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# 10-5 Absolute Value in Open Sentences

**Objective:** To solve equations and inequalities involving absolute value.

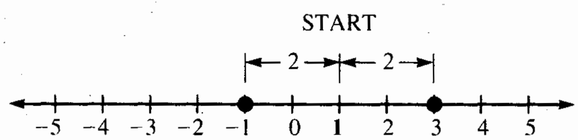
### Symbols

$|a - b| = |b - a|$ (The distance between $a$ and $b$ on a number line.)

$|a + b| = |a - (-b)|$ (The distance between $a$ and the opposite of $b$ on a number line.)

***Example 1*** Solve $|x - 1| = 2$.

***Solution 1*** To satisfy $|x - 1| = 2$, $x$ must be a number whose distance from 1 is 2. To arrive at $x$, start at 1 and move 2 units in either direction on the number line.




You arrive at 3 and $-1$ as the values of $x$. The solution set is $\{-1, 3\}$.

***Solution 2*** Note that $|x - 1| = 2$ is equivalent to the disjunction:

$$x - 1 = -2 \quad \text{or} \quad x - 1 = 2$$
$$x = -1 \quad \text{or} \quad x = 3$$

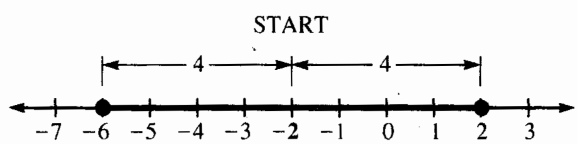
The solution set is $\{-1, 3\}$.

**Solve.**

1. $|m - 3| = 5$
2. $|k + 4| = 1$
3. $|2 + x| = 4$
4. $|7 - x| = 3$
5. $|x - 5| = 2$
6. $|6 - x| = 7$

***Example 2*** Solve $|x + 2| \le 4$ and graph its solution set.

***Solution 1*** $|x + 2| \le 4$ is equivalent to $|x - (-2)| \le 4$. The distance between $x$ and $-2$ must be no more than 4.




Starting at $-2$, numbers within 4 units in either direction will satisfy $|x + 2| \le 4$. Thus, $|x + 2| \le 4$ is equivalent to $-6 \le x \le 2$.

The solution set is $\{-6, 2$, and the real numbers between $-6$ and $2\}$. The graph is shown above.

***Solution 2*** $|x + 2| \le 4$ is equivalent to the conjunction:

$$-4 \le x + 2 \le 4$$
$$-4 - \mathbf{2} \le x + 2 - \mathbf{2} \le 4 - \mathbf{2}$$
$$-6 \le x \le 2$$

The solution set and graph are as in Solution 1.

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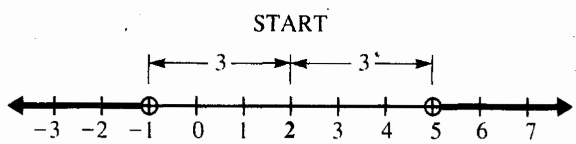
## 10–5 Absolute Value in Open Sentences *(continued)*

***Example 3*** Solve $|t - 2| > 3$ and graph its solution set.

***Solution 1*** The distance between $t$ and 2 must be greater than 3, as shown below:




Therefore, $|t - 2| > 3$ is equivalent to the disjunction

$$t < -1 \quad \text{or} \quad t > 5.$$

The solution set is {the real numbers less than −1 or greater than 5}. The graph is shown above.

***Solution 2*** $|t - 2| > 3$ is equivalent to the disjunction:

$$t - 2 < -3 \quad \text{or} \quad t - 2 > 3$$
$$t < -1 \quad \text{or} \quad t > 5$$

The solution set and graph are as in Solution 1.

**Solve each open sentence and graph its solution set.**

**7.** $|x| > 2$ **8.** $|x| \le 2$ **9.** $|x| \ge 1$

**10.** $|x - 2| < 1$ **11.** $|x - 2| > 2$ **12.** $|x + 2| \ge 1$

**13.** $|x - 1| \le 1$ **14.** $|x - 1| \ge 1$ **15.** $|x + 3| \le 1$

**16.** $|x + 1| > 1$ **17.** $|x - 3| \ge 4$ **18.** $|x - 4| < 2$

**19.** $|3 - y| \ge 5$ **20.** $|2 - x| \ge 1$ **21.** $|-2 - x| \le 4$

## Mixed Review Exercises

**Solve each inequality and graph its solution set.**

**1.** $x - 3 < 5$ **2.** $\frac{x}{3} + 6 < 2$ **3.** $8 < 4(3 + m)$

**4.** $-1 < x + 4 < 1$ **5.** $h + 2 \le 8$ or $h - 3 > 2$ **6.** $2 \le -x \le 8$

**Simplify.**

**7.** $\frac{15x}{4y^2} \div 3xy$ **8.** $\left(\frac{4a}{b}\right) \cdot \left(\frac{5b}{2a}\right)^2$

**9.** $\frac{x + 2}{3} - \frac{2x}{6}$ **10.** $2x + \frac{x}{5}$