23. $-1\frac{2}{5}, -2\frac{5}{6}$ 24. $-2\frac{3}{5}, 4\frac{1}{3}$

Example 4 If $x \in \{0, 1, 2, 3\}$, state whether $\frac{x}{2}$ increases or decreases in value as x takes on its values in increasing order.**Solution** Replace x with 0, 1, 2, and 3 in order. $\frac{x}{2}$ becomes $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}$. So $\frac{x}{2}$ increases.If $x \in \{0, 1, 2, 3\}$, state whether each fraction increases or decreases in value as x takes on its value in increasing order.

25. $\frac{x}{6}$ 26. $-\frac{x}{8}$ 27. $\frac{4}{x+1}$ 28. $\frac{x+1}{4}$ 29. $\frac{7-2x}{5}$ 30. $\frac{12}{10-3x}$

Mixed Review Exercises

Solve each inequality and graph its solution.

1. $3y + 1 \leq 7$ 2. $|0.2 + x| < 8$ 3. $6 + 4|2 - k| \geq 14$
4. $|x + 1| \geq 3$ 5. $6 \leq 3x + 6 < 9$ 6. $5 \leq 3 - 2m$