

11-4 Irrational Square Roots

Objective: To simplify radicals and to find decimal approximations of irrational square roots.

Vocabulary

Irrational numbers Real numbers that can't be expressed in the form $\frac{a}{b}$,

where a and b are integers. Their exact values can't be expressed as either terminating or repeating decimals.

Property

Property of Completeness Every decimal represents a real number, and every real number can be represented by a decimal.

Example 1 Simplify: a. $\sqrt{256}$ b. $\sqrt{50}$ c. $2\sqrt{80}$ d. $\sqrt{704}$

Solution a. $\sqrt{256} = \sqrt{4 \cdot 64}$ Factor within the radical sign.
 $= \sqrt{4} \cdot \sqrt{64}$ Use the product property of square roots.
 $= 2 \cdot 8$ Simplify.
 $= 16$

b. $\sqrt{50} = \sqrt{25 \cdot 2}$
 $= \sqrt{25} \cdot \sqrt{2}$
 $= 5\sqrt{2}$

c. $2\sqrt{80} = 2\sqrt{16 \cdot 5}$
 $= 2 \cdot 4\sqrt{5}$
 $= 8\sqrt{5}$

d. $\sqrt{704} = \sqrt{64 \cdot 11}$
 $= 8\sqrt{11}$

Simplify.

- | | | | | |
|-------------------|------------------|-------------------|-------------------|--------------------|
| 1. $\sqrt{27}$ | 2. $\sqrt{20}$ | 3. $\sqrt{72}$ | 4. $\sqrt{32}$ | 5. $\sqrt{48}$ |
| 6. $\sqrt{45}$ | 7. $\sqrt{196}$ | 8. $\sqrt{80}$ | 9. $2\sqrt{63}$ | 10. $4\sqrt{98}$ |
| 11. $7\sqrt{28}$ | 12. $4\sqrt{40}$ | 13. $\sqrt{441}$ | 14. $\sqrt{289}$ | 15. $3\sqrt{50}$ |
| 16. $12\sqrt{50}$ | 17. $\sqrt{729}$ | 18. $\sqrt{432}$ | 19. $8\sqrt{75}$ | 20. $2\sqrt{90}$ |
| 21. $\sqrt{147}$ | 22. $\sqrt{288}$ | 23. $\sqrt{4225}$ | 24. $5\sqrt{800}$ | 25. $5\sqrt{1025}$ |

11-4 Irrational Square Roots (continued)

Example 2 Approximate $\sqrt{396}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$$\begin{aligned}\sqrt{396} &= \sqrt{2^2 \cdot 3^2 \cdot 11} \\ &= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{11} \\ &= 6\sqrt{11}\end{aligned}$$

From the table: $\sqrt{11} \approx 3.317$
 $6\sqrt{11} \approx 6(3.317) \approx 19.902$

Therefore $\sqrt{396} \approx 19.90$.

Example 3 Approximate $\sqrt{0.6}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$$\sqrt{0.6} = \frac{\sqrt{60}}{\sqrt{100}} = \frac{\sqrt{60}}{10} \approx \frac{7.746}{10} = 0.7746$$

Therefore $\sqrt{0.6} \approx 0.77$.

In Exercises 26–37, use your calculator or the table at the back of the book. Approximate each square root to the nearest tenth.

26. $\sqrt{600}$

27. $\sqrt{200}$

28. $-\sqrt{800}$

29. $-\sqrt{500}$

30. $-\sqrt{2700}$

31. $-\sqrt{2200}$

32. $\pm\sqrt{6600}$

33. $\pm\sqrt{4800}$

Approximate each square root to the nearest hundredth.

34. $\sqrt{56}$

35. $\sqrt{32}$

36. $-\sqrt{0.7}$

37. $-\sqrt{0.2}$

Mixed Review Exercises

Find the indicated square roots.

1. $\sqrt{100}$

2. $-\sqrt{144}$

3. $\sqrt{\frac{9}{25}}$

4. $-\sqrt{\frac{36}{121}}$

5. $\sqrt{154^2}$

6. $\sqrt{\left(\frac{2}{5}\right)^2}$

Simplify.

7. $(13x)^2$

8. $(2y^3z^6)^2$

9. $(x + 2y)^2$

10. $[10(a + 1)]^2$

11. $(9a^3b^7c)^2$

12. $(4z^2 + 3y^3)(4z^2 - 3y^3)$