

8-9 Direct Variation

Objective: To use direct variation to solve problems.

Vocabulary

Direct variation A function defined by an equation of the form $y = kx$, where k is a nonzero constant. For example, $y = 5x$.

Constant of variation The nonzero constant k in a direct variation defined by $y = kx$. Also called the *constant of proportionality*.

Symbols $y = kx$ (y varies directly as x).

Example 1 Given that m varies directly as n and that $m = 75$ when $n = 25$, find the following:
a. the constant of variation b. the value of m when $n = 15$

Solution Let $m = kn$.

a. Substitute $m = 75$ and $n = 25$:

$$\begin{aligned}75 &= k \cdot 25 \\ 3 &= k\end{aligned}$$

b. Substitute $k = 3$ and $n = 15$: $m = 3 \cdot 15 = 45$

In Exercises 1-6, find the constant of variation.

- y varies directly as x , and $y = 18$ when $x = 3$.
- y varies directly as x , and $y = 52$ when $x = 13$.
- t varies directly as s , and $t = -36$ when $s = -4$.
- h varies directly as m , and $h = 368$ when $m = 23$.
- y varies directly as x , and $y = 252$ when $x = 18$.
- t varies directly as s , and $t = 490$ when $s = 14$.

Solve.

- y varies directly as x , and $y = 300$ when $x = 5$. Find y when $x = 15$.
- y varies directly as x , and $y = 10$ when $x = 2$. Find y when $x = 9$.
- h varies directly as a , and $a = 20$ when $h = 4$. Find a when $h = 3$.
- h varies directly as a , and $a = 24$ when $h = 8$. Find a when $h = 4$.
- y varies directly as x , and $y = 240$ when $x = 25$. Find y when $x = 40$.
- h varies directly as a , and $a = 6$ when $h = 15$. Find a when $h = 5$.

8-9 Direct Variation (continued)

Example 2 The amount of interest earned on savings is directly proportional to the amount of money saved. If \$26 interest is earned on \$325, how much interest will be earned on \$900 in the same period of time?

Solution 1

Step 1 The problem asks for the interest earned on \$900 if the interest on \$325 is \$26.

Step 2 Let i , in dollars, be the interest on d dollars.
$$\begin{array}{l} i_1 = 26 \quad i_2 = ? \\ d_1 = 325 \quad d_2 = 900 \end{array}$$

Step 3 An equation can be written in the form $\frac{i_1}{d_1} = \frac{i_2}{d_2}$.

$$\frac{26}{325} = \frac{i_2}{900}$$

Step 4 $26(900) = 325i_2$

$$23,400 = 325i_2$$

$$72 = i_2$$

Step 5 The check is left for you. The interest earned on \$900 will be \$72.

Solution 2 To solve Example 2 by the method shown in Example 1, first write the equation $i = kd$. Then solve for the constant of variation, k , by using the fact that $i = 26$ when $d = 325$. Use the value of k to find the value of i when $d = 900$. You may wish to complete the problem this way.

Solve.

- An employee's wages are directly proportional to the time worked. If an employee earns \$120 for 8 h, how much will the employee earn for 20 h?
- A certain car used 21 gal of gasoline in 7 h. If the rate of gasoline used is constant, how much gasoline will the car use on a 6-hour trip?
- The distance traveled by a bus at a constant speed varies with the length of time it travels. If a bus travels 192 mi in 4 h, how far will it travel in 9 h?
- The number of words typed is directly proportional to the time spent typing. If a typist can type 325 words in 5 min, how long will it take the typist to type a 1040-word report?

Mixed Review Exercises**Multiply.**

1. $(2x - 3)(3x - 1)$

2. $(3x - 2)(x^2 + x - 3)$

3. $-2x(3 - 5x)$

4. $(2x + 5)(2x - 5)$

5. $(t - 2)(3t + 5)$

6. $(5y - 3)(2y + 3)$