NAME _____ DATE _____

9–2 The Substitution Method

Objective: To use the substitution method to solve systems of linear equations.

Example 1 Solve by the substitution method:

$$x + y = 9$$
$$2x + 3y = 20$$

Solution

1. Solve the first equation for y.

$$x + y = 9$$
$$y = 9 - x$$

2. Substitute this expression for y in the other equation, and solve for x.

$$2x + 3(9 - x) = 20$$
$$2x + 27 - 3x = 20$$
$$-x + 27 = 20$$
$$-x = -7$$
$$x = 7$$

3. Substitute the value for x in the equation in Step 1, and solve for y.

$$y = 9 - x$$
$$y = 9 - 7$$
$$y = 2$$

4. Check $x = 7$ and $y = 2$ in *both* equations.

$$x + y = 9 \qquad 2x + 3y = 20$$
$$7 + 2 \stackrel{?}{=} 9 \qquad 2(7) + 3(2) \stackrel{?}{=} 20$$
$$9 = 9 \surd \qquad 14 + 6 \stackrel{?}{=} 20$$
$$20 = 20 \surd$$

The solution is (7, 2).

Solve by the substitution method.

1. $y = 3x$
$x + y = 12$

2. $y = 2x$
$5x - y = 12$

3. $a = 4b$
$a - b = 9$

4. $m = 5n$
$3m - 2n = 26$

5. $y = x - 1$
$2x + y = 5$

6. $y = 4x - 1$
$x + y = 4$

7. $x + y = 3$
$2x - y = 6$

8. $x - y = 2$
$x - 2y = -1$

9. $3x - y = -9$
$4x + y = -5$

10. $2x + y = 1$
$3x + 2y = 3$

11. $3x + y = 7$
$2x - 5y = -1$

12. $x - 3y = -5$
$2x - 5y = -9$

13. $4x - 2y = 5$
$x - 4y = 3$

14. $2x + y = 3$
$3x + 2y = 5$

15. $3y - x = -8$
$5y + 2x = -6$

16. $3x + y = 2$
$2x + 3y = -8$

17. $x + 2y = 7$
$2x - y = 4$

18. $x - 3y = 2$
$x = -y - 6$

19. $x - 5 = y$
$5x + 2y = 4$

20. $y - 3 = -2x$
$3x - 2y = -20$

21. $x + 8 = 2y$
$4x + y = 13$

22. $3u + v = 8$
$\frac{u}{4} - \frac{v}{2} = 3$

23. $2x - y = 2$
$x = \frac{2}{3}y$

24. $5x - 4y = -10$
$x = \frac{3}{5}y$

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9–2 The Substitution Method *(continued)*

Example 2 Solve by the substitution method:
$$2x - 6y = 8$$
$$x - 3y = 10$$

Solution
$$x - 3y = 10$$
$$x = 10 + 3y$$
$$2x - 6y = 8$$
$$2(10 + 3y) - 6y = 8$$
$$20 + 6y - 6y = 8$$
$$20 = 8 \leftarrow \text{False}$$

The *false statement* indicates that there is *no* ordered pair (x, y) that satisfies both equations. (If you graph the equations, you'll see that *the lines are parallel.*)

The system has *no solution.*

Example 3 Solve by the substitution method:
$$\frac{y}{3} = 3 - x$$
$$3x + y = 9$$

Solution
$$\frac{y}{3} = 3 - x$$ Multiply both sides by 3 to solve for y.
$$y = 9 - 3x$$
$$3x + y = 9$$
$$3x + (9 - 3x) = 9$$
$$3x + 9 - 3x = 9$$
$$9 = 9 \leftarrow \text{True}$$

The *true statement* indicates that *every* ordered pair (x, y) that satisfies one of the equations also satisfies the other. (If you graph the equations, you'll see that *the lines coincide.*)

The system has *infinitely many solutions.*

Solve by the substitution method.

25. $x - 3y = -2$
$y = 2x - 1$

26. $x + 2y = 7$
$2x + 4y = 8$

27. $y = 2x - 3$
$2y = -3x + 8$

28. $\frac{x}{2} = 3 - y$
$x + 2y = 6$

29. $9x - 5y = 105$
$\frac{1}{4}x - \frac{2}{5}y = -1$

30. $\frac{x}{3} = 2 + y$
$3x - 9y = -4$

Mixed Review Exercises

Write an equation in slope-intercept form for each line described.

1. slope $\frac{1}{2}$, passes through $(-2, 4)$

2. slope $\frac{2}{3}$, passes through $(3, -3)$

3. slope 3, y-intercept 2

4. passes through $(2, 7)$ and $(0, -3)$

5. passes through $(2, -4)$ and $(-1, 1)$

6. slope 0, y-intercept -3