

2-8 The Reciprocal of a Real Number

Objective: To simplify expressions involving reciprocals.

Vocabulary

Reciprocals Two numbers whose product is 1 are called reciprocals of each other.

For example, 5 and $\frac{1}{5}$ are reciprocals.

Symbols $\frac{1}{a}$ (the reciprocal of a) $-\frac{1}{a}$ (the reciprocal of $-a$)

Properties	Examples
<p>Property of Reciprocals Every <i>nonzero</i> real number a has a reciprocal $\frac{1}{a}$, such that</p> $a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1.$	$3 \cdot \frac{1}{3} = 1 \quad \text{and} \quad \frac{1}{3} \cdot 3 = 1$
<p>Property of the Reciprocal of the Opposite of a Number For every <i>nonzero</i> number a,</p> $\frac{1}{-a} = -\frac{1}{a}.$	$\frac{1}{-3} = -\frac{1}{3}$
<p>Property of the Reciprocal of a Product For all <i>nonzero</i> numbers a and b,</p> $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}.$	$\frac{1}{2 \cdot 3} = \frac{1}{2} \cdot \frac{1}{3}$

CAUTION 0 has no reciprocal; 1 is its own reciprocal; and -1 is its own reciprocal.

Example 1 Simplify: a. $\frac{1}{3} \cdot \frac{1}{-5}$ b. $3y \cdot \frac{1}{3}$ c. $(-6xy)\left(-\frac{1}{2}\right)$

Solution a. $\frac{1}{3} \cdot \frac{1}{-5} = \frac{1}{3(-5)} = \frac{1}{-15} = -\frac{1}{15}$

 b. $3y \cdot \frac{1}{3} = \left(3 \cdot \frac{1}{3}\right)y = 1y = y$

 c. $(-6xy)\left(-\frac{1}{2}\right) = (-6)\left(-\frac{1}{2}\right)(xy) = 3xy$

Simplify each expression.

1. $\frac{1}{3}(-12)$

2. $-\frac{1}{8}(24)$

3. $-50\left(\frac{1}{5}\right)$

4. $-30\left(\frac{1}{3}\right)$

5. $(-20)\left(-\frac{1}{4}\right)$

6. $(-42)\left(-\frac{1}{7}\right)$

7. $-36\left(-\frac{1}{4}\right)\left(\frac{1}{3}\right)$

8. $60\left(-\frac{1}{5}\right)\left(-\frac{1}{12}\right)$

9. $72\left(-\frac{1}{8}\right)\left(-\frac{1}{9}\right)$

10. $-54\left(-\frac{1}{6}\right)\left(-\frac{1}{9}\right)$

11. $\frac{1}{-2}(24)\left(\frac{1}{4}\right)$

12. $-60\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$

2-8. The Reciprocal of a Real Number (continued)

Simplify each expression.

13. $6r\left(-\frac{1}{6}\right)$

14. $32p\left(-\frac{1}{8}\right)$

15. $\frac{1}{x}(8x), x \neq 0$

16. $9x\left(\frac{1}{x}\right), x \neq 0$

17. $21xy\left(\frac{1}{7}\right)$

18. $72ab\left(\frac{1}{9}\right)$

19. $18xy\left(\frac{1}{6}\right)$

20. $(-54xy)\left(\frac{1}{-9}\right)$

21. $15xy\left(\frac{1}{-3}\right)$

22. $6cd\left(\frac{1}{-2}\right)$

23. $(-8pq)\left(\frac{1}{-2}\right)$

24. $(-42ac)\left(\frac{1}{-7}\right)$

Example 2 Simplify: a. $\frac{1}{2}(8m - 4n)$ b. $(-21a - 63b)\left(-\frac{1}{7}\right)$ **Solution** a. $\frac{1}{2}(8m - 4n) = \frac{1}{2}(8m) - \frac{1}{2}(4n)$ Use the distributive property.

$$= \left(\frac{1}{2} \cdot 8\right)m - \left(\frac{1}{2} \cdot 4\right)n$$
 Use the associative property.

$$= 4m - 2n$$
 Simplify.

b. $(-21a - 63b)\left(-\frac{1}{7}\right) = (-21a)\left(-\frac{1}{7}\right) - (63b)\left(-\frac{1}{7}\right)$

$$= (-21)\left(-\frac{1}{7}\right)a - (63)\left(-\frac{1}{7}\right)b$$

$$= (3)a - (-9)b$$

$$= 3a + 9b$$

Simplify each expression.

25. $\frac{1}{2}(-8a + 10)$

26. $\frac{1}{3}(9y - 21)$

27. $-\frac{1}{5}(-25c + 10d)$

28. $-\frac{1}{4}(24g - 32h)$

29. $(-21m - 14n)\left(-\frac{1}{7}\right)$

30. $(-26e - 52f)\left(-\frac{1}{13}\right)$

31. $(40x - 56y)\left(-\frac{1}{8}\right)$

32. $(-5a + 30b)\left(\frac{1}{-5}\right)$

Mixed Review Exercises

Translate each sentence into an equation.

1. Three more than six times a number is 21.

2. Twelve less than a number is 200.

3. The sum of two consecutive integers is 71.

4. The product of two consecutive integers is 90.

Simplify.

5. $(-8)(-3)(-5)$

6. $-24(25)(-4)$

7. $5(-7)(-6)$

8. $-8(2a - 5b)$

9. $-2(2 + x) - 2(x - 2)$

10. $10(x - 1) + 4(3 - x)$