NAME ________________________________ DATE ________________

5-9 Factoring Pattern for $ax^2 + bx + c$

Objective: To factor general quadratic trinomials with integral coefficients.

Patterns

Factoring pattern for $ax^2 + bx + c$: $(px + r)(qx + s)$.

Example 1 Factor $2x^2 - 3x - 9$.

Solution

Clue 1 Because the trinomial has a negative constant term, one of r and s will be negative and the other will be positive.

Clue 2 You can list the possible factors of the quadratic term, $2x^2$, and the possible factors of the constant term, -9.

Factors of $2x^2$	Factors of -9	
$2x, x$	$1, -9$	$-1, 9$
	$3, -3$	$-3, 3$
	$9, -1$	$-9, 1$

Make a chart to test the possibilities to see which produces the correct linear term, $-3x$.

Since $(2x + 3)(x - 3)$ gives the correct linear term, $2x^2 - 3x - 9 = (2x + 3)(x - 3)$.

Possible factors	Linear Term
$(2x + 1)(x - 9)$	$(-18 + 1)x = -17x$
$(2x + 3)(x - 3)$	$(-6 + 3)x = -3x$ ←
$(2x + 9)(x - 1)$	$(-2 + 9)x = 7x$
$(2x - 1)(x + 9)$	$(18 - 1)x = 17x$
$(2x - 3)(x + 3)$	$(6 - 3)x = 3x$
$(2x - 9)(x + 1)$	$(2 - 9)x = -7x$

Example 2 Factor $10x^2 - 11x + 3$.

Solution

Clue 1 Because the trinomial has a positive constant term and a negative linear term, both r and s will be negative.

Clue 2 List the factors of the quadratic term, $10x^2$, and the negative factors of the constant term, 3.

Factors of $10x^2$	Factors of 3
$x, 10x$	$-3, -1$
$2x, 5x$	$-1, -3$

Test the possibilities to see which produces $-11x$. Since $(2x - 1)(5x - 3)$ gives the correct linear term, $10x^2 - 11x + 3 = (2x - 1)(5x - 3)$.

Possible factors	Linear term
$(x - 3)(10x - 1)$	$(-1 - 30)x = -31x$
$(x - 1)(10x - 3)$	$(-3 - 10)x = -13x$
$(2x - 3)(5x - 1)$	$(-2 - 15)x = -17x$
$(2x - 1)(5x - 3)$	$(-6 - 5)x = -11x$ ←

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

1. $2x^2 + 5x + 2$
2. $2n^2 - 7n + 3$
3. $5y^2 - 9y - 2$
4. $3a^2 + 7a + 2$
5. $4y^2 - 5y + 1$
6. $2a^2 + 11a + 5$
7. $5a^2 - 11a + 2$
8. $7y^2 - 9y + 2$

5–9 Factoring Pattern for $ax^2 + bx + c$ *(continued)*

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

9. $2k^2 - 5k - 1$ **10.** $12k^2 - 8k + 1$ **11.** $4x^2 + 17x - 15$ **12.** $2a^2 + 7a + 5$

13. $8y^2 + 6y - 9$ **14.** $9x^2 + 3x - 2$ **15.** $7k^2 - 11k - 6$ **16.** $4u^2 - 8u - 5$

Example 3 Factor $5 - 7x - 6x^2$.

Solution

$5 - 7x - 6x^2 = -6x^2 - 7x + 5$ Arrange the terms by decreasing degree.
$= (-1)(6x^2 + 7x - 5)$ Factor -1 from each term.
$= (-1)(2x - 1)(3x + 5)$ Factor the resulting trinomial.
$= -(2x - 1)(3x + 5)$

Note: If you factor $5 - 7x - 6x^2$ directly, you will get $(5 + 3x)(1 - 2x)$. Since $(1 - 2x) = -(2x - 1)$, the two answers are equivalent.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

17. $10 - 9y - 2y^2$ **18.** $10 - x - 3x^2$ **19.** $3 - x - 10x^2$

20. $3 - 7x - 6x^2$ **21.** $10 - u - 2u^2$ **22.** $5 + 8x - 4x^2$

Example 4 Factor $5a^2 + 2ab - 7b^2$.

Solution

$5a^2 + 2ab - 7b^2 = (a \quad)(5a \quad)$ Write the factors of $5a^2$.
$= (a - ?)(5a + ?)$ Test possibilities.
$= (a - b)(5a + 7b)$

Note: If you write $(a + ?)(5a - ?)$ as the second step, you will not find a combination of factors that produces the desired linear term.

Factor. Check by multiplying the factors.

23. $x^2 - xy - 20y^2$ **24.** $4a^2 - 4ab - 3b^2$ **25.** $3a^2 - 5ab - 12b^2$

26. $5a^2 + 2ab - 7b^2$ **27.** $2x^2 - xy - 3y^2$ **28.** $8y^2 - 6yz - 9z^2$

Mixed Review Exercises

Factor.

1. $x^2 - 196$ **2.** $x^2 - 7x + 12$ **3.** $r^2 - 5r - 36$

4. $c^2 - 10c + 25$ **5.** $9y^2 - 121x^2$ **6.** $4a^2 - 25$

7. $y^2 + 13y + 36$ **8.** $p^2 + 14p + 49$ **9.** $9y^2 + 12y + 4$

10. $m^2 - m - 56$ **11.** $n^2 + 13n + 36$ **12.** $b^2 - 3b - 54$