NAME ______________________________ DATE ______________

3–8 Proof in Algebra

Objective: To prove statements in algebra.

Vocabulary

Theorem A statement that is shown to be true using a logically developed argument.

Proof Logical reasoning that uses given facts, definitions, properties, and other already proved theorems to show that a theorem is true. You may refer to the Chapter Summary, on page 88 of your textbook, for listings of properties and theorems that you can use as reasons in your proofs.

Example 1 *Prove:* If $a + b = 0$, then $b = -a$.

Proof

Statements	Reasons
1. $a + b = 0$	1. Given
2. $-a + (a + b) = -a + 0$	2. Addition property of equality
3. $(-a + a) + b = -a + 0$	3. Associative property of addition
4. $0 + b = -a + 0$	4. Property of opposites
5. $b = -a$	5. Identity property of addition

Example 2 *Prove:* $a \cdot 0 = 0$

Proof

Statements	Reasons
1. $0 = 0 + 0$	1. Identity property of addition
2. $a \cdot 0 = a(0 + 0)$	2. Multiplication property of equality
3. $a \cdot 0 = a \cdot 0 + a \cdot 0$	3. Distributive property
4. $a \cdot 0 = a \cdot 0 + 0$	4. Identity property of addition
5. $a \cdot 0 + a \cdot 0 = a \cdot 0 + 0$	5. Substitution principle
6. $a \cdot 0 = 0$	6. Subtraction property of equality

Write the missing reasons. Assume that each variable represents any real number.

1. *Prove:* For all real numbers a and b, $-b + (a + b) = a$.

Proof:

Statements	Reasons
1. $-b + (a + b) = -b + (b + a)$	1. ?
2. $= (-b + b) + a$	2. ?
3. $= 0 + a$	3. ?
4. $= a$	4. ?

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3–8 Proof in Algebra *(continued)*

Write the missing reasons. Assume that each variable represents any real number.

2. *Prove:* For all real numbers a and b, $-a(b + c) = -ab - ac$.

Proof:

Statements	Reasons
1. $-a(b + c) = (-a)b + (-a)c$	1. ?
2. $= -ab + (-ac)$	2. ?
3. $= -ab - ac$	3. ?

3. *Prove:* For all real numbers a and b, $b \neq 0$, $(ab) \div b = a$.

Proof:

Statements	Reasons
1. $(ab) \div b = (ab) \cdot \frac{1}{b}$	1. ?
2. $= a\left(b \cdot \frac{1}{b}\right)$	2. ?
3. $= a \cdot 1$	3. ?
4. $= a$	4. ?

4. *Prove:* For all real numbers a and b, $b \neq 0$, $\frac{a + b}{b} = 1 + \frac{a}{b}$.

Proof:

Statements	Reasons
1. $\frac{a + b}{b} = (a + b) \cdot \frac{1}{b}$	1. ?
2. $= (a) \cdot \frac{1}{b} + (b) \cdot \frac{1}{b}$	2. ?
3. $= (a) \cdot \frac{1}{b} + 1$	3. ?
4. $= \frac{a}{b} + 1$	4. ?
5. $= 1 + \frac{a}{b}$	5. ?

Mixed Review Exercises

Simplify.

1. $8(10 - 3) \div 4 + 2$

2. $-3(-12 + 4)$

3. $-6x - x + 9x$

4. $-\frac{1}{4}(8 + 4a)$

5. $\frac{1}{3}(3b + 9c)$

6. $12 \div \frac{1}{3}$

Evaluate if $a = 4$, $b = 3$, $c = 2$, and $x = 12$.

7. $a(x - b)$

8. $2|b - x|$

9. $x - |c - a|$

10. $\frac{x + 2a}{|1 - b|c}$

11. $\frac{4a + x + 7}{b + c}$

12. $\frac{1}{3}(x - b) + a$