NAME ______________________ DATE ______________

12 Quadratic Functions

12–1 Quadratic Equations with Perfect Squares

Objective: To solve quadratic equations involving perfect squares.

Vocabulary

Perfect square An expression such as x^2, $(x - 1)^2$, or $(2x + 5)^2$.

Roots of $x^2 = k$ An equation in the form "perfect square $= k$" $(k \geq 0)$ can be solved by the method shown in Examples 1 and 2.

If $k > 0$, then $x^2 = k$ has **2** real roots: $x = \pm\sqrt{k}$.
If $k = 0$, then $x^2 = k$ has **1** real root: $x = 0$.
If $k < 0$, then $x^2 = k$ has **no** real roots.

Example 1 Solve: **a.** $m^2 = 36$ **b.** $3r^2 = 48$ **c.** $x^2 - 11 = 0$ **d.** $m^2 = -25$

Solution

a. $m^2 = 36$
$m = \pm\sqrt{36}$
$m = \pm 6$
The solution set is $\{6, -6\}$.

b. $3r^2 = 48$
$r^2 = 16$
$r = \pm\sqrt{16} = \pm 4$
The solution set is $\{4, -4\}$.

c. $x^2 - 11 = 0$
$x^2 = 11$
$x = \pm\sqrt{11}$
The solution set is $\{\sqrt{11}, -\sqrt{11}\}$.

d. $m^2 = -25$
Since the square of any real number is always a nonnegative number, there is *no real solution.*

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution.*

1. $x^2 = 49$ **{7, −7}**
2. $2x^2 = 18$ **{3, −3}**
3. $x^2 = \frac{25}{36}$ **3.** $\left\{\frac{5}{6}, -\frac{5}{6}\right\}$
4. $a^2 = -16$ **No sol.**
5. $2x^2 = 128$ **{8, −8}**
6. $5x^2 = 125$ **{5, −5}**
7. $9x^2 = 81$ **{3, −3}**
8. $x^2 - 81 = 0$ **{9, −9}**
9. $x^2 + 25 = 0$ **No sol.**
10. $m^2 - 10 = 0$ $\{\sqrt{10}, -\sqrt{10}\}$
11. $0 = 6x^2 - 24$ **{2, −2}**
12. $0 = 3m^2 - 75$ **{5, −5}**

Example 2 Solve $(x + 3)^2 = 49$

Solution

$(x + 3)^2 = 49$ — Note that $(x + 3)^2$ is a perfect square.
$x + 3 = \pm\sqrt{49}$ — Find the *positive or negative* square root of each side.
$x = -3 \pm 7$ — Subtract 3 from each side.
$x = 4$ or $x = -10$

Check: $(4 + 3)^2 \stackrel{?}{=} 49$, $7^2 \stackrel{?}{=} 49$, $49 = 49$ ✓ $\quad (-10 + 3)^2 \stackrel{?}{=} 49$, $(-7)^2 \stackrel{?}{=} 49$, $49 = 49$ ✓

The solution set is $\{4, -10\}$.

NAME ______________________ DATE ______________

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution.*

13. $(x - 3)^2 = 0$ **{3}**
14. $(z - 1)^2 = 16$ **{−3, 5}**
15. $(r - 5)^2 = 100$ **{−5, 15}**
16. $(x - 1)^2 = 25$ **{−4, 6}**
17. $(2x + 9)^2 = 225$ **{−12, 3}**
18. $(t - 4)^2 = 9$ **{7, 1}**

Example 3 Solve: **a.** $3(x - 2)^2 = 21$ **b.** $y^2 + 10y + 25 = 36$

Solution

a. $3(x - 2)^2 = 21$
$(x - 2)^2 = 7$
$x - 2 = \pm\sqrt{7}$
$x = 2 \pm \sqrt{7}$
$x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$
The check is left to you.
The solution set is $\{2 + \sqrt{7}, 2 - \sqrt{7}\}$.

b. $y^2 + 10y + 25 = 36$
$(y + 5)^2 = 36$
$y + 5 = \pm\sqrt{36}$
$y + 5 = \pm 6$
$y = -5 \pm 6$
$y = 1$ or $y = -11$
The check is left to you.
The solution set is $\{1, -11\}$.

Note: Example 3(b) could also have been solved by factoring.

Solve. Express irrational solutions in simplest radical form. If the equation has no solution, write *No solution.*

19. $9m^2 - 1 = 35$ **{2, −2}**
20. $27 = 2r^2 - 5$ **{4, −4}**
21. $3x^2 - 9 = 33$ $\{\sqrt{14}, -\sqrt{14}\}$
22. $64 = 2t^2 - 8$ **{6, −6}**
23. $2n^2 + 6 = 38$ **{4, −4}**
24. $7x^2 + 1 = 64$ **{3, −3}**
25. $3(m - 2)^2 = 15$ **25.** $\{2 + \sqrt{5}, 2 - \sqrt{5}\}$
26. $400 = 4(z - 2)^2$ **{12, −8}**
27. $2(x - 5)^2 = 98$ **{−2, 12}**
28. $25 = (2x + 1)^2$ **{−3, 2}**
29. $5(m - 3)^2 = 80$ **{−1, 7}**
30. $6(z + 5)^2 = 216$ **{−11, 1}**
31. $3(x - 1)^2 = -24$ **No sol.**
32. $(3x - 1)^2 + 12 = 4$ **No sol.**
33. $6(x + 5)^2 = 24$ **{−7, −3}**
34. $7(x + 2)^2 = 112$ **{−6, 2}**
35. $(x - 2)^2 - 1 = 35$ **{8, −4}**
36. $2(3n - 1)^2 = 8$ $\left\{1, -\frac{1}{3}\right\}$
37. $3(2x - 1)^2 = 27$ **{2, −1}**
38. $2(x + 3)^2 - 4 = 68$
39. $5(x - 1)^2 + 3 = 23$ **{−1, 3}**
40. $x^2 - 2x + 1 = 9$ **{−2, 4}**
41. $x^2 + 18x + 81 = 98$
42. $x^2 - 12x + 36 = 64$
43. $x^2 - 4x + 4 = 16$ **{−2, 6}**
44. $x^2 + 10x + 25 = 81$ **{−14, 4}**
45. $n^2 - 8n + 16 = 36$ **{10, −2}**

Mixed Review Exercises

38. $\{-9, 3\}$ **41.** $\{-9 + 7\sqrt{2}, -9 - 7\sqrt{2}\}$ **42.** $\{14, -2\}$

Express each square as a trinomial.

1. $(x - 8)^2$ $x^2 - 16x + 64$
2. $(2x + 1)^2$ $4x^2 + 4x + 1$
3. $(4x - 3)^2$ $16x^2 - 24x + 9$
4. $(-2c + 3)^2$ $4c^2 - 12c + 9$
5. $\left(x + \frac{1}{4}\right)^2$ $x^2 + \frac{1}{2}x + \frac{1}{16}$
6. $\left(x - \frac{1}{5}\right)^2$ $x^2 - \frac{2}{5}x + \frac{1}{25}$
7. $\left(\frac{1}{2}x + \frac{1}{3}\right)^2$ $\frac{1}{4}x^2 + \frac{1}{3}x + \frac{1}{9}$
8. $\left(\frac{1}{4}x - \frac{2}{3}\right)^2$ $\frac{1}{16}x^2 - \frac{1}{3}x + \frac{4}{9}$
9. $(x + 11)^2$ $x^2 + 22x + 121$

NAME ______________________ DATE ______________

12–2 Completing The Square

Objective: To solve quadratic equations by completing the square.

Method of Completing the Square

A method of transforming a quadratic equation so that it is in the form

perfect square $= k \quad (k \geq 0)$.

For $x^2 + bx + \underline{?}$:

1. Find half the coefficient of x: $\frac{b}{2}$
2. Square the result of Step 1: $\left(\frac{b}{2}\right)^2$
3. Add the result of Step 2 to $x^2 + bx$: $x^2 + bx + \left(\frac{b}{2}\right)^2$
4. You have completed the square: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Example 1 Complete the square:

a. $x^2 + 6x + \underline{?}$ b. $x^2 - 4x + \underline{?}$

Solution a. $x^2 + 6x + 9 = (x + 3)^2$, $\left(\frac{6}{2}\right)^2 = 9$ b. $x^2 - 4x + 4 = (x - 2)^2$, $\left(-\frac{4}{2}\right)^2 = 4$

Complete the square.

1. $x^2 - 8x + \underline{?}$ **$x^2 - 8x + 16 = (x - 4)^2$**
2. $x^2 - 10x + \underline{?}$ **$x^2 - 10x + 25 = (x - 5)^2$**
3. $v^2 - 20v + \underline{?}$ **$v^2 - 20v + 100 = (v - 10)^2$**
4. $c^2 - 12c + \underline{?}$ **$c^2 - 12c + 36 = (c - 6)^2$**
5. $x^2 - 18x + \underline{?}$ **$x^2 - 18x + 81 = (x - 9)^2$**
6. $x^2 + 2x + \underline{?}$ **$x^2 + 2x + 1 = (x + 1)^2$**

Example 2 Solve $x^2 + 3x - 10 = 0$ by completing the square.

Solution

$x^2 + 3x = 10$ — Move the constant term to the right side.

$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$ — Half of the coefficient of x is $\frac{3}{2}$.

$x^2 + 3x + \frac{9}{4} = 10 + \frac{9}{4}$ — Square it and add the result to *both* sides.

perfect square $\left(x + \frac{3}{2}\right)^2 = \frac{49}{4}$ constant

$x + \frac{3}{2} = \pm \frac{7}{2}$

$x = -\frac{3}{2} \pm \frac{7}{2}$

$= \frac{-3 \pm 7}{2}$

$x = 2$ or $x = -5$

The check is left for you. The solution set is $\{-5, 2\}$.

NAME ______________________ DATE ______________

12–2 Completing the Square *(continued)*

Solve by completing the square.

7. $x^2 + 8x = -15$ **$\{-3, -5\}$**
8. $x^2 + 6x = -8$ **$\{-2, -4\}$**
9. $x^2 + 4x = 5$ **$\{-5, 1\}$**
10. $x^2 - 7x - 8 = 0$ **$\{-1, 8\}$**
11. $x^2 - 3x - 4 = 0$ **$\{-1, 4\}$**
12. $x^2 + x - 12 = 0$ **$\{-4, 3\}$**
13. $y^2 + 7y = -12$ **$\{-3, -4\}$**
14. $n^2 - 5n = 36$ **$\{-4, 9\}$**
15. $x^2 + 3x = 10$ **$\{-5, 2\}$**
16. $y^2 - 9y + 8 = 0$ **$\{8, 1\}$**
17. $x^2 - 5x - 50 = 0$ **$\{-5, 10\}$**
18. $x^2 - 3x - 40 = 0$ **$\{-5, 8\}$**

Example 3 Solve $3x^2 - 5x - 2 = 0$ by completing the square.

Solution

$3x^2 - 5x = 2$ — Move the constant term to the right side.

$x^2 - \frac{5}{3}x = \frac{2}{3}$ — Divide both sides by 3 so that the coefficient of x^2 will be 1.

$x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2 = \frac{2}{3} + \left(\frac{5}{6}\right)^2$ — Complete the square.

$x^2 - \frac{5}{3}x + \frac{25}{36} = \frac{2}{3} + \frac{25}{36}$

$\left(x - \frac{5}{6}\right)^2 = \frac{49}{36}$

$x - \frac{5}{6} = \pm \frac{7}{6}$

$x = \frac{5}{6} \pm \frac{7}{6}$

$x = 2$ or $x = -\frac{1}{3}$

The check is left for you. The solution set is $\left\{2, -\frac{1}{3}\right\}$.

Solve by completing the square.

19. $2n^2 - n - 6 = 0$ **$\left\{-\frac{3}{2}, 2\right\}$**
20. $2x^2 + x - 1 = 0$ **$\left\{-1, \frac{1}{2}\right\}$**
21. $2x^2 - 5x = 12$ **$\left\{-\frac{3}{2}, 4\right\}$**
22. $2x^2 - 5x - 3 = 0$ **$\left\{-\frac{1}{2}, 3\right\}$**
23. $3x^2 + 11x = 4$ **$\left\{-4, \frac{1}{3}\right\}$**
24. $5x^2 + 4x - 1 = 0$ **$\left\{-1, \frac{1}{5}\right\}$**

Solve the equations by (a) completing the square and (b) by factoring.

25. $x^2 - 12x + 35 = 0$ **$\{5, 7\}$**
26. $x^2 + 18x + 45 = 0$ **26. $\{-3, -15\}$**
27. $a^2 + 17a - 60 = 0$ **$\{-20, 3\}$**
28. $3x^2 - 5x = 2$ **$\left\{-\frac{1}{3}, 2\right\}$**
29. $2x^2 + x = 10$ **$\left\{-\frac{5}{2}, 2\right\}$**
30. $2x^2 - 11x = -5$ **$\left\{\frac{1}{2}, 5\right\}$**

Solve. Write irrational roots in simplest radical form.

31. $y^2 + 8y = -10$ **$\{-4 + \sqrt{6}, -4 - \sqrt{6}\}$**
32. $4x^2 - 4x = 7$ **$\left\{\frac{1 + 2\sqrt{2}}{2}, \frac{1 - 2\sqrt{2}}{2}\right\}$**
33. $3x^2 - 6x - 2 = 0$ **$\left\{\frac{3 + \sqrt{15}}{3}, \frac{3 - \sqrt{15}}{3}\right\}$**

Mixed Review Exercises

Simplify.

1. $\sqrt{140}$ **$2\sqrt{35}$**
2. $\sqrt{1200}$ **$20\sqrt{3}$**
3. $\sqrt{16x^4y^7}$ **$4x^2|y^3|\sqrt{y}$**
4. $\sqrt{60a^6b^5c^3}$ **$2|a^3|b^2|c|\sqrt{15bc}$**
5. $\frac{x}{3 - x} + \frac{4}{x + 3}$ **$\frac{x^2 - x + 12}{(3 - x)(3 + x)}$**
6. $1 + \frac{n^2}{n^2 - 1}$ **$\frac{2n^2 - 1}{n^2 - 1}$**
7. $2(3x - 1) + (x + 5)(3x + 1)$ **$3x^2 + 22x + 3$**

NAME ______________________ DATE ____________

12–3 The Quadratic Formula

Objective: To learn the quadratic formula and use it to solve equations.

The Quadratic Formula

The solutions of a quadratic equation in the form of $ax^2 + bx + c = 0$, $a \neq 0$ and $b^2 - 4ac \geq 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1 Use the quadratic formula to solve $3x^2 + 5x - 2 = 0$.

Solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 3$, $b = 5$, and $c = -2$.

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$ Substitute the given values of a, b, and c.

$= \frac{-5 \pm \sqrt{25 + 24}}{6}$

$= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6}$

$x = \frac{-5 + 7}{6} = \frac{2}{6} = \frac{1}{3}$ or $x = \frac{-5 - 7}{6} = \frac{-12}{6} = -2$

The check is left to you. The solution set is $\left\{\frac{1}{3}, -2\right\}$.

Use the quadratic formula to solve each equation.

1. $x^2 + 3x - 10 = 0$ **$\{-5, 2\}$**
2. $x^2 - 8x + 7 = 0$ **$\{7, 1\}$**
3. $x^2 + 2x - 3 = 0$ **$\{-3, 1\}$**
4. $x^2 - 14x + 24 = 0$ **$\{2, 12\}$**
5. $n^2 + 5n - 6 = 0$ **$\{-6, 1\}$**
6. $x^2 - 6x - 40 = 0$ **$\{-4, 10\}$**
7. $2x^2 + 3x - 2 = 0$ **$\left\{-2, \frac{1}{2}\right\}$**
8. $3u^2 - 5u - 2 = 0$ **$\left\{2, -\frac{1}{3}\right\}$**
9. $3x^2 - 10x - 8 = 0$ **$\left\{4, -\frac{2}{3}\right\}$**
10. $3x^2 - 2x - 1 = 0$ **$\left\{-\frac{1}{3}, 1\right\}$**
11. $2x^2 - 5x - 7 = 0$ **$\left\{-1, \frac{7}{2}\right\}$**
12. $5x^2 + 6x - 8 = 0$ **$\left\{-2, \frac{4}{5}\right\}$**

Example 2 Use the quadratic formula to solve $x^2 = x - 6$.

Solution $x^2 - x + 6 = 0$ Rewrite the equation in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$, $b = -1$, and $c = 6$.

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(6)}}{2(1)} = \frac{1 \pm \sqrt{1 - 24}}{2} = \frac{1 \pm \sqrt{-23}}{2}$

Since $\sqrt{b^2 - 4ac} = \sqrt{-23}$ and $\sqrt{-23}$ isn't a real number, there is *no real solution.*

NAME ______________________ DATE ____________

12–3 The Quadratic Formula *(continued)*

Use the quadratic formula to solve each equation.

13. $x^2 - 4x + 6 = 0$ **No sol.**
14. $2x^2 = 3x - 1$ **$\left\{\frac{1}{2}, 1\right\}$**
15. $x^2 - 4x = 30$ **$\{2 + \sqrt{34}, 2 - \sqrt{34}\}$**
16. $2x^2 + 2x + 5 = 0$ **No sol.**
17. $4x^2 + 20x = -9$ **17. $\left\{-\frac{1}{2}, -\frac{9}{2}\right\}$**
18. $3x^2 - 3x + 4 = 0$ **No sol.**

Example 3 Use the quadratic formula to solve $2x^2 - 3x - 4 = 0$. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

Solution $2x^2 - 3x - 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 2$, $b = -3$, and $c = -4$.

$x = \frac{3 \pm \sqrt{9 - 4(2)(-4)}}{2(2)}$ Substitute the given values of a, b, and c.

$= \frac{3 \pm \sqrt{9 + 32}}{4}$ Simplify.

$= \frac{3 \pm \sqrt{41}}{4}$

Since $\sqrt{41} \approx 6.40$, $x \approx \frac{3 + 6.4}{4} = 2.35 \approx 2.4$

or $x \approx \frac{3 - 6.4}{4} = -0.85 \approx -0.9$

The check is left to you.

The solution set is $\left\{\frac{3 + \sqrt{41}}{4}, \frac{3 - \sqrt{41}}{4}\right\}$ or $\{2.4, -0.9\}$.

Use the quadratic formula to solve each equation. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator. **19–24. Answers given at the back of this Answer Key.**

19. $2x^2 = 8x - 5$
20. $3x^2 + 2x = 2$
21. $x^2 - 4x - 10 = 0$
22. $x^2 - 4x - 2 = 0$
23. $2x^2 - 4x + 1 = 0$
24. $3x^2 - 8x + 2 = 0$
25. $2x^2 + 1 = 3x$ **$\left\{\frac{1}{2}, 1\right\}$**
26. $3x^2 + x = 2$ **$\left\{-1, \frac{2}{3}\right\}$**
27. $4x^2 - 11x = 3$ **$\left\{3, -\frac{1}{4}\right\}$**

Mixed Review Exercises

1. $\{-3, 7$, and the real numbers between -3 and $7\}$
3. {the real numbers between -4 and 1}
4. $\{3$ and the real numbers between 0 and $3\}$

Solve each open sentence and graph its solution set. Graphs given at the back of this Answer Key.

1. $|x - 2| \leq 5$
2. $2|y + 5| = 4$ **$\{-3, -7\}$**
3. $|2n + 3| < 5$
4. $1 < 2z + 1 \leq 7$
5. $\sqrt{x} = 5$ **$\{25\}$**
6. $\sqrt{5n + 1} = 6$ **$\{7\}$**
7. $2\sqrt{2x} = 12$ **$\{18\}$**
8. $|3 + 2k| = 11$ **$\{4, -7\}$**
9. $3|2 - m| = 12$ **$\{-2, 6\}$**

Solve by completing the square.

10. $x^2 - 8x + 12 = 0$ **$\{6, 2\}$**
11. $3x^2 + 6x = 0$ **$\{0, -2\}$**
12. $c^2 - c = 12$ **$\{-3, 4\}$**

NAME ______________________ DATE ______________

12–4 Graphs of Quadratic Equations: The Discriminant

Objective: To use the discriminant to find the number of roots of the equation $ax^2 + bx + c = 0$ and the number of x-intercepts of the graph of the related equation $y = ax^2 + bx + c$.

Vocabulary

x-intercept The x-coordinate of a point where the curve of a parabola intersects the x-axis.

Discriminant For a quadratic equation in the form $ax^2 + bx + c = 0$, the value of $b^2 - 4ac$ is called the discriminant.

	Value of $b^2 - 4ac$	Number of different real roots of $ax^2 + bx + c = 0$	Number of x-intercepts of the graph of $y = ax^2 + bx + c$
Case 1	positive	2	2
Case 2	zero	1 (a double root)	1
Case 3	negative	0	0

Example 1 Write the value of the discriminant of each equation. Then use it to decide how many different real-number roots the equation has. (Do not solve.)

a. $x^2 - 4x - 5 = 0$ **b.** $x^2 - 6x + 9 = 0$ **c.** $x^2 - 3x + 5 = 0$

Solution

a. $x^2 - 4x - 5 = 0$

Substitute $a = 1$, $b = -4$, and $c = -5$ in the discriminant formula.

$b^2 - 4ac = (-4)^2 - 4(1)(-5) = 16 - 4(-5) = 16 + 20 = 36$

Since the discriminant is positive, $x^2 - 4x - 5 = 0$ has two real roots.

b. $x^2 - 6x + 9 = 0$

Substitute $a = 1$, $b = -6$, and $c = 9$ in the discriminant formula.

$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

Since the discriminant is zero, $x^2 - 6x + 9 = 0$ has one real root.

c. $x^2 - 3x + 5 = 0$

Substitute $a = 1$, $b = -3$, and $c = 5$ in the discriminant formula.

$b^2 - 4ac = (-3)^2 - 4(1)(5) = 9 - 20 = -11$

Since the discriminant is negative, $x^2 - 3x + 5 = 0$ has no real roots.

NAME ______________________ DATE ______________

Write the value of the discriminant of each equation. Then use it to decide how many different real-number roots the equation has. (Do not solve.)

6. −11; none
12. −32; none

1. $x^2 - 3x + 2 = 0$ **1; two**
2. $x^2 - 3x + 5 = 0$ **−11; none**
3. $x^2 - 8x + 16 = 0$ **0; one**
4. $2x^2 - 5x - 4 = 0$ **57; two**
5. $4y^2 - 12y + 9 = 0$ **0; one**
6. $3t^2 - 5t + 3 = 0$
7. $2n^2 + n - 6 = 0$ **49; two**
8. $5y^2 - 8y + 4 = 0$
9. $2x^2 - 7x + 5 = 0$ **9; two**
10. $4m^2 - 20m + 25 = 0$
11. $3x^2 - 7x + 2 = 0$ **25; two**
12. $-3b^2 + 2b - 3 = 0$
13. $-2x^2 + 6x - 3 = 0$
14. $3x^2 - 4x = 6$ **88; two**
15. $x^2 - x + 1 = 0$ **−3; none**

8. −16; none 10. 0; one 13. 12; two

Example 2 Determine (a) how many x-intercepts the parabola $y = 4x - x^2 + 5$ has and (b) whether its vertex lies above, below, or on the x-axis. (Do not draw the graph.)

Solution

a. The x-intercepts of the graph are the roots of the equation

$0 = 4x - x^2 + 5$, or $-x^2 + 4x + 5 = 0$.

Its discriminant is $b^2 - 4ac = (4)^2 - 4(-1)(5) = 36$, which is positive.

The equation has two real roots.
The parabola has two x-intercepts.

b. Since the coefficient of x^2 is negative, the parabola opens downward. Its vertex must be above the x-axis (otherwise, the parabola would not intersect the x-axis in two points).

Without drawing the graph of the given equation, determine (a) how many x-intercepts the parabola has and (b) whether its vertex lies above, below, or on the x-axis.

16. $y = x^2 - x - 6$ **two; below**
17. $y = -x^2 + 3x + 4$ **two; above**
18. $y = x^2 + 9 - 6x$ **one; on**
19. $y = 2x^2 + x + 3$ **none; above**
20. $y = 5x - 2 + 3x^2$ **two; below**
21. $y = 4x^2 + 4x + 1$ **one; on**

Mixed Review Exercises

Simplify. Assume no denominator equals zero.

1. $\frac{\sqrt{2} - 1}{\sqrt{3}}$ **$\frac{\sqrt{6} - \sqrt{3}}{3}$**
2. $\sqrt{\frac{8c^3}{3}} \cdot \sqrt{\frac{6c}{25}}$ **$\frac{4c^2}{5}$**
3. $3\sqrt{27} - 10\sqrt{3} + \sqrt{12}$ **$\sqrt{3}$**
4. $\frac{2x + 6}{x^3 + 5x^2 + 6x}$ **$\frac{2}{x(x + 2)}$**
5. $\frac{x}{x - 1} + \frac{3}{2x + 2}$ **5. $\frac{2x^2 + 5x - 3}{2(x - 1)(x + 1)}$**
6. $(3.1 \cdot 10^3)(4.2 \cdot 10^2)$ **1.302×10^6**

Find the vertex and the axis of symmetry of the graph of each equation.

7. $y = 2x^2$ **(0, 0); x = 0**
8. $y = x^2 + 6x + 9$ **8. (−3, 0); x = −3**
9. $y = 2x^2 + 5$ **(0, 5); x = 0**
10. $y = x^2 - 2x$ **(1, −1); x = 1**
11. $y = -x^2 + 4x$ **(2, 4); x = 2**
12. $y = x^2 + 4x - 5$ **(−2, −9); x = −2**

NAME ______________________ DATE ______________

12-5 Methods of Solution

Objective: To choose the best method for solving a quadratic equation.

Methods for Solving a Quadratic Equation	When to Use the Method
1. Using the quadratic formula	1. If an equation is in the form $ax^2 + bx + c = 0$, especially if you use a calculator.
2. Factoring	2. If an equation is in the form $ax^2 + bx = 0$, or if the factors are easily seen.
3. Using the property of square roots of equal numbers	3. If an equation is in the form $ax^2 + c = 0$.
4. Completing the square	4. If an equation is in the form $x^2 + bx + c = 0$ and b is an even number.

Example Solve each quadratic equation using the most appropriate method.

a. $6x^2 - 54 = 0$ b. $2x^2 - 7x + 5 = 0$

c. $2t^2 - 28t = 0$ d. $n^2 + 6n - 16 = 0$

Solution

a. $6x^2 - 54 = 0$ The equation has the form $ax^2 + c = 0$.
$6x^2 = 54$ Therefore, use the property of square roots of equal numbers.
$x^2 = 9$
$x = \pm 3$

The solution set is $\{-3, 3\}$.

b. $2x^2 - 7x + 5 = 0$ The equation has the form $ax^2 + bx + c = 0$.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(5)}}{2(2)}$$ Therefore, use the quadratic formula.

$$= \frac{7 \pm \sqrt{49 - 40}}{4}$$

$$= \frac{7 \pm \sqrt{9}}{4}$$

$$= \frac{7 \pm 3}{4}$$

$$x = \frac{7+3}{4} = \frac{10}{4} \quad \text{or} \quad x = \frac{7-3}{4} = \frac{4}{4}$$

The solution set is $\left\{\frac{5}{2}, 1\right\}$.

c. $2t^2 - 28t = 0$ The equation has the form $ax^2 + bx = 0$.
$2t(t - 14) = 0$ Therefore, factor.
$2t = 0$ or $t - 14 = 0$
$t = 0$ or $t = 14$

The solution set is $\{0, 14\}$.

NAME ______________________ DATE ______________

12-5 Methods of Solution *(continued)*

d. $n^2 + 6n - 16 = 0$ The equation has the form $x^2 + bx + c = 0$.
$n^2 + 6n = 16$ Therefore, complete the square.
$n^2 + 6n + 9 = 16 + 9$
$(n + 3)^2 = 25$
$n + 3 = \pm\sqrt{25}$
$n = -3 \pm 5$

The solution set is $\{-8, 2\}$.

Solve each quadratic equation using the most appropriate method. Write irrational answers in simplest radical form. You may wish to use a calculator.

5. $\{-3 + \sqrt{11}, -3 - \sqrt{11}\}$
11. $\{2 + \sqrt{2}, 2 - \sqrt{2}\}$
16. $\{1 + \sqrt{2}, 1 - \sqrt{2}\}$

1. $x^2 + 3x + 2 = 0$ **{−1, −2}**
2. $x^2 - 2x = 24$ **{6, −4}**
3. $6x^2 = 96$ **{−4, 4}**
4. $6x^2 - 12x = 0$ **{0, 2}**
5. $x^2 + 6x - 2 = 0$
6. $2x^2 + x = 10$ $\left\{-\frac{5}{2}, 2\right\}$
7. $x^2 + 5x = 6$ **{−6, 1}**
8. $2x^2 - 7x + 3 = 0$ $\left\{3, \frac{1}{2}\right\}$
9. $4x^2 - 36x = 0$ **{0, 9}**
10. $(x - 2)^2 = 9$ **{5, −1}**
11. $x^2 - 4x + 2 = 0$
12. $4x^2 + 4x - 3 = 0$ $\left\{\frac{1}{2}, -\frac{3}{2}\right\}$
13. $x^2 - 4x = 12$ **{6, −2}**
14. $x^2 - 11x + 24 = 0$ **{3, 8}**
15. $x^2 + 3x = 18$ **{−6, 3}**
16. $x^2 - 2x = 1$
17. $m^2 - 2m = 35$ **{−5, 7}**
18. $2x^2 - 8x + 8 = 0$ **{2}**
19. $2x^2 + 5x = -2$ $\left\{-\frac{1}{2}, -2\right\}$
20. $3x^2 - 7x = 6$ $\left\{-\frac{2}{3}, 3\right\}$
21. $4x^2 + 4x - 15 = 0$
22. $6y^2 - 13y - 8 = 0$
23. $5n^2 + 14n - 3 = 0$
24. $2x^2 + 3x = 27$ $\left\{-\frac{9}{2}, 3\right\}$
25. $2x^2 - 9x + 7 = 0$ $\left\{1, \frac{7}{2}\right\}$
26. $\frac{1}{x} = \frac{2x - 5}{3}$ $\left\{-\frac{1}{2}, 3\right\}$
27. $\frac{x - 1}{2x + 1} = \frac{x + 1}{3x - 1}$ **{0, 7}**

21. $\left\{-\frac{5}{2}, \frac{3}{2}\right\}$ **22.** $\left\{-\frac{1}{2}, \frac{8}{3}\right\}$ **23.** $\left\{-3, \frac{1}{5}\right\}$

Mixed Review Exercises

Evaluate if $x = 1$, $y = 3$, and $z = -4$. Write irrational expressions in simplest radical form.

1. $\pm\sqrt{y^2 - 4xz}$ **± 5**
2. $-\sqrt{y^2 + 4x}$ **$-\sqrt{13}$**
3. $\sqrt{z^2 - 4xy}$ **2**
4. $\sqrt{z^2 + 4xy}$ **$2\sqrt{7}$**
5. $\pm\sqrt{x^2 - 4yz}$ **± 7**
6. $\sqrt{x^2 + 4yz}$ **Not a real number**

Solve. Write irrational roots in simplest radical form.

7. $4x^2 + 4x - 15 = 0$ $\left\{-\frac{5}{2}, \frac{3}{2}\right\}$
8. $2d^2 - 3d - 77 = 0$ $\left\{7, -\frac{11}{2}\right\}$
9. $2(x + 1)(x + 3) = (2x + 1)(x + 5) - 2$ **{1}**
10. $c^2 - 3c - 1 = 0$ $\left\{\frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}\right\}$
11. $3p^2 + 4p + 1 = 0$ $\left\{-\frac{1}{3}, -1\right\}$
12. $8y^2 + 14y = -3$ $\left\{-\frac{1}{4}, -\frac{3}{2}\right\}$

NAME ____________________ DATE ____________

12-6 Solving Problems Involving Quadratic Equations

Objective: To use quadratic equations to solve problems.

Example A landscaper wishes to design a rectangular formal garden that will be 6 m longer than the width. If the area of the garden is to be 135 m^2, find the length and the width.

Solution

Step 1 The problem asks for the length and the width of the garden.

Step 2 Let x = the width in meters.
Then $x + 6$ = the length in meters.

Step 3 Use the formula for the area of a rectangle to write an equation.

$$\text{Length} \times \text{Width} = \text{Area}$$
$$x(x + 6) = 135$$

Step 4 Solve.

$$x^2 + 6x = 135$$
$$x^2 + 6x + 9 = 135 + 9$$
$$(x + 3)^2 = 144$$
$$x + 3 = \pm\sqrt{144}$$
$$x + 3 = \pm 12$$
$$x = -3 \pm 12$$
$$x = 9 \quad \text{or} \quad x = -15$$

Step 5 Disregard the negative root since a negative length has no meaning.

Check: 9: $9(9 + 6) \stackrel{?}{=} 135$
$9(15) \stackrel{?}{=} 135$
$135 = 135 \checkmark$

The width of the garden is 9 m and the length is 15 m.

Solve. Give irrational roots to the nearest tenth. Use your calculator or a table of square roots as necessary.

1. The sum of a number and its square is 72. Find the number. **8 or −9**
2. The sum of a number and its square is 30. Find the number. **5 or −6**
3. The difference of a number and its square is 110. Find the number. **11 or −10**
4. The difference of a number and its square is 132. Find the number. **12 or −11**
5. The width of a rectangular garden is 4 m shorter than the length. If the area of the garden is 320 m^2, find the length and the width. **length, 20 m; width, 16 m**
6. An architect wants to design a rectangular building such that the area of the floor is 400 yd^2. The length of the floor is to be 10 yd longer than the width. Find the length and the width of the floor. **length, 25.6 yd; width, 15.6 yd**

NAME ____________________ DATE ____________

12-6 Solving Problems Involving Quadratic Equations (continued)

Solve. Give irrational roots to the nearest tenth. Use your calculator or a table of square roots as necessary.

7. The length of a rectangle is 3 times the width. The area of the rectangle is 48 cm^2. Find the length and the width. **length, 12 cm; width, 4 cm**
8. The length of a rectangular park is 5 m longer than the width. If the area of the park is 150 m^2, find the length and the width. **length, 15 m; width, 10 m**
9. The length of a rectangle is twice the width. The area of the rectangle is 72 m^2. Find the length and the width. **length, 12 m; width, 6 m**
10. The length of the base of a triangle is twice its altitude. If the area of the triangle is 144 cm^2, find the altitude. (*Hint:* Area of a triangle = $\frac{1}{2} \times$ base $\times$ height.) **12 cm**
11. The altitude of a triangle is 5 m less than its base. The area of the triangle is 80 m^2. Find the base. **15.4 m**
12. If the sides of a square are increased by 3 cm, its area becomes 144 cm^2. Find the length of the sides of the original square. **9 cm**
13. If the sides of a square are increased by 4 cm, its area becomes 196 cm^2. Find the length of the sides of the original square. **10 cm**
14. If you square a positive number, add ten times the number and subtract 84, the result is 180. What is the number? **12**
15. Oscar has a rectangular garden that measures 18 m by 15 m. Next year he wants to increase the area to 340 m^2 by increasing the width and the length by the same amount. What will be the dimensions of the garden next year? **20 m by 17 m**
16. Working alone, Carrie can paint a house in 3 h less than Walter. Together Carrie and Walter can paint a house in 8 h. How long does it take Walter to paint a house alone? **17.6 h**
17. Andrea can ride her bike 2 mi/h faster than Ruby. It takes Andrea 48 min less to travel 50 mi than it does Ruby. What is Andrea's rate in mi/h? **12.2 mi/h**

Mixed Review Exercises

Solve.

1. $\frac{\sqrt{n}}{2} = \frac{\sqrt{5}}{1}$ **{20}**
2. $\frac{\sqrt{x - 2}}{4} = \frac{3}{2}$ **{38}**
3. $\frac{\sqrt{3t}}{8} = \frac{1}{4}$ **$\left\{\frac{4}{3}\right\}$**
4. $\frac{12}{5} = \frac{36}{n}$ **{15}**
5. $m^2 = 12m - 32$ **{4, 8}**
6. $\frac{x - 2}{x + 3} + x = 2$ **{2, −4}**

In Exercises 7–9, (x_1, y_1) and (x_2, y_2) are ordered pairs of the same direct variation. Find the missing value.

7. $x_1 = 1, y_1 = 3$; $x_2 = 6, y_2 = \underline{?}$ **18**
8. $x_1 = 8, y_1 = \underline{?}$ **10**; $x_2 = 4, y_2 = 5$
9. $x_1 = 12, y_1 = 16$; $x_2 = \underline{?}$ **6**, $y_2 = 8$

NAME ____________________ DATE ____________

12–7 Direct and Inverse Variations Involving Squares

Objective: To use quadratic direct variation and inverse variation as a square in problem solving.

Vocabulary

Quadratic direct variation A function of the form

$$y = kx^2, \text{ where } k \text{ is a nonzero constant.}$$

You say that y *varies directly as* x^2 or that y *is directly proportional to* x^2.
If (x_1, y_1) and (x_2, y_2) are ordered pairs of the same quadratic variation, and neither x_1 nor x_2 is zero, then

$$\frac{y_1}{x_1^2} = \frac{y_2}{x_2^2}.$$

Inverse variation as the square A function of the form

$$x^2y = k, \quad \text{where } k \text{ is a nonzero constant,}$$

$$\text{or} \quad y = \frac{k}{x^2}, \quad \text{where } x \neq 0.$$

You say that y *varies inversely as* x^2 or y *is inversely proportional to* x^2.

If (x_1, y_1) and (x_2, y_2) are ordered pairs of the function defined by $x^2y = k$, then

$$x_1^2y_1 = x_2^2y_2.$$

Example 1 Given that a varies directly as the square of d, and $a = 18$ when $d = 3$, find the value of d when $a = 8$.

Solution 1 Use $a = kd^2$: $18 = 9k$; $2 = k$; $a = 2d^2$

For $a = 8$: $8 = 2d^2$; $4 = d^2$; $\pm 2 = d$

Solution 2 $\frac{a_1}{d_1^2} = \frac{a_2}{d_2^2}$; $\frac{18}{3^2} = \frac{8}{d_2^2}$; $18d_2^2 = 72$; $d_2^2 = 4$; $d_2 = \pm 2$

CAUTION Remember that in a particular situation you must check each root to see if it makes sense.

Solve. Give roots to the nearest tenth. You may wish to use a calculator.

1. Given that d varies directly as the square of t and $d = 16$ when $t = 2$, find the value of d when $t = 3$. **36**
2. Given that A varies directly as the square of e and $A = 32$ when $e = 2$, find the value of e when $A = 8$. **1 or −1**
3. The price of a diamond varies directly as the square of its mass in carats. If a 1.5 carat diamond costs $2700, find the cost of a 3 carat diamond. **$10,800**

NAME ____________________ DATE ____________

12–7 Direct and Inverse Variations Involving Squares *(continued)*

Example 2 Given that h varies inversely as the square of r, and that $r = 3$ when $h = 18$, find the value of r when $h = \frac{3}{10}$.

Solution 1 Use $h = \frac{k}{r^2}$: $18 = \frac{k}{9}$; $162 = k$; $h = \frac{162}{r^2}$

For $h = \frac{3}{10}$: $\frac{3}{10} = \frac{162}{r^2}$; $3r^2 = 1620$; $r^2 = 540$; $r = \pm\sqrt{540}$; $= \pm 6\sqrt{15}$

Solution 2 Use $r_1^2h_1 = r_2^2h_2$: $(3)^2 18 = r_2^2\left(\frac{3}{10}\right)$; $162 = \frac{3r_2^2}{10}$; $3r_2^2 = 1620$; $r_2^2 = 540$; $r_2 = \pm\sqrt{540}$; $r_2 = \pm 6\sqrt{15}$

Solve. Give roots to the nearest tenth. You may wish to use a calculator.

4. Given that h varies inversely as the square of r, and that $r = 4$ when $h = 25$, find the value of r when $h = 4$. **10 or −10**
5. Given that h varies inversely as the square of r, and that $r = 5$ when $h = 36$, find the value of h when $r = 15$. **4**
6. Light intensity varies inversely as the square of the distance from the source. At 2 m, an intensity scale has a reading of 6. What will be the reading at 6 m? **0.7**
7. The distance it takes an automobile to stop varies directly as the square of its speed. If the stopping distance for a car traveling at 80 km/h is 175 m, what is the stopping distance for a car traveling at 48 km/h? **63 m**
8. The height of a cylinder of a given volume is inversely proportional to the square of the radius. A cylinder of radius 4 cm has a height of 12 cm. What is the radius of a cylinder of equal volume whose height is 48 cm? **2 cm**
9. The length of a pendulum varies directly as the square of the time in seconds it takes to swing from one side to the other. If it takes a 100 cm pendulum 1 s to swing from one side to the other, how many seconds does it take a 50 cm pendulum to swing? **0.7 s**

Mixed Review Exercises

Solve for the indicated variable. State any restrictions.

1. $S = C(1 - r)$; r **$r = \frac{C - S}{C}$; $C \neq 0$**
2. $d = \frac{a(2t - v)}{2}$; v **$v = \frac{2at - 2d}{a}$; $a \neq 0$**
3. $A = \frac{h(b + c)}{2}$; c **$c = \frac{2A - hb}{h}$; $h \neq 0$**

Write an equation, in standard form, of the line passing through the given points.

4. (2, 5), (4, 6) **$x - 2y = -8$**
5. (0, 2), (−4, 1) **$x - 4y = -8$**
6. (2, 3), (−1, 0) **$x - y = -1$**

NAME ______________________ DATE ______________

12–8 Joint and Combined Variation

Objective: To solve problems involving joint variation and combined variation.

Vocabulary

Joint variation When a variable varies directly as the product of two or more other variables. You can express the relationship in the forms

$$z = kxy, \quad \text{where } k \text{ is a nonzero constant,}$$

and

$$\frac{z_1}{x_1y_1} = \frac{z_2}{x_2y_2}.$$

You say that *z varies jointly as x and y.*

Combined variation When a variable varies directly as one variable and inversely as another. You can express the relationship in the forms

$$zy = kx \quad \left(\text{or } z = \frac{kx}{y}\right), \quad \text{where } k \text{ is a nonzero constant,}$$

and

$$\frac{z_1y_1}{x_1} = \frac{z_2y_2}{x_2}.$$

You say that *z varies directly as x and inversely as y.*

Example 1 The volume of a right circular cone varies jointly as the height, h, and the square of the radius, r. If $V_1 = 1848\pi$, $h_1 = 9$, $r_1 = 14$, $h_2 = 12$, and $r_2 = 7$, find V_2.

Solution

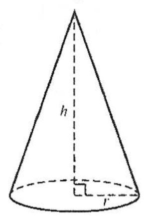
$\frac{V_1}{h_1r_1^2} = \frac{V_2}{h_2r_2^2}$ Use the form for joint variation.

$\frac{1848\pi}{9(14)^2} = \frac{V_2}{12(7)^2}$ Substitute the given values.

$9(196)V_2 = 12(49)(1848\pi)$ Solve for V_2.

$1764V_2 = 1{,}086{,}624\pi$

$V_2 = 616\pi$



Solve. Give roots to the nearest tenth. You may wish to use a calculator.

1. d varies jointly as r and t. If $d = 75$ when $r = 30$ and $t = 10$, find d when $r = 18$ and $t = 6$. **27**
2. m varies jointly as v and the square of u. If $m = 60$ when $v = 25$ and $u = 12$, find u when $m = 9$ and $v = 15$. **6 or −6**
3. The volume of a pyramid varies jointly as the height and the area of the base. A pyramid 21 cm high has a base area of 25 cm^2 and a volume of 175 cm^3. What is the volume if the height is 15 cm and the area of the base is 36 cm^2? **180 cm^3**

NAME ______________________ DATE ______________

Example 2 The power, P, of an electric current varies directly as the square of the voltage, V, and inversely as the resistance, R. If 6 volts applied across a resistance of 3 ohms produces 12 watts of power, how much voltage applied across a resistance of 6 ohms will produce 24 watts of power?

Solution

$\frac{P_1R_1}{V_1^2} = \frac{P_2R_2}{V_2^2}$ Use the form for combined variation.

$\frac{12(3)}{6^2} = \frac{24(6)}{V_2^2}$ Let $P_1 = 12$, $V_1 = 6$, $R_1 = 3$, $P_2 = 24$, and $R_2 = 6$.

$36V_2^2 = 36(24)(6)$ Solve for V_2.

$V_2^2 = 144$

$V_2 = \pm\sqrt{144} = \pm 12$ Discard the negative root since a negative voltage has no meaning.

12 volts will produce the required power.

Solve. Give roots to the nearest tenth. You may wish to use a calculator.

4. c varies directly as a and inversely as b. If $c = 16$ when $a = 96$ and $b = 16$, find c when $a = 60$ and $b = 8$. **20**
5. c varies inversely as the square of h and directly as n. If $c = 1$ when $h = 12$ and $n = 25$, find h when $c = 4.84$ and $n = 4$. **2.2 or −2.2**
6. w varies directly as u and inversely as v^2. If $w = 8$ when $u = 2$ and $v = 3$, find u when $v = 2$ and $w = 27$. **3**
7. If Q varies directly as the square of t and inversely as the cube of r, and $Q = 108$ when $t = 12$ and $r = 2$, find Q when $t = 15$ and $r = 3$. **50**
8. The power, P, of an electric current varies directly as the square of the voltage, V, and inversely as the resistance, R. If 6 volts applied across a resistance of 3 ohms produces 12 watts of power, how much power will 12 volts applied across a resistance of 9 ohms produce? **16 watts**
9. The volume of a cone varies jointly as its height and the square of its radius. A certain cone has a volume of 154π cm^3, a height of 3 cm, and a radius of 7 cm. Find the radius of another cone that has a height of 7 cm and a volume of 264π cm^3. **6 cm**
10. The distance a car travels from rest varies jointly as its acceleration and the square of the time of motion. A car travels 500 m from rest in 5 s at an acceleration of 40 m/s^2. How many seconds will it take the car to travel 150 m at an acceleration of 3 m/s^2? **10 s**

Mixed Review Exercises

Solve each system.

1. $2x - y = -1$, $x - y = -2$ **(1, 3)**
2. $x + y = 6$, $x - 2y = 12$ **(8, −2)**
3. $2x - 5y = 2$, $-x + 5y = 4$ **(6, 2)**
4. $3x + 2y = 3$, $x + 2y = 9$ **(−3, 6)**