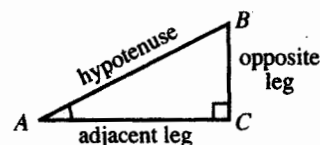# Trigonometric Ratios

Objective: To find the sine, cosine, and tangent of an acute angle.

Vocabulary

Trigonometric ratios Ratios of the lengths of the sides of a right triangle.

The trigonometric ratios have special names. In right triangle ABC,   **Symbols**

sine of $\angle A$	$=$	$\dfrac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$	$=$	$\dfrac{BC}{AB}$	$\sin A$
cosine of $\angle A$	$=$	$\dfrac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$	$=$	$\dfrac{AC}{AB}$	$\cos A$
tangent of $\angle A$	$=$	$\dfrac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$	$=$	$\dfrac{BC}{AC}$	$\tan A$

Trigonometric functions The trigonometric ratios thought of as the values of three functions ($\sin A$, $\cos A$, $\tan A$) each having the set of acute angles as its domain.
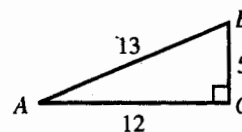

Example 1   Find the sine, cosine, and tangent of $\angle A$ and $\angle B$.

Solution

$\sin A = \dfrac{5}{13}$	$\sin B = \dfrac{12}{13}$
$\cos A = \dfrac{12}{13}$	$\cos B = \dfrac{5}{13}$
$\tan A = \dfrac{5}{12}$	$\tan B = \dfrac{12}{5}$

Example 2   Find the sine, cosine, and tangent of $\angle Q$.
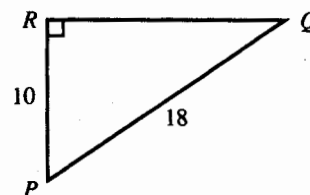
Solution   Use the Pythagorean theorem to find RQ.

$$a^2 + 10^2 = 18^2$$
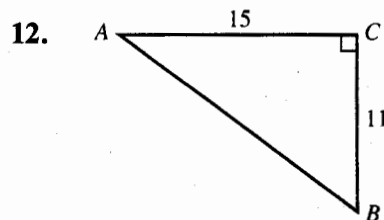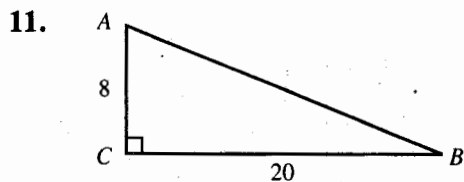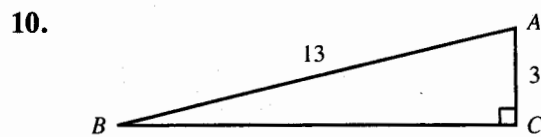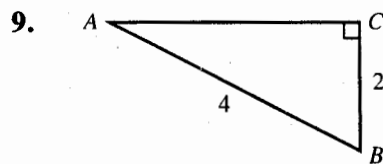$$a^2 + 100 = 324$$
$$a^2 = 224$$
$$a = 4\sqrt{14}$$

$$\sin Q = \frac{10}{18} = \frac{5}{9} \qquad \cos Q = \frac{4\sqrt{14}}{18} = \frac{2\sqrt{14}}{9} \qquad \tan Q = \frac{10}{4\sqrt{14}} = \frac{5\sqrt{14}}{28}$$

CAUTION   The trigonometric ratios are defined only for acute angles in a right triangle.

Trigonometric Ratios (continued)

For each right triangle shown, find sin A, cos A, tan A, sin B, cos B, and tan B.
Write irrational answers in simplest radical form.

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11.

12.
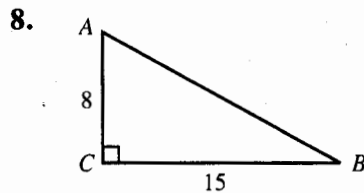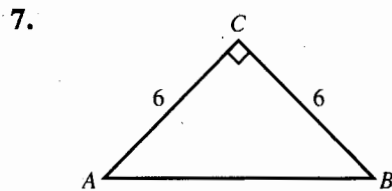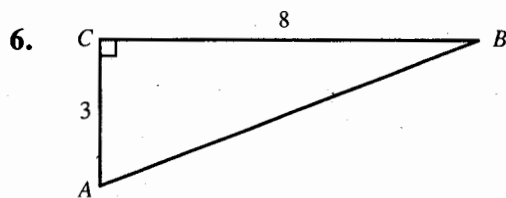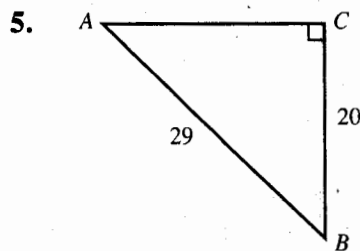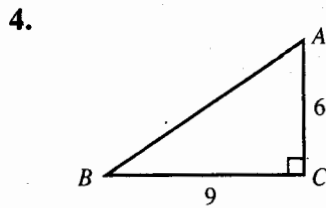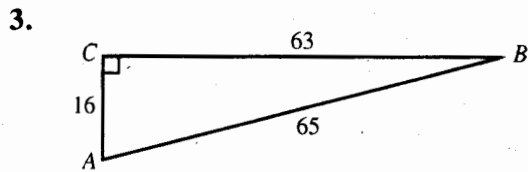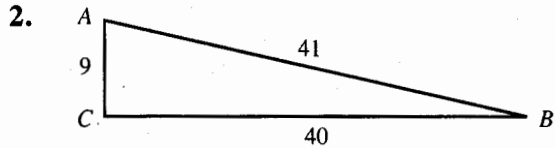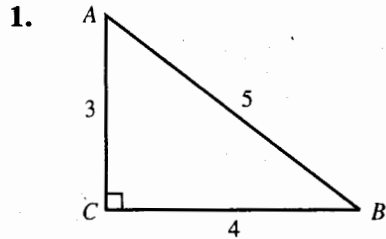
Values of Trigonometric Ratios

Objective: To find values of trigonometric ratios for given angles, and measures of angles for given trigonometric ratios.

Example 1   Find the values of:

a. sin 56°   b. cos 56°   c. tan 56°.

Solution 1   **Using a Table**

It is not possible to give *exact* trigonometric ratios for most angles. You can use a table or a scientific calculator to find approximate values of trigonometric ratios.

Locate 56° in the left-hand column of the portion of the table shown at the right, then read across the row to find:

a. sin 56° ≈ 0.8290

b. cos 56° ≈ 0.5592

c. tan 56° ≈ 1.4826

For convenience, you may write = instead of ≈.

Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175
56°	.8290	.5592	1.4826
57°	.8387	.5446	1.5399
58°	.8480	.5299	1.6003
59°	.8572	.5150	1.6643
60°	.8660	.5000	1.7321

Solution 2   **Using a Calculator**

a. To find the value for sin 56°, first enter 56. Then press the sin key to get 0.8290376.

   To the nearest ten-thousandth, sin 56° = 0.8290.

b. Enter 56 and then press the cos key to get:

   cos 56° ≈ 0.5591929, or cos 56° = 0.5592.

c. Enter 56 and then press the tan key to get:

   tan 56° ≈ 1.482561, or tan 56° = 1.4826.

Use a calculator or a table to find sin A, cos A, and tan A for the given measure of angle A.

1. 12°	2. 42°	3. 75°	4. 24°	5. 35°
6. 66°	7. 16°	8. 79°	9. 31°	10. 52°
11. 8°	12. 58°	13. 70°	14. 48°	15. 30°

Values of Trigonometric Ratios (continued)

Example 2   Find the measure of $\angle A$ to the nearest degree.

   a. $\cos A = 0.8290$ **b.** $\sin A = 0.7320$.

Solution 1   **Using a Table**

   a. Locate the value 0.8290 in the cosine column, if possible.

   Since it is there read across the row to the left-hand column to find that the angle has a measure of 34°.

   b. Since 0.7320 is not listed in the sine column, locate the entries between which 0.7320 lies.

   Since sin 47° = 0.7314 and sin 48° = 0.7431, the measure of $\angle A$ must be between 47° and 48°.

   Since 0.7320 is closer to 0.7314 than it is to 0.7431, $\angle A = 47°$ to the nearest degree.

Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175
31°	.5150	.8572	.6009
32°	.5299	.8480	.6249
33°	.5446	.8387	.6494
34°	.5592	.8290	.6745
35°	.5736	.8192	.7002
46°	.7193	.6947	1.0355
47°	.7314	.6820	1.0724
48°	.7431	.6691	1.1106
49°	.7547	.6561	1.1504
50°	.7660	.6428	1.1918

Solution 2   **Using a Calculator**

   Most calculators have the inverse keys ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, or inv sin, inv cos, inv tan) that give the measure of an acute angle.

   a. Enter 0.8290, then press the $\cos^{-1}$ key to get 34.00385. To the nearest degree, $\angle A = 34°$.

   b. Enter 0.7320, then press the $\sin^{-1}$ key to get 47.054324. To the nearest degree, $\angle A = 47°$.

Use a calculator or a table to find the measure of angle A to the nearest degree.

16. $\sin A = 0.9903$ **17.** $\cos A = 0.9063$

18. $\cos A = 0.6428$ **19.** $\tan A = 1.7321$

20. $\tan A = 0.4456$ **21.** $\sin A = 0.6558$

22. $\tan A = 1.2851$ **23.** $\cos A = 0.9607$

24. $\tan A = 3.2608$ **25.** $\sin A = 0.9560$

26. $\sin A = 0.7777$ **27.** $\cos A = 0.6683$

28. $\sin A = 0.8330$ **29.** $\sin A = 0.6440$

30. $\tan A = 5.6708$ **31.** $\cos A = 0.0860$

Problem Solving Using Trigonometry

Objective: To use trigonometric ratios to solve problems.

Trigonometric ratios can be used to solve practical problems involving right triangles. You can find values for these ratios by using a table or a scientific calculator.

Example 1   An observation tower is 75 m high. A support wire is attached to the tower 20 m from the top. If the support wire and the ground form an angle of 46°, what is the length of the support wire?

Solution   Draw a triangle and label the different values. First, find how high on the tower the support wire is attached.

$$75 - 20 = 55$$

Since $\triangle ABC$ is a right triangle,

$$\sin 46° = \frac{55}{x}, \text{ or } x = \frac{55}{\sin 46°}.$$

From a table or a calculator, sin 46° = 0.7193.

Then $x = \frac{55}{0.7193} \approx 76.5$.

To the nearest tenth of a meter, the support wire is 76.5 m long.
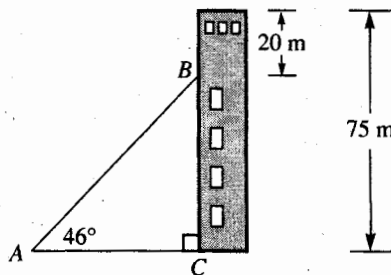
Example 2   A 12 ft ladder rests against the side of a building. If the foot of the ladder is 4 ft from the building, find, to the nearest degree, the angle the ladder makes with the ground.
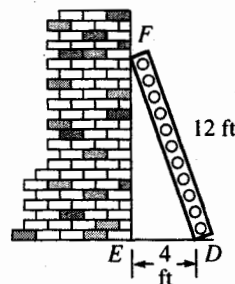
Solution   Draw a triangle and label the different values.
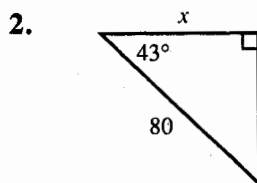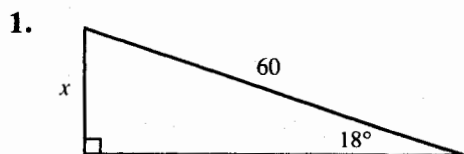Since $\triangle DEF$ is a right triangle,

$$\cos \angle D = \frac{DE}{DF}$$

$$\cos \angle D = \frac{4}{12}$$
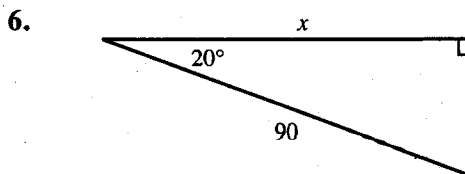
$$\cos \angle D = 0.3333$$
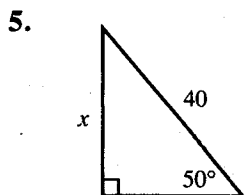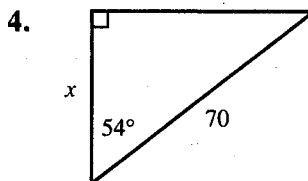
Therefore $\angle D = 71°$.

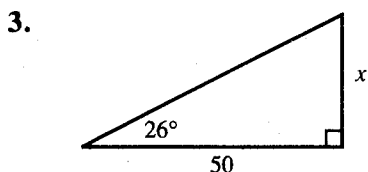
Use a table or calculator as needed to find the value of x to the nearest whole number.

1.

2.

Problem Solving Using Trigonometry *(continued)*

Use a table or calculator as needed to find the value of x to the nearest
whole number.

3.

4.

5.

6.

In a right $\triangle\ ABC,\ \angle C\ =\ 90°$. Find the lengths of the other sides of
the triangle to the nearest whole number.

7. $\angle A\ =\ 35°,\ AB\ =\ 48$

8. $\angle B\ =\ 23°,\ AB\ =\ 16$

9. $\angle A\ =\ 56°,\ BC\ =\ 40$

10. $\angle A\ =\ 44°,\ BC\ =\ 30$

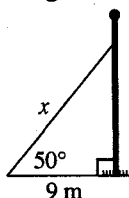**11.** $\angle A\ =\ 11°,\ AC\ =\ 17$

12. $\angle B\ =\ 30°,\ AB\ =\ 62$

Solve each problem, drawing a sketch for each. Express distances to
the nearest unit and angle measures to the nearest degree. Use a table
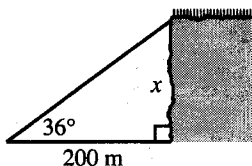or a calculator as needed.

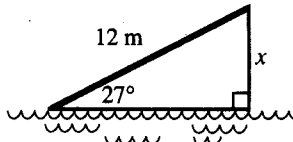**13.** How far is it across the river?
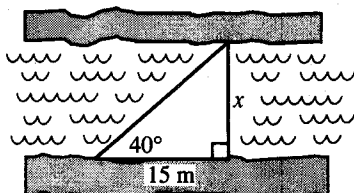
14. How long is the cable that supports the pole?

15. How high is the cliff?

16. How high is the water ski jump?

17. A kite is flying 115 ft above the ground. The length of the string to
the kite is 150 ft, measured from the ground. Find the angle to the
nearest degree that the string makes with the ground.

18. To the nearest degree what is the angle formed with the ground by a
32 ft ladder if it is leaning against a wall at a height of 28 ft?