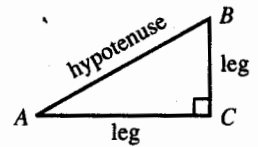


Triangles

Objective: To learn some properties of triangles.

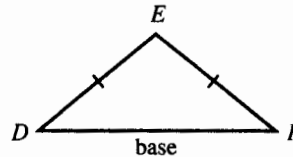
Vocabulary

Triangle A figure formed by three segments joining three points not on the same line. Each segment is a *side* of the triangle. Each of the three points is a *vertex* of the triangle. *Angles* of the triangle are formed by two sides and a vertex. The sum of the angles of a triangle is 180° .

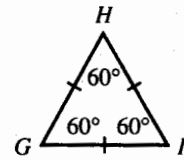


Right triangle A triangle having one right angle.

Isosceles triangle A triangle having two sides equal in length. The third side is called the *base*. The angles on either side of the base are called *base angles* and have equal measures.



Equilateral triangle A triangle with all sides of equal length. The angles of an equilateral triangle all measure 60° .



Example 1 The measures of two angles of a triangle are 26° and 64° . Find the measure of the third angle.

Solution In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$.

Let $x =$ the measure of the third angle.

$$\begin{aligned} x + 26 + 64 &= 180 \\ x + 90 &= 180 \\ x &= 90 \end{aligned}$$

The check is left for you.

The third angle has a measure of 90° .

The measure of two angles of a triangle are given. Find the measure of the third angle.

1. $35^\circ, 65^\circ$

2. $110^\circ, 40^\circ$

3. $78^\circ, 48^\circ$

4. $90^\circ, 21^\circ$

5. $25^\circ, 36^\circ$

6. $116^\circ, 23^\circ$

Example 2 Use the converse of the Pythagorean theorem to determine whether a triangle with sides 8, 15, and 17 is a right triangle.

Solution $a^2 + b^2 = c^2$ (in a right triangle)

$$8^2 + 15^2 \stackrel{?}{=} 17^2$$

$$64 + 225 \stackrel{?}{=} 289$$

$$289 = 289 \checkmark \quad \text{A triangle with sides 8, 15, and 17 is a right triangle.}$$

Triangles (continued)

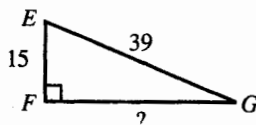
In Exercises 7–12, use the converse of the Pythagorean theorem to determine whether or not the triangle is a right triangle.

7. $\triangle ABC$: $AB = 16$, $BC = 12$, $AC = 20$
8. $\triangle DEF$: $EF = 29$, $FD = 21$, $DE = 20$
9. $\triangle GHI$: $GH = HI = 10$, $GI = 15$
10. $\triangle JKL$: $JK = 7$, $KL = 10$, $JL = 13$
11. $\triangle MNO$: $MN = 10$, $MO = 8$, $NO = 16$
12. $\triangle PQR$: $PQ = 16$, $QR = 34$, $PR = 30$

Example 3 If $\triangle EFG$ is a right triangle with $\angle F = 90^\circ$, $EF = 15$, and $EG = 39$, find FG .

Solution Draw a sketch to help you solve the problem. Note that EG is the hypotenuse.

$$\begin{aligned} EF^2 + FG^2 &= EG^2 \\ 15^2 + FG^2 &= 39^2 \\ 225 + FG^2 &= 1521 \\ FG^2 &= 1296 \\ FG &= 36 \end{aligned}$$



13. If $\triangle STU$ is a right triangle with $\angle T = 90^\circ$, $ST = 12$, and $SU = 15$, find TU .
14. If $\triangle XYZ$ is a right triangle with $\angle Z = 90^\circ$, $XZ = 24$, and $YZ = 10$, find XY .
15. If $\triangle ABC$ is a right triangle with $\angle A = 90^\circ$, $AB = 9$, and $AC = 40$, find BC .
16. If $\triangle DEF$ is isosceles, $DE = DF$, and $\angle D = 46^\circ$, find $\angle E$.
17. If $\triangle GHI$ is isosceles, $GH = GI$ and $\angle H = 30^\circ$, find $\angle G$.
18. If $\triangle ABC$ is a right isosceles triangle and $\angle C = 90^\circ$, find the measures of $\angle A$ and $\angle B$.

In Exercises 19–24, $\angle C = 90^\circ$ in $\triangle ABC$. Given the lengths of the other two sides, find the length of the third side in simplest radical form. Use the diagram shown below to help you.

19. $AC = 6$, $BC = 12$
20. $AC = 10$, $BC = 24$
21. $AC = 24$, $AB = 25$
22. $BC = 9$, $AC = 40$
23. $BC = 8$, $AB = 12$
24. $AC = 8$, $AB = 17$

