3 Solving Equations and Problems

3-1 *Transforming Equations: Addition and Subtraction*

Objective: To solve equations using addition and subtraction.

Properties

Addition Property of Equality If the same number is added to equal numbers, the sums are equal.

Subtraction Property of Equality If the same number is subtracted from equal numbers, the differences are equal.

Vocabulary

Equivalent equations Equations that have the same solution set over a given domain.

Transformations Operations on an equation that produce a simpler equivalent equation.

By substitution You can substitute an equivalent expression for any expression in an equation. You do this when you simplify an expression in an equation.

By addition You can add the same number to each side of an equation.

By subtraction You can subtract the same number from each side of an equation.

CAUTION To check your work, you should check that each solution of the final equation satisfies the *original equation*.

Example 1 Solve $x - 6 = 11$.

Solution

$x - 6 = 11$
$x - 6 + 6 = 11 + 6$ — To get x alone on one side, add 6 to each side and then simplify.
$x = 17$

Check: $x - 6 = 11$ ← Original equation.
$17 - 6 \stackrel{?}{=} 11$ — Substitute 17 for x.
$11 = 11 \checkmark$

The solution set is $\{17\}$.

Solve.

1. $a - 9 = 11$ **{20}**
2. $b - 5 = 13$ **{18}**
3. $x - 20 = -19$ **{1}**
4. $d - 14 = 5$ **{19}**
5. $x - 15 = 0$ **{15}**
6. $v - 27 = -54$ **{−27}**
7. $x - 6 = 27$ **{33}**
8. $q - 7 = 11$ **{18}**
9. $q - 9 = -16$ **{−7}**

19

Example 2 Solve $-9 = n + 11$.

Solution

$-9 = n + 11$
$-9 - 11 = n + 11 - 11$ — To get n alone on one side, subtract 11 from each side.
$-20 = n$ — Simplify.

Check: $-9 = n + 11$ ← Original equation
$-9 \stackrel{?}{=} -20 + 11$ — Substitute -20 for n.
$-9 = -9 \checkmark$ — The solution set is $\{-20\}$.

Solve.

14. **{−40}**
15. **{−75}**

10. $-6 = m + 6$ **{−12}**
11. $21 = x + 15$ **{6}**
12. $-26 + m = 24$ **{50}**
13. $-37 + n = 63$ **{100}**
14. $p + 18 = -22$
15. $a + 60 = -15$
16. $14 + t = 0$ **{−14}**
17. $29 = y - 12$ **{41}**
18. $35 = x + 16$ **{19}**
19. $-4 = u - 6$ **{2}**
20. $22 = y + 3$ **{19}**
21. $c - 8 = -10$ **{−2}**
22. $x + 1.5 = 6.8$ **{5.3}**
23. $-1 + a = 0.5$ **{1.5}**
24. $3.9 = y - 1.4$ **{5.3}**
25. $7.5 = w - 2.5$ **{10}**

Example 3 Solve $-x + 5 = 4$.

Solution

$-x + 5 = 4$
$-x + 5 - 5 = 4 - 5$ — To get x alone on one side, subtract 5 from each side and simplify.
$-x = -1$
$x = 1$ — If the opposite of a number is -1, the number must be 1.

Check: $-x + 5 = 4$ ← Original equation
$-1 + 5 \stackrel{?}{=} 4$ — Substitute 1 for x.
$4 = 4 \checkmark$ — The solution set is $\{1\}$.

Solve.

26. $-x + 3 = 5$ **{−2}**
27. $-y + 7 = 17$ **{−10}**
28. $12 - x = 18$ **{−6}**
29. $7 - y = 11$ **{−4}**
30. $9 = -x + 16$ **{7}**
31. $13 = 22 - y$ **{9}**
32. $-5 - y = 7$ **{−12}**
33. $10 = -12 - e$ **{−22}**
34. $15 = -y + 10$ **{−5}**

Mixed Review Exercises

Evaluate if $a = 3$, $b = -6$, $c = -4$, and $d = 2$.

1. $a - |b - c|$ **1**
2. $(|c| - d) - (|b| - a)$ **−1**
3. $3|c| - (-b)$ **6**
4. $\frac{a - 2b}{a + d}$ **3**
5. $\frac{3b + c - d}{ad}$ **−4**
6. $\frac{2ab}{c + d}$ **18**

Simplify.

7. $(-3)(-4)(8)$ **96**
8. $(-7 \cdot 16) + (-7 \cdot 24)$ **−280**
9. $252 \div (-36)$ **−7**

20

NAME ______________________ DATE ______________

3–2 Transforming Equations: Multiplication and Division

Objective: To solve equations using multiplication or division.

Properties

Multiplication Property of Equality If equal numbers are multiplied by the same number, the products are equal.

Division Property of Equality If equal numbers are divided by the same *nonzero* number, the quotients are equal.

Transformations

By multiplication You can multiply each side of an equation by the same *nonzero* real number.

By division You can divide each side of an equation by the same *nonzero* real number.

CAUTION 1 When you transform an equation, never multiply or divide by zero.

CAUTION 2 When you multiply or divide by a negative number, be careful with the sign of your answer.

Example 1 Solve $4x = 128$.

Solution

$$\frac{4x}{4} = \frac{128}{4}$$
$$x = 32$$

To get x alone on one side, divide each side by 4 (or multiply by $\frac{1}{4}$, the reciprocal of 4).

Check: $4x = 128$

$4(32) \stackrel{?}{=} 128$

$128 = 128 \checkmark$

The solution set is $\{32\}$.

Solve.

1. $7m = 140$ **{20}**
2. $12n = 240$ **{20}**
3. $-8x = 96$ **{−12}**
4. $-11f = -143$ **{13}**
5. $-720 = -24g$ **{30}**
6. $330 = -15u$ **{−22}**
7. $108 = -9x$ **{−12}**
8. $45k = -270$ **{−6}**
9. $26n = -520$ **{−20}**

Example 2 Solve $12 = -\frac{3}{4}n$.

Solution

$$-\frac{4}{3}(12) = -\frac{4}{3}\left(-\frac{3}{4}n\right)$$
$$-16 = n$$

To get n alone on one side, multiply each side by $-\frac{4}{3}$, the reciprocal of $-\frac{3}{4}$.

Check: $12 = -\frac{3}{4}n$

$12 \stackrel{?}{=} -\frac{3}{4}(-16)$

$12 = 12 \checkmark$

The solution set is $\{-16\}$.

NAME ______________________ DATE ______________

Solve.

10. $\frac{2}{3}m = 6$ **{9}**
11. $\frac{3}{5}y = -15$ **{−25}**
12. $-\frac{5}{8}x = 40$ **{−64}**
13. $-\frac{4}{5}y = -20$ **{25}**
14. $\frac{2}{5}d = -40$ **{−100}**
15. $\frac{3}{4}g = -24$ **{−32}**
16. $\frac{7}{8}y = -56$ **{−64}**
17. $-\frac{7}{10}e = 140$ **{−200}**
18. $-\frac{2}{7}n = -28$ **{98}**

Example 3 Solve: **a.** $\frac{x}{2} = -6$ **b.** $\frac{1}{2}n = 3\frac{1}{2}$

Solution

a.

$$2\left(\frac{x}{2}\right) = 2(-6)$$
$$x = -12$$

Check: $\frac{x}{2} = -6$

$\frac{-12}{2} \stackrel{?}{=} -6$

$-6 = -6 \checkmark$

The solution set is $\{-12\}$.

b.

$$\frac{1}{2}n = 3\frac{1}{2}$$
$$\frac{1}{2}n = \frac{7}{2}$$
$$2\left(\frac{1}{2}\right)n = 2\left(\frac{7}{2}\right)$$
$$n = 7$$

Check: $\frac{1}{2}n = 3\frac{1}{2}$

$\frac{1}{2}(7) \stackrel{?}{=} \frac{7}{2}$

$\frac{7}{2} = \frac{7}{2} \checkmark$

The solution set is $\{7\}$.

Solve.

19. $\frac{c}{6} = -24$ **{−144}**
20. $\frac{y}{5} = -25$ **{−125}**
21. $-\frac{u}{12} = 12$ **{−144}**
22. $-\frac{x}{3} = 15$ **{−45}**
23. $-28 = \frac{c}{7}$ **{−196}**
24. $-\frac{1}{4}x = 2\frac{1}{4}$ **{−9}**
25. $\frac{1}{5}f = 3\frac{1}{5}$ **{16}**
26. $\frac{1}{2}b = 2\frac{1}{2}$ **{5}**
27. $-\frac{1}{3}y = 3\frac{2}{3}$ **{−11}**

Mixed Review Exercises

Evaluate if $a = -2$, $b = 3$, and $c = -6$.

1. $6b - 2a$ **22**
2. $(2b - 5c)a$ **−72**
3. $|c| + |a| - b$ **5**
4. $|b| - |a + c|$ **−5**
5. $\frac{-(7ab)}{c}$ **−7**
6. $\frac{8 + a}{c}$ **−1**

Simplify.

7. $6a + 5 + 7a$ **13a + 5**
8. $7n - 6 + 6$ **7n**
9. $9p - p + 3$ **8p + 3**
10. $-3(m + 4)$ **−3m − 12**
11. $(x + 5)6$ **6x + 30**
12. $2(3y - 4)$ **6y − 8**

NAME ______ DATE ______

3-3 Using Several Transformations

Objective: To solve equations by using more than one transformation.

Vocabulary

Inverse operations Operations that "undo" each other. For example, multiplication and division are inverse operations. Likewise, addition and subtraction are inverse operations.

Tips for solving an equation in which the variable is on one side.

1. Simplify each side of the equation as needed.
2. Use inverse operations to "undo" the operations in the equation.

Example 1 Solve $3n - 7 = 8$.

Solution

	Use inverse operations:
$3n - 7 + 7 = 8 + 7$	To undo the subtraction of 7 from $3n$, add 7 to each side.
$3n = 15$	
$\frac{3n}{3} = \frac{15}{3}$	To undo the multiplication of n by 3, divide each side by 3.
$n = 5$	The solution set is $\{5\}$.

Example 2 Solve $\frac{1}{2}x + 1 = 7$.

Solution

	Use inverse operations:
$\frac{1}{2}x + 1 - 1 = 7 - 1$	Subtract 1 from each side.
$\frac{1}{2}x = 6$	
$2\left(\frac{1}{2}x\right) = 6 \cdot 2$	Multiply each side by 2, the reciprocal of $\frac{1}{2}$.
$x = 12$	The solution set is $\{12\}$.

Solve.

1. $2y + 1 = 15$ **{7}**
2. $2x - 7 = 13$ **{10}**
3. $26 = 5y + 1$ **{5}**
4. $58 = 3y - 2$ **{20}**
5. $-11 + 4y = 25$ **{9}**
6. $13 + 6y = -23$ **{−6}**
7. $\frac{1}{2}x - 3 = 5$ **{16}**
8. $\frac{1}{3}x + 5 = 7$ **{6}**
9. $3 = \frac{1}{4}x - 1$ **{16}**
10. $6 = \frac{1}{5}x + 2$ **{20}**
11. $\frac{x}{2} + 7 = 1$ **{−12}**
12. $\frac{x}{5} - 2 = 4$ **{30}**

Example 3 Solve $\frac{x-2}{3} = 4$.

Solution

$3\left(\frac{x-2}{3}\right) = 3 \cdot 4$	Multiply each side by 3.
$x - 2 = 12$	
$x - 2 + 2 = 12 + 2$	Add 2 to each side.
$x = 14$	The solution set is $\{14\}$.

NAME ______ DATE ______

3-3 Using Several Transformations *(continued)*

Solve.

13. $\frac{x-1}{2} = 5$ **{11}**
14. $\frac{3-x}{4} = 2$ **{−5}**
15. $\frac{x-6}{6} = -1$ **{0}**
16. $-3 = \frac{x-1}{5}$ **{−14}**
17. $\frac{2-x}{3} = -4$ **{14}**
18. $-2 = \frac{1-x}{7}$ **{15}**

Example 4 Solve $28 = 9a + 5a$.

Solution

$28 = 9a + 5a$	Combine $9a$ and $5a$.
$28 = 14a$	
$\frac{28}{14} = \frac{14a}{14}$	Divide each side by 14.
$2 = a$	The solution set is $\{2\}$.

Solve.

19. $4w - w = -12$ **{−4}**
20. $20 = 2a + 3a$ **{4}**
21. $y - 4y = -18$ **{6}**
22. $5t + 3t = -16$ **{−2}**
23. $-7v + 3v = -12$ **{3}**
24. $24 = -3n + 9n$ **{4}**

Example 5 Solve $3(y + 2) - 1 = 11$.

Solution

$3(y + 2) - 1 = 11$	Use the distributive property to rewrite the left side.
$3y + 6 - 1 = 11$	
$3y + 5 = 11$	
$3y + 5 - 5 = 11 - 5$	Subtract 5 from each side.
$3y = 6$	
$\frac{3y}{3} = \frac{6}{3}$	Divide each side by 3.
$y = 2$	The solution set is $\{2\}$.

Solve.

25. $2(x - 1) = 16$ **{9}**
26. $3(y - 5) = 12$ **{9}**
27. $20 = 4(x + 3)$ **{2}**
28. $5(n + 2) - 3 = -18$ **{−5}**
29. $6(x - 4) + 5 = 11$ **{5}**
30. $-3 = 7(h - 2) + 11$ **{0}**

Mixed Review Exercises

Solve.

1. $\frac{1}{4}x = -17$ **{−68}**
2. $\frac{x}{6} = \frac{2}{3}$ **{4}**
3. $\frac{1}{4}x = 2\frac{1}{4}$ **{9}**
4. $-4 + x = -1$ **{3}**
5. $x + 7 = 16$ **{9}**
6. $30 = y + 12$ **{18}**
7. $-10 + x = -18$ **{−8}**
8. $24 - x = 26$ **{−2}**
9. $0.5x = -5$ **{−10}**
10. $3.2 = n + 3$ **{0.2}**
11. $0 = 5x$ **{0}**
12. $14y = 280$ **{20}**

22

NAME ______________________ DATE ______________

3–4 *Using Equations to Solve Problems*

Objective: To use the five-step plan to solve word problems.

Example 1 The sum of 25 and twice a number is 93. Find the number.

Solution

Steps 1, 2 Let n = the number. Then $2n$ = twice the number.

Step 3 The sum of 25 and twice a number is 93.

$$25 + 2n = 93$$

Step 4 Solve.

$$25 - 25 + 2n = 93 - 25$$
$$2n = 68$$
$$n = 34$$

Step 5 *Check in the words of the problem:* Is the sum of 25 and twice 34 equal to 93?

$$25 + 2(34) \stackrel{?}{=} 93$$
$$25 + 68 \stackrel{?}{=} 93$$
$$93 = 93 \surd$$

The number is 34.

Solve each problem using the five-step plan to help you.

1. The sum of 17 and twice a number is 87. Find the number. **35**
2. The sum of 8 and three times a number is 128. Find the number. **40**
3. Seven more than twice a number is 175. Find the number. **84**
4. Four less than half a number is 15. Find the number. **38**
5. When one half of a number is decreased by 13, the result is 62. Find the number. **150**
6. Six less than two thirds of a number is 18. Find the number. **36**

Example 2 Find four consecutive even integers whose sum is 44.

Solution

Steps 1, 2 Let n = the first integer. Then $n + 2$ = the second integer, $n + 4$ = the third integer, and $n + 6$ = the fourth integer.

Step 3 The sum of the four consecutive even integers is 44.

$$n + (n + 2) + (n + 4) + (n + 6) = 44$$

Step 4 Solve.

$$4n + 12 = 44$$
$$4n = 32$$

{If you're careful, you can subtract 12 from each side in your head.

$n = 8$ ←the first integer
$n + 2 = 10$ ←the second integer
$n + 4 = 12$ ←the third integer
$n + 6 = 14$ ←the fourth integer

Step 5 *Check:* $8 + 10 + 12 + 14 \stackrel{?}{=} 44$

$$44 = 44 \surd$$

The numbers are 8, 10, 12, and 14.

NAME ______________________ DATE ______________

3–4 *Using Equations to Solve Problems (continued)*

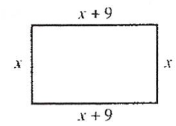
Solve each problem using the five-step plan to help you.

7. Find three consecutive integers whose sum is 138. **45, 46, 47**
8. Find three consecutive odd integers whose sum is 87. **27, 29, 31**
9. Find three consecutive even integers whose sum is 150. **48, 50, 52**
10. Find four consecutive odd integers whose sum is 144. **33, 35, 37, 39**
11. Find five consecutive integers whose sum is 160. **30, 31, 32, 33, 34**
12. Otto has \$140. If he saves \$2.50 per week, how long will it take him to have \$200? **24 weeks**

Example 3 The length of a rectangle is 9 cm more than the width. The perimeter is 78 cm. Find the length and the width.

Solution

Step 1 Draw a diagram to help you understand the problem.



Step 2 Let x = the width. Then $x + 9$ = the length.

Step 3

$$\text{perimeter} = 78$$
$$x + (x + 9) + x + (x + 9) = 78$$

Step 4 Solve.

$$4x + 18 = 78$$
$$4x = 60$$
$$x = 15 \quad \text{and} \quad x + 9 = 24$$

Step 5 *Check:* Is the sum of the lengths of the sides 78 cm?

$$15 + 24 + 15 + 24 \stackrel{?}{=} 78$$
$$78 = 78 \surd$$

The width is 15 cm. The length is 24 cm.

Solve each problem using the five-step plan. Draw a diagram to help you.

13. The length of a rectangle is 11 cm more than the width. The perimeter is 90 cm. Find the length and width of the rectangle. **length, 28 cm; width, 17 cm**
14. The width of a rectangle is 12 cm less than the length. The perimeter is 120 cm. Find the length and width of the rectangle. **length, 36 cm; width, 24 cm**
15. The perimeter of a rectangle is 232 cm and the width is 56 cm. Find the length of the rectangle. **60 cm**
16. The perimeter of a rectangle is 340 cm and the length is 90 cm. Find the width of the rectangle. **80 cm**

Mixed Review Exercises

Solve.

1. $-3 + y = 2$ **{5}**
2. $x - 1.2 = 6$ **{7.2}**
3. $y + 6 = 15$ **{9}**
4. $\frac{2}{3}y = 6$ **{9}**
5. $-15 = \frac{c}{2}$ **{−30}**
6. $-\frac{1}{5}x = 12$ **{−60}**
7. $31 = y - 9$ **{40}**
8. $x - 15 = 16$ **{31}**
9. $0.25y = 8$ **{32}**
10. $3y + 2 = 17$ **{5}**
11. $2x - 3 = 15$ **{9}**
12. $3(a - 1) + 5 = 32$ **{10}**

NAME ______________________ DATE ______________

3–5 Equations with the Variable on Both Sides

Objective: To solve equations with the variable on both sides.

Example 1 Solve $5x = 2x + 15$.

Solution

$5x - 2x = 2x + 15 - 2x$ Subtract $2x$ from each side.

$3x = 15$

$x = 5$

The solution set is $\{5\}$.

Check: $5(5) \stackrel{?}{=} 2(5) + 15$

$25 \stackrel{?}{=} 10 + 15$

$25 = 25 \checkmark$

Example 2 Solve $4x = 30 - x$.

Solution

$4x + x = 30 - x + x$ Add x to each side.

$5x = 30$

$x = 6$ The solution set is $\{6\}$.

Solve.

1. $5n = 3n + 8$ **{4}**
2. $7a = 2a + 30$ **{6}**
3. $y = 20 - 3y$ **{5}**
4. $3b = 80 - 5b$ **{10}**
5. $10n = 36 - 2n$ **{3}**
6. $2x = 20 - 8x$ **{2}**
7. $21a = 56 + 7a$ **{4}**
8. $30 + 6x = 12x$ **{5}**
9. $-9a = -12a - 45$ **{−15}**
10. $33c + 60 = 21c$ **{−5}**
11. $72 - 4n = -22n$ **{−4}**
12. $-11a = -12a - 21$ **{−21}**

Example 3 Solve $\frac{2}{5}x + 12 = x$.

Solution

$\frac{2}{5}x + 12 - \frac{2}{5}x = x - \frac{2}{5}x$ Subtract $\frac{2}{5}x$ from each side.

$12 = \frac{5}{5}x - \frac{2}{5}x$ Rewrite $1x$ as $\frac{5}{5}x$.

$12 = \frac{3}{5}x$

$\frac{5}{3} \cdot \frac{12}{1} = \frac{5}{3}\left(\frac{3}{5}x\right)$ Multiply each side by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$.

$20 = x$ The solution set is $\{20\}$.

Example 4 Solve $\frac{6 + x}{3} = x$.

Solution

$3\left(\frac{6 + x}{3}\right) = 3 \cdot x$ Multiply each side by 3, the reciprocal of $\frac{1}{3}$.

$6 + x = 3x$

$6 + x - x = 3x - x$ Subtract x from each side.

$6 = 2x$

$3 = x$ The solution set is $\{3\}$.

NAME ______________________ DATE ______________

3–5 Equations with the Variable on Both Sides *(continued)*

Solve.

13. $\frac{2}{3}x - 5 = x$ **{−15}**
14. $\frac{3}{4}x - 8 = x$ **{−32}**
15. $x = \frac{1}{2}x + 7$ **{14}**
16. $x = \frac{4}{5}x - 9$ **{−45}**
17. $\frac{x - 2}{3} = x$ **{−1}**
18. $\frac{3 + y}{4} = y$ **{1}**
19. $y = \frac{7 - 2y}{5}$ **{1}**
20. $x = \frac{9 + x}{4}$ **{3}**

Vocabulary

Empty set or null set The set with no members.

Identity An equation that is true for every value of the variable(s).

Symbol ϕ (empty set, or the null set)

CAUTION An equation may have no solution, or it may be satisfied by every real number.

Example 5 Solve: **a.** $5(a - 2) - 3 = 3a + 7 + 2a$ **b.** $\frac{1}{3}(24x - 15) = 8x - 5$

Solution

a. $5a - 10 - 3 = 5a + 7$

$5a - 13 = 5a + 7$

$-13 = 7$ ← **False**

The equation has *no solution.*

b. $8x - 5 = 8x - 5$ ← **Identity**

An identity is true for every value of the variable.

The solution set is {real numbers}.

Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution*.

21. $2(x - 3) = 5x$ **{−2}**
22. $4(y - 5) = 9y$ **{−4}**
23. $3n = 6(3 - n)$ **{2}**
24. $-3m = 5(2 - m)$ **{5}**
25. $2(a - 1) = 2a + 3$ **No solution**
26. $\frac{1}{4}(28x - 8) = 7x - 2$ **Identity**
27. $\frac{1}{3}(3x - 3) + 2 = 2x$ **{1}**
28. $4(a - 1) - 5 = 3a + 7$ **{16}**
29. $3(5 + y) - y = 2y + 15$ **Identity**
30. $4a + 7 + a = 3(a - 1)$ **{−5}**
31. $\frac{3n - 15}{4} = 2n$ **{−3}**
32. $\frac{2n - 9}{2} = n$ **No solution**

Mixed Review Exercises

Simplify.

1. $3 + \left(-\frac{1}{3}\right) + \left(-\frac{5}{3}\right)$ **1**
2. $-2\frac{3}{4} + 1\frac{1}{4}$ **$-1\frac{1}{2}$**
3. $-115 - (-10)$ **−105**
4. $15x + (-3x) - 2$ **12x − 2**
5. $-4y + 5 + 18y + 23$ **14y + 28**
6. $6(-2)(-5)(-4)$ **−240**

Solve.

7. $-2 - x = 5$ **{−7}**
8. $4 + (1 + k) = 2$ **{−3}**
9. $3x = -276$ **{−92}**
10. $\frac{1}{2}x = 3\frac{1}{2}$ **{7}**
11. $\frac{x}{6} = 7$ **{42}**
12. $-10\frac{2}{3} = -\frac{1}{3}x$ **{32}**

3–6 Problem Solving: Using Charts

Objective: To organize the facts of a problem in a chart.

Example 1 Organize the given information in a chart: In game 1 Jesse scored twice as many points as Ramon. In game 2 Jesse scored six fewer points than he did in game 1. In game 2 Ramon scored eight more points than he did in game 1.

Solution

	Game 1 points	Game 2 points
Jesse	$2n$	$2n - 6$
Ramon	n	$n + 8$

Example 2 Solve the problem using the two given facts:
Find the number of Calories in a banana and in a peach.
(1) A banana contains 65 Calories more than a peach.
(2) Ten peaches have 50 fewer Calories than 4 bananas.

Solution

Step 1 The problem asks for the number of Calories in a banana and in a peach.

Step 2 Let p = the number of Calories in a peach.
Then $p + 65$ = the number of Calories in a banana.

	Calories per fruit ×	Number of fruit =	Total Calories
Peach	p	10	$10p$
Banana	$p + 65$	4	$4(65 + p)$

Step 3 Calories in 10 peaches = Calories in 4 bananas − 50
$10p = 4(p + 65) - 50$

Step 4 Solve.
$10p = 4p + 260 - 50$
$6p = 210$
$p = 35$ and $p + 65 = 100$

Step 5 *Check:* (1) 100 Calories is 65 more than 35 Calories. (2) Ten peaches have 10 · 35, or 350, Calories and four bananas have 4 · 100, or 400, Calories. 350 = 400 − 50 √
There are 35 Calories in a peach and 100 Calories in a banana.

Solve each problem using the two given facts. If a chart is given, complete the chart to help you solve the problem.

1. Find the number of full 8 hour shifts that Cleo worked last month. **18 eight hour shifts**
(1) He worked three times as many 8 hour shifts as 6 hour shifts.
(2) He worked a total of 180 hours.

	Hours per Shift ×	No. of Shifts =	Total hours worked
6 h shift	? **6**	x	? **6x**
8 h shift	? **8**	? **3x**	? **24x**

3–6 Problem Solving: Using Charts *(continued)*

2. Find the total weight of the boxes of cheddar cheese in a shipment of 3 lb boxes of cheddar cheese and 2 lb boxes of Swiss cheese. **330 lb**
(1) There were 20 fewer 2 lb boxes of Swiss cheese than 3 lb boxes of cheddar cheese.
(2) The total weight of the shipment was 510 lb.

	Weight per box ×	Number of boxes =	Total weight	
Cheddar	? **3**	x	?	**3x**
Swiss	? **2**	? **x − 20**	?	**2(x − 20)**

3. Find the number of 20-ride tickets sold. **20 twenty-ride tickets**
(1) Twenty times as many 8-ride tickets as 20-ride tickets were sold.
(2) The total number of tickets represented 3600 rides.

	Rides per ticket ×	Number of tickets sold =	Total rides
20-ride tickets	? **20**	n	? **20n**
8-ride tickets	? **8**	? **20n**	? **160n**

4. Find the amount of time Maurice spent taking bowling lessons. **24 h**
(1) He took three times as many 2 h bowling lessons as he did 1 h tennis lessons.
(2) He spent a total of 28 h taking bowling lessons and tennis lessons.

	Hours per lesson ×	Number of lessons =	Total time
Bowling	? **2**	? **3x**	? **6x**
Tennis	? **1**	? **x**	? **x**

5. Find the number of Calories in a grapefruit and an orange. **grapefruit, 50 Calories;**
(1) An orange has 15 more Calories than a grapefruit. **orange, 65 Calories**
(2) Twenty oranges and ten grapefruit have 1800 Calories.

6. Find the number of Calories in a honeydew and in a cantaloupe. **honeydew, 90 Calories;**
(1) A honeydew has 20 more Calories than a cantaloupe. **cantaloupe, 70 Calories**
(2) Six honeydew and three cantaloupes have 750 Calories.

Mixed Review Exercises

Solve.

1. $15x = 360$ **{24}**
2. $6 = \frac{3}{5}x$ **{10}**
3. $9z - 5z = 0$ **{0}**
4. $165 = 3x$ **{55}**
5. $6y + 5 = 35$ **{5}**
6. $-10 + 3y = -28$ **{−6}**
7. $4x - x = 21$ **{7}**
8. $3(x + 2) = 4x$ **{6}**
9. $6x - 7 = 2x + 41$ **{12}**
10. $21 - x = 1 - 6x$ **{−4}**
11. $-x = 3x - 52$ **{13}**
12. $5(y + 1) + 3 = 3y - 20$ **{−14}**

3-7 Cost, Income, and Value Problems

Objective: To solve problems involving cost, income, and value.

Formulas

Cost = number of items × price per item

Income = hours worked × wage per hour

Total value = number of items × value per item

Example 1 Tickets for a concert cost $8 for adults and $4 for students. A total of 920 tickets worth $5760 were sold. How many adult tickets were sold?

Solution

Step 1 The problem asks for the number of adult tickets sold.

Step 2 Let x = the number of adult tickets sold.
Then $920 - x$ = the number of student tickets sold.
Make a chart.

	Number	× Price per ticket =	Cost
Adult	x	8	$8x$
Student	$920 - x$	4	$4(920 - x)$

Step 3 The only fact not recorded in the chart is that the total cost of the tickets was $5760. Write an equation using this fact.

Adult ticket cost + Student ticket cost = 5760

$8x + 4(920 - x) = 5760$

Step 4

$$8x + 4(920 - x) = 5760$$
$$8x + 3680 - 4x = 5760$$
$$4x + 3680 = 5760$$
$$4x = 2080$$
$$x = 520 \longleftarrow \text{adult tickets}$$
$$920 - x = 400 \longleftarrow \text{student tickets}$$

Step 5 *Check:* 520 adult tickets at $8 each cost $4160.
400 student tickets at $4 each cost $1600.
The total number of tickets is 520 + 400, or 920. √
The total cost of the tickets is $4160 + $1600, or $5760. √

520 adult tickets were sold.

Solve. Complete the chart first.

1. Forty students bought caps at the baseball game. Plain caps cost $4 each and deluxe ones cost $6 each. If the total bill was $236, how many students bought the deluxe cap? **38 students**

	Number ×	Price =	Cost
Deluxe	d	? **6**	? **6d**
Plain	?	?	?

40 − d **4** **4(40 − d)**

3-7 Cost, Income, and Value Problems *(continued)*

Solve. Complete the chart first.

2. Adult tickets for the game cost $6 each and student tickets cost $3 each. A total of 1040 tickets worth $5400 were sold. How many student tickets were sold? **280 student tickets**

1040 − s **6(1040 − s)**

	Number ×	Price =	Cost
Adult	?	? **6**	?
Student	s	? **3**	? **3s**

3. A collection of 60 dimes and nickels is worth $4.80. How many dimes are there? (*Hint:* In your equation, use 480¢, instead of $4.80.) **36 dimes**

	Number ×	Value of coin =	Total value
Dimes	d	? **10**	? **10d**
Nickels	?	? **5**	?

60 − d **5(60 − d)**

4. A collection of 54 dimes and nickels is worth $3.80. How many nickels are there? (*Hint:* In your equation, use 380¢ instead of $3.80.) **32 nickels**

	Number ×	Value of coin =	Total value
Dimes	?	? **10**	?
Nickels	n	? **5**	? **5n**

54 − n **10(54 − n)**

5. Henry paid $.80 for each bag of peanuts. He sold all but 20 of them for $1.50 and made a profit of $54. How many bags did he buy? (*Hint:* Profit = selling price − buying price.) **120 bags**

	Number ×	Price (¢)	= Cost (¢)
Bought	b	? **80**	? **80b**
Sold	?	? **150**	?

b − 20 **150(b − 20)**

6. Paula paid $4 for each stadium cushion. She sold all but 12 of them for $8 each and made a profit of $400. How many cushions did she buy? (*Hint:* Profit = selling price − buying price.) **124 cushions**

	Number ×	Price (¢)	= Cost (¢)
Bought	b	? **4**	? **4b**
Sold	?	? **8**	?

b − 12 **8(b − 12)**

Solve. Make and complete a chart first.

7. I have three times as many dimes as quarters. If the coins are worth $6.60, how many quarters are there? **12 quarters**

8. I have 12 more nickels than quarters. If the coins are worth $5.40, how many nickels are there? **28 nickels**

Mixed Review Exercises

Simplify.

1. $\frac{30 \div 5 + 2}{13 - 5}$ **1**

2. $24 \div \frac{1}{4}$ **96**

3. $\frac{1}{4}(28y - 12) + 6$ **7y + 3**

4. $(-5)(4)(-2)$ **40**

5. $3(2x + 5) + 4(-x)$ **2x + 15**

6. $6(x - y) + 5(2y + x)$ **11x + 4y**

Evaluate if $a = 2$, $b = 3$, and $c = 8$.

7. $\frac{3a + b}{c - 5}$ **3**

8. $\frac{bc}{2a}$ **6**

9. $2(c - a) - b \div 3$ **11**

NAME ______________________ DATE ____________

3–8 Proof in Algebra

Objective: To prove statements in algebra.

Vocabulary

Theorem A statement that is shown to be true using a logically developed argument.

Proof Logical reasoning that uses given facts, definitions, properties, and other already proved theorems to show that a theorem is true. You may refer to the Chapter Summary, on page 88 of your textbook, for listings of properties and theorems that you can use as reasons in your proofs.

Example 1 *Prove:* If $a + b = 0$, then $b = -a$.

Proof

Statements	Reasons
1. $a + b = 0$	1. Given
2. $-a + (a + b) = -a + 0$	2. Addition property of equality
3. $(-a + a) + b = -a + 0$	3. Associative property of addition
4. $0 + b = -a + 0$	4. Property of opposites
5. $b = -a$	5. Identity property of addition

Example 2 *Prove:* $a \cdot 0 = 0$

Proof

Statements	Reasons
1. $0 = 0 + 0$	1. Identity property of addition
2. $a \cdot 0 = a(0 + 0)$	2. Multiplication property of equality
3. $a \cdot 0 = a \cdot 0 + a \cdot 0$	3. Distributive property
4. $a \cdot 0 = a \cdot 0 + 0$	4. Identity property of addition
5. $a \cdot 0 + a \cdot 0 = a \cdot 0 + 0$	5. Substitution principle
6. $a \cdot 0 = 0$	6. Subtraction property of equality

Write the missing reasons. Assume that each variable represents any real number.

1. *Prove:* For all real numbers a and b, $-b + (a + b) = a$.

Proof:

Statements	Reasons
1. $-b + (a + b) = -b + (b + a)$	1. ? **Comm. prop. of add.**
2. $= (-b + b) + a$	2. ? **Assoc. prop. of add.**
3. $= 0 + a$	3. ? **Prop. of opposites**
4. $= a$	4. ? **Identity prop. of add.**

NAME ______________________ DATE ____________

3–8 Proof in Algebra *(continued)*

Write the missing reasons. Assume that each variable represents any real number.

2. *Prove:* For all real numbers a and b, $-a(b + c) = -ab - ac$.

Proof:

Statements	Reasons
1. $-a(b + c) = (-a)b + (-a)c$	1. ? **Distributive prop.**
2. $= -ab + (-ac)$	2. ? **Prop. of opposites in products**
3. $= -ab - ac$	3. ? **Def. of subtr.**

3. *Prove:* For all real numbers a and b, $b \neq 0$, $(ab) \div b = a$.

Proof:

Statements	Reasons
1. $(ab) \div b = (ab) \cdot \frac{1}{b}$	1. ? **Def. of div.**
2. $= a\left(b \cdot \frac{1}{b}\right)$	2. ? **Assoc. prop. of mult.**
3. $= a \cdot 1$	3. ? **Prop. of reciprocals**
4. $= a$	4. ? **Identity prop. of mult.**

4. *Prove:* For all real numbers a and b, $b \neq 0$, $\frac{a + b}{b} = 1 + \frac{a}{b}$.

Proof:

Statements	Reasons
1. $\frac{a + b}{b} = (a + b) \cdot \frac{1}{b}$	1. ? **Def. of div.**
2. $= (a) \cdot \frac{1}{b} + (b) \cdot \frac{1}{b}$	2. ? **Distributive prop.**
3. $= (a) \cdot \frac{1}{b} + 1$	3. ? **Prop. of reciprocals**
4. $= \frac{a}{b} + 1$	4. ? **Def. of div.**
5. $= 1 + \frac{a}{b}$	5. ? **Comm. prop. of add.**

Mixed Review Exercises

Simplify.

1. $8(10 - 3) \div 4 + 2$ **16**
2. $-3(-12 + 4)$ **24**
3. $-6x - x + 9x$ **2x**
4. $-\frac{1}{4}(8 + 4a)$ **−2 − a**
5. $\frac{1}{3}(3b + 9c)$ **b + 3c**
6. $12 \div \frac{1}{3}$ **36**

Evaluate if $a = 4$, $b = 3$, $c = 2$, and $x = 12$.

7. $a(x - b)$ **36**
8. $2|b - x|$ **18**
9. $x - |c - a|$ **10**
10. $\frac{x + 2a}{|1 - b|c}$ **5**
11. $\frac{4a + x + 7}{b + c}$ **7**
12. $\frac{1}{3}(x - b) + a$ **7**