NAME ______________________ DATE ____________

11–2 Decimal Forms of Rational Numbers

Objective: To express rational numbers as decimals or fractions.

Vocabulary

Terminating decimal The result when a common fraction is written as a decimal by dividing the numerator by the denominator and the remainder is zero. Also called *ending decimal* or *finite decimal.* For example, $\frac{3}{8} = 0.375$.

Nonterminating decimal The result when a common fraction is written as a decimal by dividing the numerator by the denominator and a digit or a block of digits repeat endlessly as the remainder. Also called *unending, infinite, repeating,* or *periodic decimals.* For example, $\frac{7}{11} = 0.6363\ldots = 0.\overline{63}$. Dots or an overbar are used to indicate the repeating block of digits.

Example 1 Express $\frac{5}{8}$ as a decimal.

Solution

The division at the right shows that $\frac{5}{8}$ can be expressed as the terminating decimal 0.625.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Example 2 Express each rational number as a decimal: **a.** $\frac{1}{6}$ **b.** $\frac{2}{11}$ **c.** $2\frac{1}{7}$

Solution If you don't reach a remainder of zero when dividing the numerator by the denominator, continue to divide until the remainders begin to repeat.

a. $\frac{1}{6} \rightarrow$
$$\begin{array}{r} 0.166 \\ 6\overline{)1.000} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

$\frac{1}{6} = 0.166\ldots = 0.1\overline{6}$

b. $\frac{2}{11} \rightarrow$
$$\begin{array}{r} 0.1818 \\ 11\overline{)2.0000} \\ \underline{1\,1} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

$\frac{2}{11} = 0.1818\ldots = 0.\overline{18}$

c. $2\frac{1}{7} = \frac{15}{7} \rightarrow$
$$\begin{array}{r} 2.142857 \\ 7\overline{)15.000000} \\ \underline{14} \\ 1\,0 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 1 \end{array}$$

$2\frac{1}{7} = 2.142857\ldots = 2.\overline{142857}$

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11–2 *Decimal Forms of Rational Numbers* *(continued)*

Express each rational number as a terminating or repeating decimal.

1. a. $\frac{1}{3}$ **b.** $\frac{1}{30}$ **2. a.** $\frac{5}{2}$ **b.** $\frac{5}{200}$ **3. a.** $-\frac{2}{9}$ **b.** $-\frac{2}{9000}$ **4. a.** $-\frac{2}{5}$ **b.** $-\frac{2}{50}$

5. $\frac{13}{8}$ **6.** $\frac{5}{12}$ **7.** $\frac{7}{27}$ **8.** $-\frac{5}{18}$ **9.** $3\frac{3}{20}$ **10.** $2\frac{4}{11}$ **11.** $-5\frac{3}{4}$ **12.** $\frac{11}{27}$

Example 3 Express each terminating decimal as a fraction in simplest form.

a. 0.24 **b.** 0.325

Solution **a.** $0.24 = \frac{24}{100} = \frac{6}{25}$ **b.** $0.325 = \frac{325}{1000} = \frac{13}{40}$

Example 4 Express $0.5\overline{21}$ as a fraction in simplest form.

Solution Let $N =$ the number $0.5\overline{21}$ and $n =$ the number of digits in the block of repeating digits.

Multiply N by 10^n. Since $0.5\overline{21}$ has 2 digits in the repeating block, $n = 2$. Therefore, multiply both sides of the equation $N = 0.5\overline{21}$ by 10^2 or 100.

$$100N = 100(0.5\overline{21}).$$

Since $0.5\overline{21} = 0.52121\ldots$, $0.5\overline{21}$ can also be written as $0.521\overline{21}$.

Then $100(0.5\overline{21}) = 100(0.521\overline{21})$
$= 52.1\overline{21}$

Subtract N from $100N$. $100N = 52.1\overline{21}$
$N = 0.5\overline{21}$

Solve for N. $99N = 51.6$

$N = \frac{51.6}{99} = \frac{516}{990} = \frac{86}{165}$ So $0.5\overline{21} = \frac{86}{165}$.

Express each rational number as a fraction in simplest terms.

13. 0.3	**14.** 0.88	**15.** 0.225	**16.** 2.6	**17.** 4.26
18. $0.\overline{2}$	**19.** $1.8\overline{3}$	**20.** $0.\overline{074}$	**21.** $2.\overline{21}$	**22.** $0.\overline{09}$
23. $0.\overline{37}$	**24.** $0.\overline{63}$	**25.** $0.08\overline{3}$	**26.** $0.\overline{08}$	**27.** $-2.\overline{18}$

Mixed Review Exercises

Find the prime factorization of each number.

1. 120 **2.** 50 **3.** 484 **4.** 1125 **5.** 196 **6.** 288

Solve.

7. $(y + 2)(y - 3) = 0$ **8.** $(a + 2)^2 = 16$ **9.** $x^2 = -9$

10. $k^3 - 25k = 0$ **11.** $|x + 2| = 6$ **12.** $k + 3 < 12$