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9 Systems of Linear Equations

9–1 *The Graphing Method*

Objective: To use graphs to solve systems of linear equations.

Vocabulary

System of equations Two or more equations in the same variables. Also called a *system of simultaneous equations.*

To solve a system of equations To find all ordered pairs (x, y) that make *both* equations true.

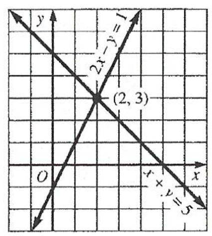
Solution of a system of equations An ordered pair that satisfies both equations at the same time.

Coincide Two lines coincide if their graphs are the same. The equations are equivalent.

Example 1 Solve the system by graphing:

$$2x - y = 1$$
$$x + y = 5$$

Solution Graph $2x - y = 1$ and $x + y = 5$ in the same coordinate plane. The only point on *both* lines is the *intersection point* (2, 3). The only solution of *both* equations is (2, 3).



Check: You can check that (2, 3) is a solution of the system by substituting $x = 2$ and $y = 3$ in both equations.

$2x - y = 1$	$x + y = 5$
$2(2) - 3 = 1$	$2 + 3 = 5$

The system has the solution (2, 3).

Solve each system by the graphing method. Graphs given at the back of this Answer Key.

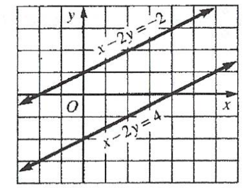
1. $x + y = 6$, $x - y = 2$ **(4, 2)**
2. $x + y = 5$, $x - y = -3$ **(1, 4)**
3. $x + y = 9$, $x - y = 3$ **(6, 3)**
4. $y = x + 2$, $y = 2x - 1$ **(3, 5)**
5. $2x - y = 0$, $x + y = 3$ **(1, 2)**
6. $2x + y = 1$, $x + y = 3$ **(−2, 5)**
7. $2x + y = 5$, $x - y = 4$ **(3, −1)**
8. $x + 2y = 5$, $x - y = -1$ **(1, 2)**
9. $x - y = 4$, $2x + y = 2$ **(2, −2)**
10. $-2x + y = -1$, $2x + y = 7$ **(2, 3)**
11. $y - 2x = -5$, $y - x = -3$ **(2, −1)**
12. $2y - x = 2$, $y + x = 4$ **(2, 2)**

Example 2 Solve the system by graphing:

$$x - 2y = 4$$
$$x - 2y = -2$$

Solution When you graph the equations in the same coordinate plane, you see that the lines have the same slope but different y-intercepts. The graphs are parallel lines. Since the lines do not intersect, there is no point that represents a solution of both equations.

The system has *no solution.*

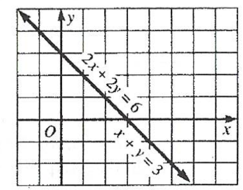


Example 3 Solve the system by graphing:

$$x + y = 3$$
$$2x + 2y = 6$$

Solution When you graph the equations in the same coordinate plane, you see that the graphs coincide. The equations are equivalent. Every point on the line represents a solution of both equations.

The system has *infinitely many solutions.*



Solve each system by the graphing method. Graphs given at the back of this Answer Key.

13. $3x - y = 8$, $x + y = 4$ **(3, 1)**
14. $2x + 3y = 5$, $y = x$ **(1, 1)**
15. $2x - 3y = 4$, $2x - y = 0$ **(−1, −2)**
16. $3x + 3y = 9$, $x + y = 3$ **Infinitely many solutions**
17. $x + 2y = -4$, $x + 2y = 8$ **No solution**
18. $3x + y = 6$, $2x - y = -1$ **(1, 3)**
19. $x - y = -6$, $x - y = 2$ **No solution**
20. $y - x = -3$, $y - 2x = -5$ **(2, −1)**
21. $2x + y = 5$, $2x + y = -1$ **No solution**
22. $2x - y = 7$, $x + 2y = 11$ **(5, 3)**
23. $4x + y = -14$, $3x = y$ **(−2, −6)**
24. $x - y = 4$, $2x - 2y = 8$ **Infinitely many solutions**

Mixed Review Exercises

Simplify. Give your answers using positive exponents.

1. $\frac{16a^2b}{8ab^2}$ $\quad\frac{2a}{b}$
2. $(a^{-2}b^3)^3$ $\quad\frac{b^9}{a^6}$
3. $\frac{15m^5n}{25m^2n^3}$ $\quad\frac{3m^3}{5n^2}$
4. $(x^3y^2)^{-2}$ $\quad\frac{1}{x^6y^4}$
5. x^4y^{-3} $\quad\frac{x^4}{y^3}$
6. $\frac{x^3y^2}{x^{-2}y}$ $\quad x^5y$

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9-2 The Substitution Method

Objective: To use the substitution method to solve systems of linear equations.

Example 1 Solve by the substitution method: $x + y = 9$
$2x + 3y = 20$

Solution

1. Solve the first equation for y.
$$x + y = 9$$
$$y = 9 - x$$

2. Substitute this expression for y in the other equation, and solve for x.
$$2x + 3(9 - x) = 20$$
$$2x + 27 - 3x = 20$$
$$-x + 27 = 20$$
$$-x = -7$$
$$x = 7$$

3. Substitute the value for x in the equation in Step 1, and solve for y.
$$y = 9 - x$$
$$y = 9 - 7$$
$$y = 2$$

4. Check $x = 7$ and $y = 2$ in *both* equations.
$$x + y = 9 \qquad 2x + 3y = 20$$
$$7 + 2 \stackrel{?}{=} 9 \qquad 2(7) + 3(2) \stackrel{?}{=} 20$$
$$9 = 9 \checkmark \qquad 14 + 6 \stackrel{?}{=} 20$$
$$20 = 20 \checkmark$$

The solution is (7, 2).

Solve by the substitution method.

1. $y = 3x$; $x + y = 12$ **(3, 9)**
2. $y = 2x$; $5x - y = 12$ **(4, 8)**
3. $a = 4b$; $a - b = 9$ **(12, 3)**
4. $m = 5n$; $3m - 2n = 26$ **(10, 2)**
5. $y = x - 1$; $2x + y = 5$ **(2, 1)**
6. $y = 4x - 1$; $x + y = 4$ **(1, 3)**
7. $x + y = 3$; $2x - y = 6$ **(3, 0)**
8. $x - y = 2$; $x - 2y = -1$ **(5, 3)**
9. $3x - y = -9$; $4x + y = -5$ **(−2, 3)**
10. $2x + y = 1$; $3x + 2y = 3$ **(−1, 3)**
11. $3x + y = 7$; $2x - 5y = -1$ **(2, 1)**
12. $x - 3y = -5$; $2x - 5y = -9$ **(−2, 1)**
13. $4x - 2y = 5$; $x - 4y = 3$ **$\left(1, -\frac{1}{2}\right)$**
14. $2x + y = 3$; $3x + 2y = 5$ **(1, 1)**
15. $3y - x = -8$; $5y + 2x = -6$ **(2, −2)**
16. $3x + y = 2$; $2x + 3y = -8$ **(2, −4)**
17. $x + 2y = 7$; $2x - y = 4$ **(3, 2)**
18. $x - 3y = 2$; $x = -y - 6$ **(−4, −2)**
19. $x - 5 = y$; $5x + 2y = 4$ **(2, −3)**
20. $y - 3 = -2x$; $3x - 2y = -20$ **(−2, 7)**
21. $x + 8 = 2y$; $4x + y = 13$ **(2, 5)**
22. $3u + v = 8$; $\frac{u}{4} - \frac{v}{2} = 3$ **(4, −4)**
23. $2x - y = 2$; $x = \frac{2}{3}y$ **(4, 6)**
24. $5x - 4y = -10$; $x = \frac{3}{5}y$ **(6, 10)**

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Example 2 Solve by the substitution method: $2x - 6y = 8$
$x - 3y = 10$

Solution
$$x - 3y = 10$$
$$x = 10 + 3y$$
$$2x - 6y = 8$$
$$2(10 + 3y) - 6y = 8$$
$$20 + 6y - 6y = 8$$
$$20 = 8 \leftarrow \text{False}$$

The *false statement* indicates that there is *no* ordered pair (x, y) that satisfies both equations. (If you graph the equations, you'll see that *the lines are parallel.*)

The system has *no solution.*

Example 3 Solve by the substitution method: $\frac{y}{3} = 3 - x$
$3x + y = 9$

Solution
$$\frac{y}{3} = 3 - x \quad \text{Multiply both sides by 3 to solve for } y.$$
$$y = 9 - 3x$$
$$3x + y = 9$$
$$3x + (9 - 3x) = 9$$
$$3x + 9 - 3x = 9$$
$$9 = 9 \leftarrow \text{True}$$

The *true statement* indicates that *every* ordered pair (x, y) that satisfies one of the equations also satisfies the other. (If you graph the equations, you'll see that *the lines coincide.*)

The system has *infinitely many solutions.*

Solve by the substitution method.

25. $x - 3y = -2$; $y = 2x - 1$ **(1, 1)**
26. $x + 2y = 7$; $2x + 4y = 8$ **No solution**
27. $y = 2x - 3$; $2y = -3x + 8$ **(2, 1)**
28. $\frac{x}{2} = 3 - y$; $x + 2y = 6$ **Infinitely many solutions**
29. $9x - 5y = 105$; $\frac{1}{4}x - \frac{2}{5}y = -1$ **(20, 15)**
30. $\frac{x}{3} = 2 + y$; $3x - 9y = -4$ **No solution**

Mixed Review Exercises

Write an equation in slope-intercept form for each line described.

1. slope $\frac{1}{2}$, passes through $(-2, 4)$ **$y = \frac{1}{2}x + 5$**
2. slope $\frac{2}{3}$, passes through $(3, -3)$ **$y = \frac{2}{3}x - 5$**
3. slope 3, y-intercept 2 **$y = 3x + 2$**
4. passes through (2, 7) and (0, −3) **$y = 5x - 3$**
5. passes through (2, −4) and (−1, 1) **$y = -\frac{5}{3}x - \frac{2}{3}$**
6. slope 0, y-intercept −3 **$y = -3$**

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9–3 Solving Problems with Two Variables

Objective: To use systems of linear equations in two variables to solve problems.

Example 1 Joel has 14 coins, all dimes and quarters, worth \$2.60. How many dimes and quarters does Joel have?

Solution

Step 1 The problem asks for the number of dimes and the number of quarters.

Step 2 Let d = the number of dimes and q = the number of quarters. Make a chart.

	Number	× Value per coin	= Total value
Dimes	d	10	$10d$
Quarters	q	25	$25q$

Step 3 The two facts not recorded in the chart are the total number of coins, 14, and the total value, \$2.60. Use these facts to write a system of equations.

$$d + q = 14$$
$$10d + 25q = 260$$

Step 4

$d = 14 - q$ Find d in terms of q.
$10(14 - q) + 25q = 260$ Substitute.
$140 - 10q + 25q = 260$
$15q = 120$
$q = 8$

$d = 14 - q$
$d = 14 - 8$
$d = 6$

Step 5 The check is left for you. Joel has 6 dimes and 8 quarters.

Solve, using two equations in two variables.

1. Rod has 40 coins, all dimes and quarters, worth \$7.60. How many dimes and how many quarters does he have? **16 dimes, 24 quarters**
2. Gayle has 36 coins, all nickels and dimes, worth \$2.40. How many dimes does she have? **12 dimes**
3. Leo has \$4.80 in dimes and quarters. He has 6 more dimes than quarters. How many quarters does he have? **12 quarters**
4. Nancy and Kerry have the same number of coins. Nancy has only dimes and Kerry has only quarters. If Kerry has \$3.00 more than Nancy, how much does she have? **\$5.00**
5. Ben has \$3.40 in nickels and dimes. He has 4 more dimes than nickels. How many dimes does he have? **24 dimes**

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Example 2 Connie has \$4000 invested in stocks and bonds. The stocks pay 6% interest and the bonds pay 8% interest. If her annual income from the stocks and bonds is \$270, how much is invested in stocks?

Solution

Step 1 The problem asks for the amount invested in stocks.

Step 2 Let s = amount invested in stocks and b = amount invested in bonds.

	Principal	× Rate	= Interest
Stocks	s	0.06	$0.06s$
Bonds	b	0.08	$0.08b$
Total	4000		270

Step 3

$s + b = 4000$ The total amount invested is \$4000.
$0.06s + 0.08b = 270$ The total amount of interest earned is \$270.
$s = 4000 - b$ Find s in terms of b.

Step 4

$0.06(4000 - b) + 0.08b = 270$ Substitute.
$6(4000 - b) + 8b = 27{,}000$ Multiply each side of the equation
$24{,}000 - 6b + 8b = 27{,}000$ by 100 to eliminate decimals.
$24{,}000 + 2b = 27{,}000$
$2b = 3000$
$b = 1500$ $s = 4000 - b$ or $s = 2500$

Step 5 The check is left for you. Connie has \$2500 invested in stocks.

Solve, using two equations in two variables.

6. Sam invests \$6000 in treasury notes and bonds. The notes pay 8% annual interest and the bonds pay 10% annual interest. If the annual income is \$550, how much is invested in bonds? **\$3500**
7. Kathleen has \$8000 invested in stocks and bonds. The stocks pay her 6% annual interest and the bonds pay 9% interest. If her annual income from the stocks and bonds is \$630, how much is invested in stocks? **\$3000**
8. Marty invested \$7000 in treasury notes and stocks. The stocks paid 7% and the notes paid 8%, giving an annual income of \$535. How much is invested in treasury notes? **\$4500**

Mixed Review Exercises

Solve.

1. $\frac{1}{3}x + 3 = 1$ **{−6}**
2. $\frac{1}{2}y = 3\frac{1}{2}$ **{7}**
3. $\frac{x + 3}{2} = 6$ **{9}**
4. $2(a + 1) = 8 - 4(a - 6)$ **{5}**
5. $-9 = n + 4$ **{−13}**
6. $3x + 15 = x + 5$ **{−5}**

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9–4 *The Addition-or-Subtraction Method*

Objective: To use addition or subtraction to solve systems of linear equations in two variables.

Vocabulary

Addition-or-subtraction method A method to solve systems of equations. You can use the addition-or-subtraction method whenever two equations have the same or opposite coefficients for one of their terms.

Example 1 **(The Addition Method)**

Solve: $4x - y = 7$
$2x + y = 5$

Solution

1. Add similar terms of the two equations.
$$\begin{array}{r} 4x - y = 7 \\ \underline{2x + y = 5} \\ 6x \quad = 12 \end{array}$$ The y-terms are eliminated.

2. Solve the resulting equation. $x = 2$

3. Substitute 2 for x in either of the original equations to find y.
$$\begin{aligned} 2x + y &= 5 \\ 2(2) + y &= 5 \\ y &= 1 \end{aligned}$$

4. Check $x = 2$ and $y = 1$ in *both* original equations.
$$\begin{aligned} 4x - y &= 7 \\ 4(2) - 1 &\stackrel{?}{=} 7 \\ 7 &= 7 \end{aligned} \qquad \begin{aligned} 2x + y &= 5 \\ 2(2) + 1 &\stackrel{?}{=} 5 \\ 5 &= 5 \end{aligned}$$

The solution is (2, 1).

Example 2 **(The Subtraction Method)**

Solve: $5c + 3d = 14$
$5c - d = 22$

Solution

1. Subtract similar terms of the two equations.
$$\begin{array}{r} 5c + 3d = 14 \\ \underline{5c - d = 22} \\ 4d = -8 \end{array}$$ The c-terms are eliminated.

2. Solve the resulting equation. $d = -2$

3. Substitute -2 for d in either of the original equations to find c.
$$\begin{aligned} 5c + 3(-2) &= 14 \\ 5c - 6 &= 14 \\ 5c &= 20 \\ c &= 4 \end{aligned}$$

4. The check in both equations is left for you.

The solution is $(4, -2)$.

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9–4 *The Addition-or-Subtraction Method (continued)*

Solve by the addition-or-subtraction method.

1. $x + y = 6$, $x - y = 2$ **(4, 2)**
2. $m + n = 12$, $m - n = 6$ **(9, 3)**
3. $2x + y = 3$, $x - y = 3$ **(2, −1)**
4. $2x + y = 5$, $x + y = 4$ **(1, 3)**
5. $3m - 2n = 11$, $5m + 2n = 13$ **(3, −1)**
6. $12m + 3n = 0$, $5m + 3n = 7$ **(−1, 4)**
7. $6x - 7y = 14$, $-6x + 3y = -6$ **(0, −2)**
8. $4a - 5b = 10$, $2a - 5b = 0$ **(5, 2)**
9. $2c + 3d = 3$, $2c + d = -3$ **(−3, 3)**
10. $4x - 3y = -10$, $2x + 3y = 4$ **(−1, 2)**
11. $2x - y = 7$, $3x + y = 8$ **(3, −1)**
12. $6x - 5y = 1$, $2x - 5y = 17$ **(−4, −5)**
13. $9x + 2y = -22$, $9x - 10y = 2$ **(−2, −2)**
14. $5m + 12n = -1$, $8m + 12n = 20$ **(7, −3)**
15. $3a + 2c = 30$, $5a - 2c = 2$ **(4, 9)**
16. $3m + 4n = 7$, $-3m + 9n = 6$ **(1, 1)**
17. $4x - 2y = -8$, $4x + 5y = 6$ **(−1, 2)**
18. $6a - 5b = 2$, $4a + 5b = -32$ **(−3, −4)**
19. $7x - 11y = -1$, $13x + 11y = 61$ **(3, 2)**
20. $\frac{1}{2}x + \frac{1}{3}y = 2$, $\frac{3}{2}x - \frac{1}{3}y = 2$ **(2, 3)**
21. $\frac{3}{4}x - \frac{1}{6}y = -7$, $\frac{5}{4}x - \frac{1}{6}y = -11$ **(−8, 6)**

Solve by either the substitution or the addition-or-subtraction method.

22. $a = 4b$, $a + 2b = -6$ **(−4, −1)**
23. $x - 5y = 3$, $2x + y = 6$ **(3, 0)**
24. $3x - 8y = 10$, $2x + 8y = -20$ **(−2, −2)**
25. $3(a - 2b) = 6$, $2(a + 3b) = -6$ **(0, −1)**
26. $n = 6m - 2$, $\frac{1}{2}n - m = -1$ **(0, −2)**
27. $\frac{1}{3}a - \frac{2}{3}b = -2$, $a + b - 12 = 0$ **(6, 6)**
28. $y = \frac{2}{3}x$, $2x + 3y = -24$ **(−6, −4)**
29. $\frac{a}{3} - \frac{b}{3} = 2$, $2a + b = 3$ **(3, −3)**
30. $2n - 11 = \frac{m}{4}$, $n = \frac{m}{-3}$ **(−12, 4)**

Mixed Review Exercises

Simplify.

1. $6x^3 + 4x^2 - x + 5x^2$ $\mathbf{6x^3 + 9x^2 - x}$
2. $2 \cdot 3^2$ **18**
3. $(2 \cdot 10^3) + (3 \cdot 10^2) + (5 \cdot 10)$ **2350**
4. $-3[2n - (n + 1)]$ $\mathbf{-3n + 3}$
5. $(8x^3y^2)\left(\frac{3}{4}x^2y\right)$ $\mathbf{6x^5y^3}$
6. $(2a^5)^2$ $\mathbf{4a^{10}}$
7. $(-2ab^2)^3$ $\mathbf{-8a^3b^6}$
8. $2x[3x + 2(4 - x)]$ $\mathbf{2x^2 + 16x}$
9. $(4ab)(-2ab^2)(5a^2b^3)$ $\mathbf{-40a^4b^6}$
10. $\left(-\frac{1}{12}\right)(60)\left(\frac{1}{5}\right)$ **−1**
11. $\dfrac{-6}{\frac{1}{2}}$ **−12**
12. $\frac{1}{5}(-45m + 30n)$ $\mathbf{-9m + 6n}$

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9-5 Multiplication with the Addition-or-Subtraction Method

Objective: To use multiplication with the addition-or-subtraction method to solve systems of linear equations.

Example 1 Solve: $3x - y = 9$
$2x + 5y = -11$

Solution

1. Multiply both sides of the first equation by 5 so that the y-terms are opposites.
$5(3x - y) = 5(9) \rightarrow 15x - 5y = 45$
$2x + 5y = -11 \rightarrow \underline{2x + 5y = -11}$
2. Add similar terms. $17x = 34$
3. Solve the resulting equation. $x = 2$
4. Substitute 2 for x in either original equation to find the value of y.
$3(2) - y = 9$
$6 - y = 9$
$-y = 3$
$y = -3$
5. The check is left for you.

The solution is $(2, -3)$.

CAUTION Check your solution in the original equations as a transformed equation could contain an error.

Solve each system by using multiplication with the addition-or-subtraction method.

1. $2x + y = 7$, $3x - 4y = 5$ **(3, 1)**
2. $3a + 5b = 1$, $a + 2b = 0$ **(2, −1)**
3. $2x - y = 8$, $x - 4y = -3$ **(5, 2)**
4. $m + 2n = 9$, $3m - 5n = 5$ **(5, 2)**
5. $a - 2b = 1$, $3a + b = -4$ **(−1, −1)**
6. $3x - 2y = -1$, $x + y = 3$ **(1, 2)**
7. $5x - y = -4$, $4x - 3y = -1$ **(−1, −1)**
8. $2m + 3n = 6$, $m + 2n = 5$ **(−3, 4)**
9. $2x - y = 8$, $x - 8y = 4$ **(4, 0)**
10. $x + 3y = -2$, $4x + 7y = 7$ **(7, −3)**
11. $x + 3y = 5$, $3x + 2y = -6$ **(−4, 3)**
12. $5x - 2y = -3$, $x + 3y = -4$ **(−1, −1)**
13. $3x - 2y = 5$, $x - 4y = -5$ **(3, 2)**
14. $5x - y = 14$, $4x - 3y = 20$ **(2, −4)**
15. $3x + 2y = 2$, $-7x + y = -16$ **(2, −2)**

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Example 2 Solve: $3a + 2b = 4$
$11a + 5b = 3$

Solution

1. Transform both equations by multiplication so that the b-terms are the same.
$5(3a + 2b) = 5(4) \rightarrow 15a + 10b = 20$
$2(11a + 5b) = 2(3) \rightarrow \underline{22a + 10b = 6}$
2. Subtract similar terms. $-7a = 14$
3. Solve the resulting equation. $a = -2$
4. Substitute for a in either original equation to find the value of b.
$3(-2) + 2b = 4$
$-6 + 2b = 4$
$2b = 10$
$b = 5$
5. The check is left for you. The solution is $(-2, 5)$.

Solve each system by using multiplication with the addition-or-subtraction method.

16. $3t - 8z = -2$, $7t + 4z = 18$ **(2, 1)**
17. $6a + 7c = 8$, $2a + 5c = 8$ **(−1, 2)**
18. $4x + 9y = 3$, $-7x + 3y = -24$ **(3, −1)**
19. $2x - 3y = 18$, $3x + 4y = -7$ **(3, −4)**
20. $4x + 3y = -14$, $6x - 2y = -8$ **(−2, −2)**
21. $3a + 4b = 4$, $2a - 3b = 14$ **(4, −2)**
22. $5m - 2n = -1$, $4m + 5n = -14$ **(−1, −2)**
23. $2x + 7y = 5$, $3x - 5y = 23$ **(6, −1)**
24. $4x - 3y = 10$, $5x + 6y = -7$ **(1, −2)**
25. $2x + 3y = 9$, $3x + 5y = 16$ **(−3, 5)**
26. $5x - 4y = 5$, $2x + 3y = 25$ **(5, 5)**
27. $5a - 2c = 1$, $4a + 5c = 47$ **(3, 7)**
28. $6x - 5y = 12$, $8x - 3y = 16$ **(2, 0)**
29. $7x - 5y = 20$, $3x + 2y = 21$ **(5, 3)**
30. $6x + 5y = 13$, $5x + 9y = 6$ **(3, −1)**
31. $3x + 2y = 4$, $11x + 5y = 3$ **(−2, 5)**
32. $2x + 7y = -3$, $3x + 5y = 1$ **(2, −1)**
33. $4x - 5y = 3$, $3x + 2y = -15$ **(−3, −3)**

Mixed Review Exercises

Factor completely.

1. $4 - 16x + 16x^2$ **$4(1 - 2x)^2$**
2. $6m^2n - 18mn^3$ **$6mn(m - 3n^2)$**
3. $9c^2 - 16d^2$ **$(3c + 4d)(3c - 4d)$**
4. $x^2 + 7x + 10$ **$(x + 2)(x + 5)$**
5. $2y^2 + 7y + 3$ **$(2y + 1)(y + 3)$**
6. $p^2 - 5p - 14$ **$(p - 7)(p + 2)$**

Find the constant of variation.

7. y varies directly as x, and $y = 63$ when $x = 9$. **7**
8. t varies directly as s, and $t = -24$ when $s = 96$. **$-\frac{1}{4}$**
9. p is directly proportional to n, and $p = 27$ when $n = 36$. **$\frac{3}{4}$**
10. h is directly proportional to k, and $h = 30$ when $k = 6$. **5**

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9–6 Wind and Water Current Problems

Objective: To use systems of equations to solve wind and water current problems.

Example A jet can travel the 6000 km distance between Washington, D.C. and London in 6 h with the wind. The return trip against the same wind takes 7 h 30 min. Find the rate of the jet in still air and the rate of the wind.

Solution

Step 1 The problem asks for the rate of the jet in still air and the rate of the wind.

Step 2 Let r = the rate in km/h of the jet in still air.
Let w = the rate in km/h of the wind.

The time 7 h 30 min is $7\frac{30}{60}$ h, or 7.5 h.

	Rate	× Time	= Distance
With the wind	$r + w$	6	6000
Against the wind	$r - w$	7.5	6000

Step 3 Use the information in the chart to write two equations.

$$6(r + w) = 6000, \quad \text{or } r + w = 1000$$
$$7.5(r - w) = 6000, \quad \text{or } r - w = 800$$

Step 4

$$\begin{aligned} r + w &= 1000 \\ r - w &= 800 \\ \hline 2r &= 1800 \\ r &= 900 \end{aligned}$$

$$\begin{aligned} 900 + w &= 1000 \\ w &= 100 \end{aligned}$$

Step 5 The check is left for you.

The rate of the jet is 900 km/h.
The rate of the wind is 100 km/h.

Solve.

1. A small plane traveled the 1200 km distance between two islands in 4 h with the wind. The return trip against the same wind took 5 h. Find the rate of the plane in still air and the rate of the wind. **plane, 270 km/h; wind, 30 km/h**

2. A plane traveled the 2080 km distance between two cities in 5 h with the wind. The return trip against the same wind took 6.5 h. Find the rate of the plane in still air and the rate of the wind. **plane, 368 km/h; wind, 48 km/h**

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9–6 Wind and Water Current Problems *(continued)*

Solve.

3. A small plane can travel the 3200 km distance between two cities in 10 h with the wind. Against the same wind the plane can only fly 2400 km in 10 h. Find the rate of the plane in still air and the rate of the wind. **plane, 280 km/h; wind, 40 km/h**

4. A plane can fly 4800 km in 4 h with the wind. The return trip against the same wind takes 5 h. Find the rate of the plane in still air and the rate of the wind. **plane, 1080 km/h; wind, 120 km/h**

5. The 4200 km trip from New York to San Francisco takes 7 h flying against the wind, but only 6 h returning. Find the speed of the plane in still air and the wind speed. **plane, 650 km/h; wind, 50 km/h**

6. Paddling with current, a canoeist can travel 48 km in 3 h. Against the current the canoeist takes 4 h to travel the same distance. Find the rate of the canoeist in still water and the rate of the current. **canoeist, 14 km/h; current, 2 km/h**

7. A cabin cruiser traveling with the current went 120 km in 3 h. Against the current it took 5 h to travel the same distance. Find the rate of the cabin cruiser in still water and the rate of the current. **cruiser, 32 km/h; current, 8 km/h**

8. A sailboat travels 24 mi downstream in 3 h. The return trip upstream takes 4 h. Find the speed of the sailboat in still water and the rate of the current. **boat, 7 mi/h; current, 1 mi/h**

9. A crew can row 45 km downstream in 3 h. Rowing against the same current, the crew rowed the same distance in 5 h. Find the rowing rate of the crew in still water and the rate of the current. **boat, 12 km/h; current, 3 km/h**

10. The 3600 km trip between two cities takes 6 h flying with the wind and 7.2 h against the wind. Find the speed of the plane in still air and the wind speed. **plane, 550 km/h; wind, 50 km/h**

Mixed Review Exercises

Solve each system using multiplication with the addition-or-subtraction method.

1. $2x - 3y = 12$, $x + y = 1$ **(3, −2)**

2. $x + y = 3$, $3x - 5y = 17$ **(4, −1)**

3. $2x - 3y = 6$, $3x + 4y = -25$ **(−3, −4)**

Simplify.

4. $\frac{2n^2 - 13n + 20}{2n - 5}$ **$n - 4$**

5. $\frac{3}{x - 1} + \frac{4}{1 - x}$ **$\frac{-1}{x - 1}$, or $-\frac{1}{x - 1}$**

6. $\frac{2x + 1}{6} - \frac{x + 3}{4}$ **$\frac{x - 7}{12}$**

7. $a - 1 - \frac{a + 2}{a - 3}$ **$\frac{a^2 - 5a + 1}{a - 3}$**

8. $\frac{x^2 - 10xy + 25y^2}{x - y} \div \frac{x^2 - 4xy - 5y^2}{x^2 - y^2}$ **$x - 5y$**

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9–7 Puzzle Problems

Objective: To use systems of equations to solve digit, age, and fraction problems.

Example 1 **(Digit problem)**

The sum of the digits in a two-digit number is 10. The new number obtained when the digits are reversed is 18 more than the original number. Find the original number.

Solution

Step 1 The problem asks for the original number.

Step 2 Let t = the tens digit and u = the units digit of the original number.

	Tens	Units	Value
Original number	t	u	$10t + u$
Number with digits reversed	u	t	$10u + t$

Step 3 Use the facts of the problem to write two equations.

$t + u = 10$ Sum of the digits of the original number is 10.

$(10u + t) - (10t + u) = 18$
$10u + t - 10t - u = 18$ {Difference between new number and original number is 18.
$9u - 9t = 18$
$9(u - t) = 18$
$u - t = 2$

Step 4
$u + t = 10$
$u - t = 2$ {Write the two equations as a system and solve for one variable.
$2u = 12$
$u = 6$

$u - t = 2$ Substitute 6 for u in the second equation.
$6 - t = 2$
$t = 4$

Step 5 The check is left for you.

The original number is 46.

Solve by using a system of two equations in two variables.

1. The sum of the digits in a two-digit number is 7. The new number obtained when the digits are reversed is 27 less than the original number. Find the original number. **52**
2. A two-digit number is seven times the sum of its digits. The tens digit is 3 more than the units digit. What is the number? **63**

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Example 2 **(Age problem)** Chan is three years older than Myra. Six years ago Chan was twice as old as Myra was. Find their ages now.

Solution

Steps 1, 2 Let c = Chan's age now and let m = Myra's age now.

Step 3 Use the facts of the problem to write two equations:

$c = m + 3$ {now
$c - 6 = 2(m - 6)$ {six years ago

Age	Now	6 years ago
Chan	c	$c - 6$
Myra	m	$m - 6$

Step 4 Simplify the equations and solve the system: $m = 9$, $c = 12$

Step 5 The check is left for you. Chan is 12 years old now and Myra is 9.

Example 3 **(Fraction problem)** The denominator of a fraction is 4 more than the numerator. If 2 is subtracted from each, the value of the new fraction is $\frac{1}{5}$. Find the original fraction.

Solution

Steps 1, 2 Let n = the numerator and d = the denominator of the original fraction.

Step 3 Use the facts of the problem to write two equations. $d = n + 4$
$\frac{n-2}{d-2} = \frac{1}{5}$, or $5(n - 2) = d - 2$

Step 4 Simplify the equations and solve the system: $n = 3, d = 7$

Step 5 The check is left to you. The original fraction $\frac{n}{d}$ is $\frac{3}{7}$.

Solve by using a system of two equations in two variables.

3. Max is 5 years older than Paulette. Next year he will be twice as old as she will be. How old is each now? **Max, 9 years old; Paulette, 4 years old**
4. Gloria is 20 years older than Reggie. Five years ago she was five times as old as he was. How old is each now? **Gloria, 30 years old; Reggie, 10 years old**
5. The denominator of a fraction is 7 more than the numerator. If 5 is added to both the numerator and denominator, the value of the resulting fraction is $\frac{1}{2}$. What is the original fraction? $\mathbf{\frac{2}{9}}$
6. The denominator of a fraction is 1 more than the numerator. If the numerator is decreased by 1, the value of the resulting fraction is $\frac{3}{4}$. What is the original fraction? $\mathbf{\frac{7}{8}}$

Mixed Review Exercises

Solve.

1. $\frac{x+3}{2} - \frac{x}{3} = \frac{5}{6}$ **{−4}**
2. $\frac{3n+2}{5} = \frac{n-2}{3}$ **{−4}**
3. $\frac{3+c}{2+c} = \frac{3}{4}$ **{−6}**
4. $-\frac{1}{4}x = 12$ **{−48}**