

## 8-10 Inverse Variation

**Objective:** To use inverse variation to solve problems.

### Vocabulary

**Inverse variation** A function defined by an equation of the form  $xy = k$ , where  $k$  is a nonzero constant. For example,  $xy = 6$ .

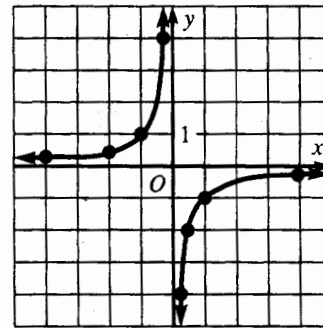
**Hyperbola** The graph of  $xy = k$  for any nonzero value of  $k$ .

**Example 1** Graph the equation  $xy = -1$ .

**Solution**

$x$	$y$
-4	$\frac{1}{4}$
-2	$\frac{1}{2}$
-1	1
$-\frac{1}{4}$	4

$x$	$y$
$\frac{1}{4}$	-4
$\frac{1}{2}$	-2
1	-1
4	$-\frac{1}{4}$



Graph each equation if the domain and the range are both the set of real numbers.

You may wish to verify your graphs on a computer or graphing calculator.

1.  $xy = 8$

2.  $xy = 16$

3.  $xy = -4$

4.  $xy = -6$

5.  $x = \frac{4}{y}$

6.  $y = \frac{6}{x}$

7.  $\frac{x}{3} = \frac{-3}{y}$

8.  $\frac{x}{2} = \frac{6}{y}$

**Example 2**  $(x_1, y_1)$  and  $(x_2, y_2)$  are ordered pairs of the same inverse variation.

Find the missing value:  $x_1 = 2, y_1 = 28, x_2 = 4, y_2 = \underline{\quad?}$ .

**Solution**

An inverse variation  $xy = k$  can also be expressed as  $x_1y_1 = x_2y_2$ .

$$2 \cdot 28 = 4 \cdot y_2 \quad \text{Replace } x_1 \text{ with 2, } y_1 \text{ with 28, and } x_2 \text{ with 4.}$$

$$56 = 4y_2 \quad \text{Solve the equation.}$$

$$14 = y_2, \text{ or } y_2 = 14.$$

$(x_1, y_1)$  and  $(x_2, y_2)$  are ordered pairs of the same inverse variation. Find the missing value.

9.  $x_1 = 6, y_1 = 5, x_2 = 2, y_2 = \underline{\quad?}$

10.  $x_1 = 8, y_1 = 24, x_2 = \underline{\quad?}, y_2 = 48$

11.  $x_1 = 5, y_1 = 8, x_2 = 10, y_2 = \underline{\quad?}$

12.  $x_1 = 6, y_1 = \underline{\quad?}, x_2 = 9, y_2 = 8$

13.  $x_1 = \underline{\quad?}, y_1 = 20, x_2 = 8, y_2 = 5$

14.  $x_1 = 8, y_1 = 9, x_2 = \underline{\quad?}, y_2 = 18$

**8–10 Inverse Variation** (continued)

**Example 3** If a 12 g mass is 60 cm from the fulcrum of a lever, how far from the fulcrum is a 45 g mass that balances the 12 g mass?

**Solution** A lever is a bar pivoted at a point called the *fulcrum*.  
If masses  $m_1$  and  $m_2$  are placed at distances  $d_1$  and  $d_2$  from the fulcrum, and the bar is balanced, then  $m_1d_1 = m_2d_2$ .

$$\text{Let } m_1 = 12, d_1 = 60, \text{ and } m_2 = 45, d_2 = \underline{\quad? \quad}.$$

$$\text{Use } m_1d_1 = m_2d_2.$$

$$12 \cdot 60 = 45 \cdot d_2.$$

$$720 = 45d_2$$

$$16 = d_2$$

The distance of the 45 g mass from the fulcrum is 16 cm.

In Exercises 15–22, refer to the lever at balance in Example 3. Find the missing value.

15.  $m_1 = 12, m_2 = 8, d_1 = 45, d_2 = \underline{\quad? \quad}$

16.  $m_1 = 60, m_2 = \underline{\quad? \quad}, d_1 = 8, d_2 = 12$

17.  $m_1 = 24, m_2 = 8, d_1 = \underline{\quad? \quad}, d_2 = 18$

18.  $m_1 = \underline{\quad? \quad}, m_2 = 40, d_1 = 5, d_2 = 7$

19.  $m_1 = 12, m_2 = 9, d_1 = \underline{\quad? \quad}, d_2 = 40$

20.  $m_1 = 108, m_2 = 60, d_1 = \underline{\quad? \quad}, d_2 = 9$

Solve.

21. Sarah weighs 105 lb and Wyatt weighs 140 lb. If Sarah sits 8 ft from the seesaw support, how far from the support must Wyatt sit to balance the seesaw?

22. Yoko weighs 120 lb and Lars weighs 180 lb. If Yoko sits 6 ft from the seesaw support, how far from the support must Lars sit to balance the seesaw?

**Mixed Review Exercises**

Show that the lines whose equations are given are parallel.

1.  $x + 2y = 3$

$$x + 2y = 5$$

2.  $2x + 6y = 7$

$$x + 3y = 1$$

3.  $x - y = 3$

$$y - x = 3$$

4.  $-6x + 9y = 2$

$$2x - 3y = 6$$

Find the constant of variation.

5.  $t$  varies directly as  $s$ , and  $t = 12$  when  $s = -3$ .

6.  $y$  varies directly as  $x$ , and  $y = 8$  when  $x = 32$ .

7.  $m$  varies directly as  $n$ , and  $m = 27$  when  $n = 3$ .