

NAME \_\_\_\_\_

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## 2 Working with Real Numbers

### 2-1 Basic Assumptions

**Objective:** To use number properties to simplify expressions.

#### Vocabulary

**Unique** One and only one

**Terms** When  $a$  and  $b$  are added,  $a$  and  $b$  are called terms.

**Factors** When  $a$  and  $b$  are multiplied,  $a$  and  $b$  are called factors.

Properties of Real Numbers	Addition	Multiplication
<b>Closure Properties</b> The sum and product of any two real numbers are also real numbers and they are unique.	$2 + 3 = 5$ and only 5	$2 \cdot 3 = 6$ and only 6
<b>Commutative Properties</b> The order in which you add or multiply any two real numbers does not affect the result.	$3 + 5 = 5 + 3$	$3 \cdot 5 = 5 \cdot 3$
<b>Associative Properties</b> When you add or multiply any three real numbers, the grouping (or association) of the numbers does not affect the result.	$(3 + 4) + 6 = 3 + (4 + 6)$	$(3 \cdot 4)5 = 3(4 \cdot 5)$

**Example 1** Simplify: a.  $75 + 13 + 25 + 47$  b.  $4 \cdot 7 \cdot 25 \cdot 3$

**Solution** Regrouping makes mental math easier.

$$\begin{aligned} \text{a. } 75 + 13 + 25 + 47 &= (75 + 25) + (13 + 47) && \text{Regroup the terms.} \\ &= 100 + 60 && \text{Simplify within the} \\ &= 160 && \text{parentheses. Add.} \end{aligned}$$

$$\begin{aligned} \text{b. } 4 \cdot 7 \cdot 25 \cdot 3 &= (4 \cdot 25)(7 \cdot 3) && \text{Regroup the factors.} \\ &= 100 \cdot 21 && \text{Simplify within the parentheses.} \\ &= 2100 && \text{Multiply.} \end{aligned}$$

**Example 2** Simplify  $1\frac{1}{3} + 16\frac{4}{5} + 2\frac{2}{3} + 3\frac{1}{5}$ .

**Solution** Regroup the fractions. Simplify within the parentheses. Add.

$$\begin{aligned} 1\frac{1}{3} + 16\frac{4}{5} + 2\frac{2}{3} + 3\frac{1}{5} &= \left(1\frac{1}{3} + 2\frac{2}{3}\right) + \left(16\frac{4}{5} + 3\frac{1}{5}\right) \\ &= 4 + 20 \\ &= 24 \end{aligned}$$

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### 2-1 Basic Assumptions (continued)

**Example 3** Simplify  $0.8 + 3.7 + 0.2 + 5.3$ .

**Solution** Regroup the decimals. Simplify within the parentheses. Add.

$$\begin{aligned} 0.8 + 3.7 + 0.2 + 5.3 &= (0.8 + 0.2) + (3.7 + 5.3) \\ &= 1 + 9 \\ &= 10 \end{aligned}$$

**Simplify.**

- $125 + 42 + 75 + 28$  **270**
- $507 + 36 + 43 + 14$  **600**
- $2 \cdot 18 \cdot 5 \cdot 4$  **720**
- $40 \cdot 3 \cdot 4 \cdot 20$  **9600**
- $50 \cdot 27 \cdot 4 \cdot 2$  **10,800**
- $4 \cdot 15 \cdot 25 \cdot 3$  **4500**
- $3\frac{1}{2} + 5\frac{2}{3} + 2\frac{1}{2} + \frac{1}{3}$  **12**
- $7\frac{2}{3} + 4\frac{3}{5} + 2\frac{1}{3} + \frac{12}{5}$  **17**
- $0.2 + 3.9 + 2.8 + 0.1$  **7**
- $0.6 + 5.2 + 0.4 + 3.8$  **10**
- $2.85 + 3.75 + 1.15 + 9.25$  **17**
- $3.25 + 1.95 + 8.75 + 11.05$  **25**

**Example 4** Simplify: a.  $6 + 8m + 4 + 7n$  b.  $(3w)(2x)(4y)(5z)$

**Solution** a.  $6 + 8m + 4 + 7n = 8m + 7n + (6 + 4)$  Regroup the terms.  
 $= 8m + 7n + 10$  Simplify.

b.  $(3w)(2x)(4y)(5z) = (3 \cdot 2 \cdot 4 \cdot 5)(wxyz)$  Regroup the factors.  
 $= 120wxyz$  Simplify.

**Simplify.**

- |                                |                                 |                                |                                |
|--------------------------------|---------------------------------|--------------------------------|--------------------------------|
| $5y + 10$                      | $5z + 20$                       | $3x + 9$                       | $2w + 7$                       |
| 13. $2 + 5y + 8$               | 14. $9 + 5z + 11$               | 15. $4 + 3x + 5$               | 16. $3 + 2w + 4$               |
| 17. $3(20a)$ <b>60a</b>        | 18. $4(5n)$ <b>20n</b>          | 19. $(5x)(6y)$ <b>30xy</b>     | 20. $(8m)(5n)$ <b>40mn</b>     |
| 21. $(6x)(y)(4z)$ <b>24xyz</b> | 22. $(2p)(3q)(5r)$ <b>30pqr</b> | 23. $(3a)(7b)(c)$ <b>21abc</b> | 24. $(e)(6f)(2g)$ <b>12efg</b> |
| 25. $a + 2 + b + 5$            | 26. $9 + x + y + 3$             | 27. $3p + 4 + 2q + 6$          | 28. $7m + 1 + 5n + 4$          |
| 29. $4 + 6x + 2 + 3y$          | 30. $6p + 3 + 2q + 37$          | 31. $(5a)(4b)(25c)(8d)$        | 32. $(4w)(2x)(5y)(5z)$         |
| $6x + 3y + 6$                  | $6p + 2q + 40$                  | <b>4000abcd</b>                | <b>200wxyz</b>                 |
|                                |                                 | 25. $a + b + 7$                | 26. $x + y + 12$               |
|                                |                                 | 27. $3p + 2q + 10$             | 28. $7m + 5n + 5$              |

#### Mixed Review Exercises

Evaluate if  $a = 2$ ,  $x = 4$ ,  $y = 6$ , and  $z = 3$ .

- $\frac{3x - a}{a + z}$  **2**
- $4z(y - a)$  **48**
- $\frac{2a + x}{3z - (y + 2)}$  **8**

**Simplify.**

- $|-3| + |-5|$  **8**
- $\left|-\frac{1}{6}\right| + 0$   **$\frac{1}{6}$**
- $|-3.2| + |3.2|$  **6.4**
- $|8| - |-8|$  **0**
- $|-4| - |-2|$  **2**
- $\left|-\frac{5}{7}\right| - \left|\frac{3}{7}\right|$   **$\frac{2}{7}$**

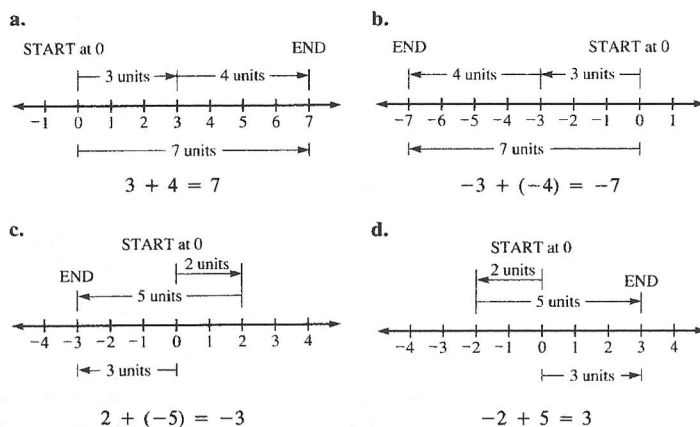
## 2-2 Addition On A Number Line

**Objective:** To add real numbers using a number line or properties about opposites.

Properties	Examples
<b>Identity Property of Addition</b> The sum of a real number and 0 is identical to the number itself. $a + 0 = a$ and $0 + a = a$	$5 + 0 = 5$ and $0 + 5 = 5$
<b>Properties of Opposites</b> Every real number has an opposite. The sum of a real number and its opposite is 0. $a + (-a) = 0$ and $(-a) + a = 0$	$3 + (-3) = 0$ and $(-3) + 3 = 0$
<b>Property of the Opposite of a Sum</b> For all real numbers $a$ and $b$ : $-(a + b) = (-a) + (-b)$	$-(2 + 3) = -5 = (-2) + (-3)$

**Example 1** Simplify: a.  $3 + 4$  b.  $-3 + (-4)$  c.  $2 + (-5)$  d.  $-2 + 5$

**Solution**



**Simplify each expression. If necessary, draw a number line to help you.**

- $4 + 2$  6
- $-4 + (-2)$  -6
- $6 + (-9)$  -3
- $-6 + 9$  3
- $3 + 5$  8
- $-3 + (-5)$  -8
- $-8 + 4$  -4
- $8 + (-4)$  4
- $(-2 + 5) + 4$  7
- $(-4 + 7) + 1$  4
- $(-3 + 1) + 2$  0
- $(-6 + 5) + 3$  2

## 2-2 Addition On A Number Line (continued)

**Simplify each expression. If necessary, draw a number line to help you.**

- $(-6 + 9) + (-2)$  1
- $(-4 + 7) + (-3)$  0
- $(-3 + 5) + 7$  9
- $(-8 + 12) + (-6)$  -2
- $[-6 + (-5)] + 6$  -5
- $[-3 + (-7)] + 3$  -7
- $25 + [7 + (-2)]$  30
- $32 + [8 + (-14)]$  26
- $[19 + (-9)] + [2 + (-2)]$  10
- $(-2 + 2) + [26 + (-6)]$  20
- $[0 + (-6)] + [-5 + (-25)]$  -36
- $(-7 + 7) + [14 + (-4)]$  10
- $-1 + (-2) + (-7)$  -10
- $(-4) + (-6) + (-9)$  -19
- $-3 + (-11) + 8 + (-5)$  -11
- $-10 + 14 + (-3) + (-12)$  -11
- $-5.3 + (-1.5) + 6.8$  0
- $4.2 + (-3.4) + (-6.9)$  -6.1
- $-\frac{3}{4} + 3 + (-\frac{9}{4})$  0
- $-\frac{11}{3} + (-4) + (-\frac{7}{3})$  -10

**Example 2** Simplify  $3 + (-2) + x + 6$ .

**Solution** Use the commutative and associative properties to regroup.

$$\begin{aligned} 3 + (-2) + x + 6 &= 1 + x + 6 \\ &= x + (1 + 6) \\ &= x + 7 \end{aligned}$$

**Simplify.**

- $2 + x + (-6)$   $x + (-4)$
- $y + (-1) + 5$   $y + 4$
- $3n + 4 + (-1)$   $3n + 3$
- $5 + 2n + (-4)$   $2n + 1$
- $12 + 4n + (-20)$   $4n + (-8)$
- $-8 + (-2z) + 11$   $-2z + 3$

### Mixed Review Exercises

**Simplify.**

- $6 + 8 \div 2$  10
- $|-5| + |2|$  7
- $9 - [ -(-1) ]$  8
- $\frac{2 + (3 \cdot 6)}{4}$  5
- $|\frac{3}{5}| + |-\frac{2}{5}|$  1
- $|\frac{-3}{4}| + |-\frac{1}{4}|$  1
- $52 + 35 + 48 + 15$  150
- $12 \cdot 5 \cdot 30$  1800
- $(2x)(4y)(5z)$   $40xyz$
- $4\frac{3}{4} + 2\frac{1}{3} + 5\frac{1}{4} + \frac{2}{3}$  13
- $0.3 + 2.6 + 5.7$  8.6
- $4 \cdot 19 \cdot 25$  1900

## 2-3 Rules for Addition

**Objective:** To add real numbers using rules for addition.

### Vocabulary

**Opposite signs** A positive and a negative number are said to have opposite signs.

Rules for Addition	Examples
If two numbers have the <i>same sign</i> , add their absolute values and put their common sign before the result.	$2 + 5 = 7$ $-2 + (-5) = -7$
If two numbers have <i>opposite signs</i> , subtract the lesser absolute value from the greater and put the sign of the number having the greater absolute value before the result.	$6 + (-4) = 6 - 4 = 2$ $(-6) + 4 = -(6 - 4) = -2$
If two numbers are <i>opposites</i> , then their sum is zero.	$3 + (-3) = 0$

**Example 1** Add  $6 + (-8) + 13 + (-9)$ .

**Solution 1** Add the numbers in order from left to right.

$$\begin{array}{r} 6 + (-8) + 13 + (-9) \\ \underline{-2} + 13 + (-9) \\ \underline{11} + (-9) \\ \underline{2} \end{array}$$

**Solution 2** 1. Add positive numbers. 2. Add negative numbers. 3. Add the results.

$$\begin{array}{r} 6 \\ 13 \\ \hline 19 \end{array} \quad \begin{array}{r} -8 \\ -9 \\ \hline -17 \end{array} \quad \begin{array}{r} 19 \\ -17 \\ \hline 2 \end{array}$$

**Add.**

1. $\begin{array}{r} 6 \\ 2 \\ \hline 8 \end{array}$	2. $\begin{array}{r} -4 \\ -7 \\ \hline -11 \end{array}$	3. $\begin{array}{r} -7 \\ 6 \\ \hline -1 \end{array}$	4. $\begin{array}{r} -3 \\ 8 \\ \hline 5 \end{array}$	5. $\begin{array}{r} 23 \\ 64 \\ \hline 87 \end{array}$	6. $\begin{array}{r} -56 \\ 31 \\ \hline -25 \end{array}$
7. $\begin{array}{r} -37 \\ -56 \\ \hline -93 \end{array}$	8. $\begin{array}{r} -35 \\ 120 \\ \hline 85 \end{array}$	9. $\begin{array}{r} 126 \\ -35 \\ -37 \\ \hline -17 \\ 37 \end{array}$	10. $\begin{array}{r} -145 \\ 309 \\ -47 \\ \hline -82 \\ 35 \end{array}$	11. $\begin{array}{r} 136 \\ -58 \\ -47 \\ \hline -23 \\ 8 \end{array}$	12. $\begin{array}{r} -162 \\ 323 \\ -47 \\ \hline -82 \\ 32 \end{array}$

**Add.**

13. $(-8 + 5) + 2 = -1$	14. $(-12 + 15) + 6 = 9$	15. $(-4 + 8) + (-3) = 1$
16. $(-2 + 6) + (-4) = 0$	17. $-5 + (-3) + 5 = -3$	18. $-4 + (-14) + 4 = -14$

## 2-3 Rules for Addition (continued)

**Add.**

19. $16 + 5 + (-8) = 13$	20. $-6 + (-24) + 6 = -24$
21. $(-3 + 3) + 7 + (-11) = -4$	22. $(-3 + 3) + 17 + (-7) = 10$
23. $-2 + (-4) + (-8) = -14$	24. $-7 + (-5) + (-6) = -18$
25. $-3 + (-9) + 7 + (-5) = -10$	26. $-15 + 10 + (-3) + (-2) = -10$

**Example 2** Simplify  $3 + (-5) + (-x) + 7$ .

**Solution**  $3 + (-5) + (-x) + 7 = -x + \underbrace{3 + 7}_{10} + (-5)$  Regroup the terms.  
 $= -x + \underbrace{10 + (-5)}_5$  Simplify.  
 $= -x + 5$

**Simplify.**

27. $-2 + x + (-6) + 3 = x + (-5)$	28. $3 + (-8) + (-y) + (-11) = -y + (-16)$
29. $-5 + 2a + 3 + (-3) = 2a + (-5)$	30. $-5 + 2a + 8 + 7 = 2a + 10$
31. $17 + 8b + (-15) + (-10) = 8b + (-8)$	32. $-[6 + (-1)] + (-c) + 2 = -c + (-3)$
33. $-(-7) + 3y + (-6) + 4 = 3y + 5$	34. $3x + [7 + (-2) + (-3)] = 3x + 2$

**Example 3** Evaluate  $x + y + (-2)$  if  $x = -2$ , and  $y = 5$ .

**Solution**  $x + y + (-2) = \underbrace{(-2) + 5}_3 + (-2)$  Substitute  $-2$  for  $x$  and  $5$  for  $y$ .  
 $= \underbrace{3 + (-2)}_1$  Add from left to right.  
 $= 1$  Simplify.

**Evaluate each expression if  $x = -2$ ,  $y = 5$ , and  $z = -3$ .**

35. $y + z + (-2) = 0$	36. $-18 + x + y = -15$
37. $-11 + (-x) + (-y) = -14$	38. $-z + (-7) + y = 1$
39. $1 + (-y) + x = -6$	40. $-x + (-y) + (-15) = -18$

## Mixed Review Exercises

**Simplify.**

1. $3 + 8 + 2 = 13$	2. $7 \cdot 5 \cdot 3 \cdot 2 = 210$	3. $(9 - 6 \div 3) \cdot 2 = 14$
4. $ -9  - 7 = 2$	5. $ -1.6  + 1.6 = 3.2$	6. $ -11  -  -5  = 6$
7. $\frac{9 \cdot 6 + 9 \cdot 4}{6 + 3} = 10$	8. $3\frac{1}{5} + 7\frac{1}{2} + 8\frac{4}{5} = 19\frac{1}{2}$	9. $2.7 + 1.0 + 3.3 = 7$
10. $[12 + (-2)] + 5 = 15$	11. $(-7 + 2) + (-3) = -8$	12. $-2 + (-8) + 7 + (-1) = -4$

## 2-4 Subtracting Real Numbers

**Objective:** To subtract real numbers and to simplify expressions involving differences.

### Definition of Subtraction

To subtract a real number  $b$ , add the opposite of  $b$ .

$$a - b = a + (-b)$$

For example,  $3 - 9 = 3 + (-9) = -6$ .

**Example 1** Simplify: a.  $2 - 7$  b.  $-6 - 3$  c.  $-2 - (-8)$

**Solution**

a.  $2 - 7 = 2 + (-7) = -5$   
 b.  $-6 - 3 = -6 + (-3) = -9$   
 c.  $-2 - (-8) = -2 + 8 = 6$

**CAUTION 1** Subtraction is *not* commutative.

$$\begin{aligned} 7 - 3 &= 4, \\ \text{but } 3 - 7 &= -4, \\ \text{so } 7 - 3 &\neq 3 - 7 \end{aligned}$$

**CAUTION 2** Subtraction is *not* associative.

$$\begin{aligned} (7 - 3) - 2 &= 4 - 2 = 2, \\ \text{but } 7 - (3 - 2) &= 7 - 1 = 6, \\ \text{so } (7 - 3) - 2 &\neq 7 - (3 - 2) \end{aligned}$$

**Simplify.**

- |                                       |  |                             |
|---------------------------------------|--|-----------------------------|
| 1. $25 - 9$ <b>16</b>                 | 2. $17 - 11$ <b>6</b>                  | 3. $9 - 13$ <b>-4</b>       |
| 4. $6 - 16$ <b>-10</b>                | 5. $0 - 5$ <b>-5</b>                   | 6. $0 - (-3)$ <b>3</b>      |
| 7. $-12 - 0$ <b>-12</b>               | 8. $-8 - 1$ <b>-9</b>                  | 9. $3 - (-3)$ <b>6</b>      |
| 10. $7 - (-5)$ <b>12</b>              | 11. $-8 - (-3)$ <b>-5</b>              | 12. $36 - 216$ <b>-180</b>  |
| 13. $143 - 270$ <b>-127</b>           | 14. $36 - (-34)$ <b>70</b>             | 15. $-25 - (-24)$ <b>-1</b> |
| 16. $-15 - (-3)$ <b>-12</b>           | 17. $-3 - (-15)$ <b>12</b>             |                             |
| 18. $-17 - (-8)$ <b>-9</b>            | 19. $-2.3 - 3.5$ <b>-5.8</b>           |                             |
| 20. $-4.2 - 5.6$ <b>-9.8</b>          | 21. $2.65 - (-2.35)$ <b>5</b>          |                             |
| 22. $-15$ decreased by $5$ <b>-20</b> | 23. $-8$ decreased by $-14$ <b>6</b>   |                             |
| 24. $18$ less than $-2$ <b>-20</b>    | 25. $10$ less than $-6$ <b>-16</b>     |                             |
| 26. $56 - (45 - 32)$ <b>43</b>        | 27. $125 - (160 - 35)$ <b>0</b>        |                             |
| 28. $214 - (54 - 66)$ <b>226</b>      | 29. $167 - (20 - 45)$ <b>192</b>       |                             |
| 30. $(25 - 32) - (44 - 55)$ <b>4</b>  | 31. $(46 - 50) - (65 - 40)$ <b>-29</b> |                             |
| 32. $(2 - 7) - (-12 + 15)$ <b>-8</b>  | 33. $(32 - 24) - (-6 + 9)$ <b>5</b>    |                             |

## 2-4 Subtracting Real Numbers (continued)

**Example 2** Simplify  $13 - 9 - 8 + 5$ .

$$\begin{aligned} \text{Solution } 13 - 9 - 8 + 5 &= 13 - 9 - 8 + 5 \\ &= \underbrace{13 + (-9)}_4 + (-8) + 5 \\ &= \underbrace{4 + (-8)}_{-4} + 5 \\ &= \underbrace{-4 + 5}_1 \end{aligned}$$

**Simplify.**

34.  $3 - 4 + 7 - 15 + 21$  **12**                      35.  $14 - 12 + 11 + 3 - 20$  **-4**  
 36.  $-5 - 18 + 6 - 7 + 10$  **-14**                      37.  $-9 - 21 + 3 - 8 + 30$  **-5**

**Example 3** Simplify: a.  $-(x - 5)$  b.  $-(3 - y)$  c.  $-(-2 + a)$

**Solution** To find the opposite of a sum or a difference, you change the sign of each term of the sum or difference.

$$\begin{aligned} \text{a. } -(x - 5) &= -x + 5 & \text{b. } -(3 - y) &= -3 + y \\ \text{c. } -(-2 + a) &= 2 - a \end{aligned}$$

**Simplify.**

38.  $-(x + 2) - x - 2$  **39.**  $-(4 - y) - 4 + y$  **40.**  $-(-7 + a) 7 - a$  **41.**  $-(x - 3) - x + 3$   
 42.  $-(y - 5) - y + 5$  **43.**  $-(8 - x) - 8 + x$  **44.**  $-(b - 6) - b + 6$  **45.**  $-(2 + n) - 2 - n$

**Example 4** Simplify  $8 - (x + 3)$ .

$$\begin{aligned} \text{Solution } 8 - (x + 3) &= 8 - x - 3 & \text{Change the sign of each term of } x + 3. \\ &= (8 - 3) - x & \text{Regroup the terms.} \\ &= 5 - x & \text{Simplify.} \end{aligned}$$

**Simplify.**

46.  $6 - (y + 4)$  **2 - y** **47.**  $4 - (q - 6)$  **10 - q** **48.**  $x - (x + 2) - 2$  **49.**  $n - (-3 + n)$  **3**

### Mixed Review Exercises

- |  |   |   |
|--|---|---|
| 1. $ -6  +  2 $ <b>8</b>   | 2. $17 \cdot 2 \cdot 3 \cdot 5$ <b>510</b>              | 3. $2 + 6x + 5y + 8$  |
| 4. $\left -\frac{3}{4}\right  - \left -\frac{1}{4}\right $ <b><math>\frac{1}{2}</math></b> | 5. $-\frac{3}{2} + \left(-\frac{5}{2}\right)$ <b>-4</b> | 6. $1\frac{1}{4} + \left(-3\frac{3}{4}\right)$ <b><math>-2\frac{1}{2}</math></b>  |
| 7. $[5 + (-9)] + 7$ <b>3</b>   | 8. $3.4 - 0.5 + (-1.4)$ <b>1.5</b>                      | 9. $-4 + [-6 + (-2)]$ <b>-12</b>  |
| 10. $-2.4 + 8.3 + (-3.6)$ <b>2.3</b>   | 11. $-27 + (-28) + 18 + 47$ <b>10</b>                   | 12. $2 + (-3) + (-10) + (-x)$ <b><math>-x - 11</math> or <math>-11 - x</math></b> |

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## 2-5 The Distributive Property

**Objective:** To use the distributive property to simplify expressions.

### Vocabulary

**Equivalent expressions** Expressions that represent the same number.

**Simplifying an expression** Replacing an expression containing variables by an equivalent expression with as few terms as possible.

Distributive Property	
<b>Distributive Property of Multiplication (with respect to addition)</b>	
For all real numbers $a$ , $b$ , and $c$ ,	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ .
For example,	$6(9 + 4) = 6 \cdot 9 + 6 \cdot 4$ and $(9 + 4)6 = 9 \cdot 6 + 4 \cdot 6$
<b>Distributive Property of Multiplication (with respect to subtraction)</b>	
For all real numbers $a$ , $b$ , and $c$ ,	$a(b - c) = ab - ac$ and $(b - c)a = ba - ca$ .
For example,	$8(12 - 2) = 8 \cdot 12 - 8 \cdot 2$ and $(12 - 2)8 = 12 \cdot 8 - 2 \cdot 8$

**CAUTION** When using the distributive properties, be sure to multiply *both* of the numbers inside the parentheses by the number outside the parentheses. For example,  
 $6(13 - 3) = 6 \cdot 13 - 6 \cdot 3$  *not*  $6 \cdot 13 - 3$ .

**Example 1** Simplify: a.  $5 \cdot 48$  b.  $8(7.5)$  c.  $6(4\frac{1}{3})$  d.  $(11 - 5)9$

**Solution** Use the distributive property to multiply.

$$\begin{aligned} \text{a. } 5 \cdot 48 &= 5(40 + 8) & \text{b. } 8(7.5) &= 8(7 + 0.5) \\ &= (5 \cdot 40) + (5 \cdot 8) & &= (8 \cdot 7) + (8 \cdot 0.5) \\ &= 200 + 40 & &= 56 + 4 \\ &= 240 & &= 60 \\ \\ \text{c. } 6(4\frac{1}{3}) &= 6(4 + \frac{1}{3}) & \text{d. } (11 - 5)9 &= (11 - 5)9 \\ &= (6 \cdot 4) + (6 \cdot \frac{1}{3}) & &= (11 \cdot 9) - (5 \cdot 9) \\ &= 24 + 2 & &= 99 - 45 \\ &= 26 & &= 54 \end{aligned}$$

**Simplify.** Use the distributive property.

- |                                 |                                 |                                 |                             |
|---------------------------------|---------------------------------|---------------------------------|-----------------------------|
| 1. $6 \cdot 35$ <b>210</b>      | 2. $5 \cdot 52$ <b>260</b>      | 3. $4(8.5)$ <b>34</b>           | 4. $8(6.25)$ <b>50</b>      |
| 5. $12(2\frac{1}{3})$ <b>28</b> | 6. $10(2\frac{1}{5})$ <b>22</b> | 7. $15(3\frac{2}{3})$ <b>55</b> | 8. $12 \cdot 25$ <b>300</b> |
| 9. $5(20 - 1)$ <b>95</b>        | 10. $6(60 - 2)$ <b>348</b>      | 11. $9(30 - 1)$ <b>261</b>      | 12. $8(40 - 3)$ <b>296</b>  |
| 13. $(9 - 4)6$ <b>30</b>        | 14. $(12 - 3)8$ <b>72</b>       | 15. $(20 - 1)5$ <b>95</b>       | 16. $(30 - 7)6$ <b>138</b>  |

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## 2-5 The Distributive Property (continued)

**Example 2** Simplify: a.  $64 \cdot 19 + 36 \cdot 19$  b.  $(3.8)(25) - (1.8)(25)$

**Solution** a.  $64 \cdot 19 + 36 \cdot 19 = (64 + 36)19 = (100)19 = 1900$     b.  $(3.8)(25) - (1.8)(25) = (3.8 - 1.8)25 = (2)25 = 50$

**Simplify.**

17.  $20 \cdot 19 + 80 \cdot 19$  **1900**    18.  $13 \cdot 53 + 87 \cdot 53$  **5300**    19.  $(17 \cdot 24) - (17 \cdot 24)$  **0**  
 20.  $(63 \cdot 71) + (37 \cdot 71)$  **7100**    21.  $(0.65)(28) + (0.35)(28)$  **28**    22.  $(4.3)(25) - (2.3)(25)$  **50**

**Example 3** Write an equivalent expression without parentheses.

a.  $5(n - 2)$     b.  $(5y + 6)2$

**Solution** a.  $5(n - 2) = 5 \cdot n - 5 \cdot 2 = 5n - 10$     b.  $(5y + 6)2 = (5y)2 + (6)2 = 10y + 12$

**For each expression write an equivalent expression without parentheses.**

23.  $2(x + 3)$   **$2x + 6$**     24.  $6(a + 5)$   **$6a + 30$**     25.  $5(n - 1)$   **$5n - 5$**     26.  $7(b - 5)$   **$7b - 35$**   
 27.  $3(6n + 2)$   **$18n + 6$**     28.  $8(5n - 3)$     29.  $3(x - y)$   **$3x - 3y$**     30.  $2(4x - y)$   **$8x - 2y$**   
 31.  $(4n - 7)4$     32.  $(3x + 4)5$     33.  $(3x + 4y)8$     34.  $(5m + 7n)2$   
 28.  **$40n - 24$**     31.  **$16n - 28$**     32.  **$15x + 20$**     33.  **$24x + 32y$**     34.  **$10m + 14n$**

**Example 4** Simplify: a.  $8x + 6x$  b.  $9y - 2y$  c.  $5n - 6 + 3n$

**Solution** a.  $8x + 6x = (8 + 6)x = 14x$   
 b.  $9y - 2y = 9y + (-2)y = (9 - 2)y = 7y$   
 c.  $5n - 6 + 3n = 5n + 3n - 6 = 8n - 6$

**Simplify.**

35.  $6a + 4a$   **$10a$**     36.  $5m + 7m$   **$12m$**     37.  $15y - 6y$   **$9y$**     38.  $3x - (-9)x$   **$12x$**   
 39.  $(-4)n + 9n$   **$5n$**     40.  $(-7)n - 8n$     41.  $2a + 9 + 5a$     42.  $7n + 1 + 3n$   
 43.  $9n - 5 + 2n$     44.  $3x + 8 - 2x$     45.  $9y - 6 + 5y$     46.  $10n - 7 + 6n$   
 40.  **$(-15)n$**     41.  **$7a + 9$**     42.  **$10n + 1$**     43.  **$11n - 5$**     44.  **$x + 8$**     45.  **$14y - 6$**     46.  **$16n - 7$**

### Mixed Review Exercises

Evaluate if  $a = -2$ ,  $b = -3$ ,  $c = 4$ ,  $x = 6$ , and  $y = 8$ .

1.  $4x + y - c$  **28**    2.  $(x \cdot x + c) \div 8$  **5**    3.  $3y - (2x \div c)$  **21**    4.  $|a| + |b| + (-y) - 3$   
 5.  $c + |a| + |-y|$  **14**    6.  $2|a| - 3|b|$  **-5**    7.  $-(x - b) + c$  **-5**    8.  $x + y + (-5)$  **9**  
 9.  $a + b + (-c)$  **-9**    10.  $c - (a - b)$  **3**    11.  $-a + b - c$  **-5**    12.  $|b - a| - c$  **-3**

## 2-6 Rules for Multiplication

**Objective:** To multiply real numbers.

Properties	Examples
<b>Identity Property of Multiplication</b> The product of a number and 1 is identical to the number itself. $a \cdot 1 = a$ and $1 \cdot a = a$	$6 \cdot 1 = 6$ and $1 \cdot 6 = 6$
<b>Multiplication Property of Zero</b> When one of the factors of a product is zero, the product itself is zero. $a \cdot 0 = 0$ and $0 \cdot a = 0$	$6 \cdot 0 = 0$ and $0 \cdot 6 = 0$
<b>Multiplication Property of -1</b> For every real number $a$ : $a(-1) = -a$ and $(-1)a = -a$	$6(-1) = -6$ and $(-1)6 = -6$ $(-5)(-1) = -(-5) = 5$ and $(-1)(-5) = -(-5) = 5$
<b>Property of Opposites in Products</b> For all real numbers $a$ and $b$ : $(-a)(b) = -ab$ $a(-b) = -ab$ $(-a)(-b) = ab$	$(-4)(5) = -20$ $4(-5) = -20$ $(-4)(-5) = 20$

### Rules for Multiplication

- If two numbers have the *same* sign, their product is positive.  
If two numbers have *opposite* signs, their product is negative.
- The product of an *even* number of negative numbers is positive.  
The product of an *odd* number of negative numbers is negative.

**Example 1** Multiply: a.  $3(6)$  b.  $(-3)(6)$  c.  $3(-6)$  d.  $(-3)(-6)$

**Solution**

a.  $3(6) = 18$  (Both factors have the same sign.)  
 b.  $(-3)(6) = -18$  (The two factors have opposite signs.)  
 c.  $3(-6) = -18$  (The two factors have opposite signs.)  
 d.  $(-3)(-6) = 18$  (Both factors have the same sign.)

**Example 2**

a.  $2(-3)(-4)(-5)$  is negative because it has 3 negative factors.  
 b.  $(-1)(-4)(-5)(6)(-7)$  is positive because it has 4 negative factors.  
 c.  $(-6)(7)(0)(-4)$  is zero because it has a zero factor.

## 2-6 Rules for Multiplication (continued)

**Multiply.**

1.  $(-12)(-3)$  **36**      2.  $18(-4)$  **-72**      3.  $2(17)$  **34**      4.  $18(0)$  **0**  
 5.  $(-2)(5)(-8)$  **80**      6.  $(4)(-7)(10)$  **-280**      7.  $(-2)(-3)(-4)$  **-24**      8.  $(-11)(-12)(0)$  **0**  
 9.  $35(-26)(0)$  **0**      10.  $5(-2)(-8)(-5)$  **-400**      11.  $(-7)(3)(-1)(2)$  **42**      12.  $(-8)(-5)(-1)(-3)$  **120**

**Example 3** Simplify: a.  $(-2x)(-6y)$       b.  $3y + (-7y)$

**Solution**

a.  $(-2x)(-6y) = (-2)x(-6)y$   
 $= (-2)(-6)xy$   
 $= 12xy$

b.  $3y + (-7y) = [3 + (-7)]y$   
 $= (-4)y$   
 $= -4y$

- Simplify.** 13.  $12ab$  14.  $30xy$  15.  $-10pq$  16.  $-28ef$  17.  $30ab$   
 18.  $-15a$  19.  $-3x$  20.  $5x$  21.  $-8y$  22. **0**
13.  $(-3a)(-4b)$  14.  $(5x)(6y)$  15.  $2p(-5q)$  16.  $(-4e)(7f)$  17.  $(-6a)(-5b)$   
 18.  $-7a + (-8a)$  19.  $2x + (-5x)$  20.  $8x + (-3x)$  21.  $(-11y) + 3y$  22.  $-4n + 4n$

**Example 4** Simplify: a.  $-3(2x - y)$       b.  $5x - 4(x - 1)$

**Solution**

a.  $-3(2x - y) = -3(2x) - (-3)(y)$   
 $= -6x - (-3y)$   
 $= -6x + 3y$

b.  $5x - 4(x - 1) = 5x - (4x - 4 \cdot 1)$   
 $= 5x - (4x - 4)$   
 $= 5x - 4x + 4$   
 $= x + 4$

**Simplify.**

23.  $-6(x - 2y)$   **$-6x + 12y$**       24.  $-5(2c + d)$   **$-10c - 5d$**       25.  $-4(3m + 2n)$   **$-12m - 8n$**   
 26.  $-7(-4y - 5)$   **$28y + 35$**       27.  $(3x - 5)(-6)$   **$-18x + 30$**       28.  $(-3 + 5y)(-2)$   **$6 - 10y$**   
 29.  $4x - 3(x - 2)$   **$x + 6$**       30.  $6x - 2(x + 3)$   **$4x - 6$**       31.  $3x - 5(x - 1)$   **$-2x + 5$**   
 32.  $(-1)(a - b + 2)$       33.  $(-1)(2x - y - 3)$       34.  $(-1)(x + y - z)$   
 35.  $4x - 2x + 7 + x$       36.  $2y - 5 - 5y + 3$       37.  $11p - 6c - 7c + 9p$   
 32.  **$-a + b - 2$**       33.  **$-2x + y + 3$**       34.  **$-x - y + z$**   
 35.  **$3x + 7$**       36.  **$-3y - 2$**       37.  **$20p - 13c$**

### Mixed Review Exercises

**Translate each sentence into an equation.**

- Three times a number is 27.  **$3n = 27$**
- The quotient of  $n$  and 4 is 15.  **$\frac{n}{4} = 15$**
- One half of a number is nine.  **$\frac{1}{2}n = 9$**
- Six less than twice a number is 14.  
 **$2n - 6 = 14$**

**Simplify.**

5.  $110 - (12 - 8)$  **106**      6.  $161 - (8 - 11)$  **164**      7.  $2 + (-5) + (-y) + 9$   **$6 - y$**   
 8.  $3(20 + 5)$  **75**      9.  $2n + (-5n)$   **$-3n$**       10.  $5(n + 1) + 7$   **$5n + 12$**

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## 2-7 Problem Solving: Consecutive Integers

**Objective:** To write equations to represent relationships among integers.

### Vocabulary

**Consecutive Integers** Numbers obtained by counting by ones from any integer.

For example,  $-2, -1, 0, 1,$  and  $2$  are consecutive integers.

**Even Integer** An integer that is the product of 2 and any integer.

For example,  $-10, -4, 2, 6,$  and  $8$  are even integers.

**Odd Integer** An integer that is not even.

For example,  $-3, -1, 7, 9,$  and  $11$  are odd integers.

**Consecutive Even Integers** Numbers obtained by counting by twos from any even integer.

For example,  $-6, -4, -2, 0,$  and  $2$  are consecutive even integers.

**Consecutive Odd Integers** Numbers obtained by counting by twos from any odd integer.

For example,  $-5, -3, -1, 1,$  and  $3$  are consecutive odd integers.

- Example 1** An integer is represented by  $n$ .
- Write the next four consecutive integers after  $n$ .
  - Write an equation to represent this relationship:  
The sum of three consecutive integers starting with  $n$  is 93.
  - Write an equation to represent this relationship:  
The product of two consecutive integers starting with  $n$  is 56.

- Solution**
- $n + 1, n + 2, n + 3, n + 4$
  - $n + (n + 1) + (n + 2) = 93$
  - $n(n + 1) = 56$

**Write an equation to represent the given relationship.**

- $n + (n + 1) + (n + 2) = 39$
- $n + (n + 1) + (n + 2) = -51$

- The sum of three consecutive integers is 39.
  - The sum of three consecutive integers is  $-51$ .
  - The product of two consecutive integers is 42.
  - The product of two consecutive integers is 30.
- $$n(n + 1) = 42 \qquad n(n + 1) = 30$$

- Example 2** Write an equation to represent this relationship:  
The sum of three consecutive odd numbers is 33.

**Solution** Let  $n$  = the first integer,  $n + 2$  = the second integer,  
and  $n + 4$  = the third integer.

$$\underbrace{n + (n + 2) + (n + 4)}_{= 33} \text{ is } 33.$$

**Write an equation to represent the given relationship.**

- $n + (n + 2) + (n + 4) = 45$
- $n + (n + 2) + (n + 4) = 72$

- The sum of three consecutive odd integers is 45.
  - The sum of three consecutive even integers is 72.
  - The sum of four consecutive integers is 90.
  - The sum of four consecutive even integers is  $-44$ .
- $$n + (n + 1) + (n + 2) + (n + 3) = 90 \qquad n + (n + 2) + (n + 4) + (n + 6) = -44$$

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## 2-7 Problem solving: Consecutive Integers (continued)

**Example 3** Write an equation to represent this relationship:  
The product of two consecutive integers is 110.

**Solution** Let  $n$  = the first integer and  $n + 1$  = the second integer.

$$\underbrace{\text{The product of the two consecutive integers}}_{n(n + 1)} \text{ is } 110. \qquad \downarrow = 110$$

**Write an equation to represent the given relationship.**

- The product of two consecutive integers is 72.  $n(n + 1) = 72$
- The product of two consecutive even integers is 80.  $n(n + 2) = 80$
- The product of two consecutive integers is 132.  $n(n + 1) = 132$
- The product of two consecutive odd integers is 195.  $n(n + 2) = 195$

**Example 4** Solve over the given domain: The sum of three consecutive odd integers is 32 more than the smallest integer. What are the integers?

Domain for the smallest integer:  $\{9, 11, 13\}$

**Solution** *Step 1* The unknowns are the three consecutive odd integers.

*Step 2* Let  $n$  = the smallest integer,  $n + 2$  = the middle integer,  
and  $n + 4$  = the largest integer.

*Step 3* 
$$\underbrace{\text{The sum}}_{n + (n + 2) + (n + 4)} \text{ is } 32 \text{ more than the smallest integer.}$$

*Step 4* Replace  $n$  in turn by 9, 11, and 13.

$n$	$n + (n + 2) + (n + 4) = n + 32$	
9	$9 + 11 + 13 = 9 + 32$	False
11	$11 + 13 + 15 = 11 + 32$	False
13	$13 + 15 + 17 = 13 + 32$	True

*Step 5* The check is left to you. The integers are 13, 15, and 17.

**Solve over the given domain.**

- The sum of three consecutive even integers is 50 more than the largest integer.  
What are the integers? Domain for the smallest integer:  $\{20, 22, 24\}$  **24, 26, and 28**
- The sum of three consecutive odd integers is 72 more than the smallest integer.  
What are the integers? Domain for the smallest integer:  $\{29, 31, 33\}$  **33, 35, and 37**

### Mixed Review Exercises

- $(30 - 3) - (43 - 20)$  **4**
- $-4.5 + 2.3 - 1.7$   **$-3.9$**
- $-2 + 3c + (-4) + 7$   **$3c + 1$**
- $\frac{7}{4} + \left(-\frac{10}{4}\right) - \frac{3}{4}$   **$-\frac{6}{4}$**
- $2\frac{2}{5} + 10 + 3\frac{3}{5}$  **16**
- $5\left(\frac{3}{4}\right) - 4\left(\frac{1}{4}\right) + 3\left(\frac{3}{4}\right)$  **5**
- $-(10 - x) - (x - 15)$  **5**
- $14a - 4a + 5a$  **15a**
- $10 + 4y + 5 + (-3)$   **$12 + 4y$**

## 2-8 The Reciprocal of a Real Number

**Objective:** To simplify expressions involving reciprocals.

### Vocabulary

**Reciprocals** Two numbers whose product is 1 are called reciprocals of each other.

For example, 5 and  $\frac{1}{5}$  are reciprocals.

**Symbols**  $\frac{1}{a}$  (the reciprocal of  $a$ )  $-\frac{1}{a}$  (the reciprocal of  $-a$ )

Properties	Examples
<b>Property of Reciprocals</b> Every nonzero real number $a$ has a reciprocal $\frac{1}{a}$ , such that $a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1.$	$3 \cdot \frac{1}{3} = 1 \quad \text{and} \quad \frac{1}{3} \cdot 3 = 1$
<b>Property of the Reciprocal of the Opposite of a Number</b> For every nonzero number $a$ , $\frac{1}{-a} = -\frac{1}{a}.$	$\frac{1}{-3} = -\frac{1}{3}$
<b>Property of the Reciprocal of a Product</b> For all nonzero numbers $a$ and $b$ , $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}.$	$\frac{1}{2 \cdot 3} = \frac{1}{2} \cdot \frac{1}{3}$

**CAUTION** 0 has no reciprocal; 1 is its own reciprocal; and  $-1$  is its own reciprocal.

**Example 1** Simplify: a.  $\frac{1}{3} \cdot \frac{1}{-5}$  b.  $3y \cdot \frac{1}{3}$  c.  $(-6xy)\left(-\frac{1}{2}\right)$

**Solution** a.  $\frac{1}{3} \cdot \frac{1}{-5} = \frac{1}{3(-5)} = \frac{1}{-15} = -\frac{1}{15}$

b.  $3y \cdot \frac{1}{3} = \left(3 \cdot \frac{1}{3}\right)y = 1y = y$

c.  $(-6xy)\left(-\frac{1}{2}\right) = (-6)\left(-\frac{1}{2}\right)(xy) = 3xy$

Simplify each expression.

- $\frac{1}{3}(-12) - 4$
- $-\frac{1}{8}(24) - 3$
- $-50\left(\frac{1}{5}\right) - 10$
- $-30\left(\frac{1}{3}\right) - 10$
- $(-20)\left(-\frac{1}{4}\right) 5$
- $(-42)\left(-\frac{1}{7}\right) 6$
- $-36\left(-\frac{1}{4}\right)\left(\frac{1}{3}\right) 3$
- $60\left(-\frac{1}{5}\right)\left(-\frac{1}{12}\right) 1$
- $72\left(-\frac{1}{8}\right)\left(-\frac{1}{9}\right) 1$
- $-54\left(-\frac{1}{6}\right)\left(-\frac{1}{9}\right) - 1$
- $\frac{1}{-2}(24)\left(\frac{1}{4}\right) - 3$
- $-60\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) - 10$

## 2-8 The Reciprocal of a Real Number (continued)

Simplify each expression.

- $6r\left(-\frac{1}{6}\right) - r$
- $32p\left(-\frac{1}{8}\right) - 4p$
- $\frac{1}{x}(8x), x \neq 0$
- $9x\left(\frac{1}{x}\right), x \neq 0$
- $21xy\left(\frac{1}{7}\right) 3xy$
- $72ab\left(\frac{1}{9}\right) 8ab$
- $18xy\left(\frac{1}{6}\right) 3xy$
- $(-54xy)\left(\frac{1}{-9}\right) 6xy$
- $15xy\left(-\frac{1}{3}\right) - 5xy$
- $6cd\left(\frac{1}{-2}\right) - 3cd$
- $(-8pq)\left(\frac{1}{-2}\right) 4pq$
- $(-42ac)\left(\frac{1}{-7}\right) 6ac$

**Example 2** Simplify: a.  $\frac{1}{2}(8m - 4n)$  b.  $(-21a - 63b)\left(-\frac{1}{7}\right)$

**Solution** a.  $\frac{1}{2}(8m - 4n) = \frac{1}{2}(8m) - \frac{1}{2}(4n)$  Use the distributive property.  
 $= \left(\frac{1}{2} \cdot 8\right)m - \left(\frac{1}{2} \cdot 4\right)n$  Use the associative property.  
 $= 4m - 2n$  Simplify.

b.  $(-21a - 63b)\left(-\frac{1}{7}\right) = (-21a)\left(-\frac{1}{7}\right) - (63b)\left(-\frac{1}{7}\right)$   
 $= (-21)\left(-\frac{1}{7}\right)a - (63)\left(-\frac{1}{7}\right)b$   
 $= (3)a - (-9)b$   
 $= 3a + 9b$

Simplify each expression.

- $\frac{1}{2}(-8a + 10) - 4a + 5$
- $\frac{1}{3}(9y - 21) 3y - 7$
- $-\frac{1}{5}(-25c + 10d) 5c - 2d$
- $-\frac{1}{4}(24g - 32h) - 6g + 8h$
- $(-21m - 14n)\left(-\frac{1}{7}\right) 3m + 2n$
- $(-26e - 52f)\left(-\frac{1}{13}\right) 2e + 4f$
- $(40x - 56y)\left(-\frac{1}{8}\right) - 5x + 7y$
- $(-5a + 30b)\left(\frac{1}{-5}\right) a - 6b$

### Mixed Review Exercises

Translate each sentence into an equation.  $6x + 3 = 21$   $x - 12 = 200$

- Three more than six times a number is 21.
- Twelve less than a number is 200.
- The sum of two consecutive integers is 71.  
 $n + (n + 1) = 71$
- The product of two consecutive integers is 90.  
 $n(n + 1) = 90$

Simplify.

- $(-8)(-3)(-5) - 120$
- $-24(25)(-4) 2400$
- $5(-7)(-6) 210$
- $-8(2a - 5b) - 16a + 40b$
- $-2(2 + x) - 2(x - 2) - 4x$
- $10(x - 1) + 4(3 - x) 6x + 2$



## 2-9 Dividing Real Numbers

**Objective:** To divide real numbers and to simplify expressions involving quotients.

### Definition of Division

To divide by a nonzero real number  $b$ , multiply by the reciprocal of  $b$ .

$$a \div b \text{ or } \frac{a}{b} = a \cdot \frac{1}{b}. \quad \text{For example, } 24 \div 3 = 24 \cdot \frac{1}{3}.$$

### Rules for Division

If two numbers have the same sign, their quotient is positive.

If two numbers have opposite signs, their quotient is negative.

**CAUTION 1** You can't divide by zero since zero has no reciprocal.

**CAUTION 2** Division is not commutative. For example,  $4 \div 2 = 2$ , but  $2 \div 4 = \frac{1}{2}$ .

**CAUTION 3** Division is not associative. For example,  $(12 \div 6) \div 2 = 2 \div 2 = 1$ , but  $12 \div (6 \div 2) = 12 \div 3 = 4$ .

**Example 1** Simplify: a.  $\frac{30}{6}$  b.  $\frac{30}{-6}$  c.  $\frac{-30}{6}$  d.  $\frac{-30}{-6}$

**Solution** a.  $\frac{30}{6} = 30 \div 6 = 30 \cdot \frac{1}{6} = 5$  b.  $\frac{30}{-6} = 30 \div (-6) = 30\left(-\frac{1}{6}\right) = -5$

c.  $\frac{-30}{6} = -30 \div 6 = -30 \cdot \frac{1}{6} = -5$  d.  $\frac{-30}{-6} = -30 \div (-6) = -30\left(-\frac{1}{6}\right) = 5$

### Simplify.

1.  $42 \div 14$  3.  $-24 \div (-6)$  4.  $-32 \div (-8)$

5.  $\frac{-144}{12}$  6.  $\frac{96}{-16}$  7.  $\frac{-100}{-5}$  8.  $\frac{-75}{-3}$

**Example 2** Simplify: a.  $8 \div \left(-\frac{4}{5}\right)$  b.  $\frac{-4}{-\frac{1}{2}}$

**Solution** a.  $8 \div \left(-\frac{4}{5}\right) = 8\left(-\frac{5}{4}\right) = -10$  b.  $\frac{-4}{-\frac{1}{2}} = (-4) \div \left(-\frac{1}{2}\right) = (-4)(-2) = 8$

### Simplify.

9.  $6 \div \left(-\frac{1}{3}\right)$  10.  $12 \div \left(-\frac{1}{4}\right)$  11.  $0 \div \frac{5}{6}$  12.  $-8 \div \left(-\frac{1}{2}\right)$  13.  $0 \div \left(-\frac{2}{7}\right)$

14.  $\frac{-12}{-\frac{1}{4}}$  15.  $\frac{8}{-\frac{1}{2}}$  16.  $\frac{-20}{\frac{1}{5}}$  17.  $\frac{0}{\frac{1}{9}}$  18.  $\frac{-8}{-\frac{1}{8}}$

## 2-9 Dividing Real Numbers (continued)

**Example 3** Simplify: a.  $\frac{32x}{-8}$  b.  $\frac{w}{12} \cdot 12$

**Solution** a.  $\frac{32x}{-8} = 32x\left(-\frac{1}{8}\right)$  Multiply by the reciprocal of  $-8$ .

$$= 32\left(-\frac{1}{8}\right)x \quad \text{Regroup the factors.}$$

$$= -4x \quad \text{Simplify.}$$

b.  $\frac{w}{12} \cdot 12 = w \cdot \frac{1}{12} \cdot 12$   
 $= w \cdot 1$   
 $= w$

### Simplify.

19.  $\frac{-18x}{3} - 6x$  20.  $\frac{-42x}{6} - 7x$  21.  $\frac{50x}{-10} - 5x$  22.  $\frac{-36x}{-6} 6x$

23.  $5 \cdot \frac{x}{5} x$  24.  $\frac{-w}{8} \cdot 8 - w$  25.  $(-6)\left(\frac{-y}{2}\right) 3y$  26.  $(-10)\left(\frac{-x}{-2}\right) 5x$

27.  $\frac{144b}{12} 12b$  28.  $\frac{121b}{-11} - 11b$  29.  $\frac{-48x}{6} - 8x$  30.  $\frac{-108x}{-36} 3x$

**Example 4** Find the average of 14,  $-2$ ,  $-8$ ,  $-12$ .

**Solution** Find the sum of the numbers and divide by the number of numbers.

$$\frac{14 + (-2) + (-8) + (-12)}{4} = \frac{-8}{4} = -2$$

Find the average of the given numbers.

31.  $-12, 5, -10, -7$  32.  $15, -21, -8, 6$  33.  $13, -5, -16, -4$

34.  $23, -13, -18, 20$  35.  $11, -15, -22, 16, 0$  36.  $23, -12, -17, 21, 5$

## Mixed Review Exercises

Solve if  $x \in \{0, 1, 2, 3, 4, 5, 6\}$ .

1.  $x + 5 = 7$  {2}

2.  $\frac{1}{2}x = 3$  {6}

3.  $x - 1 = 4$  {5}

4.  $3x = 9$  {3}

5.  $3x + 1 = 7$  {2}

6.  $x \div 3 = 1$  {3}

Solve over the domain  $\{0, 1, 2, 3, 4, 5\}$ .

7.  $\frac{1}{3}n = 1$  {3}

8.  $3y - 1 = 14$  {5}

9.  $x + 2 = 6$  {4}

10.  $2x = 2$  {1}

11.  $x \cdot x = 4$  {2}

12.  $3n = n \cdot 3$  {0, 1, 2, 3, 4, 5}