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10 Inequalities

10–1 Order of Real Numbers

Objective: To review the concept of order and to graph inequalities in one variable.

Vocabulary

Inequality A statement formed by placing an inequality symbol between numerical or variable expressions.

Solutions of an inequality The values of the domain of the variable for which the inequality is true.

Solution set of an inequality The set of all solutions of the inequality.

Graph of an inequality The graph of the numbers in the solution set of an inequality.

Symbols

Inequality symbols Symbols used to show the order of two real numbers:

$x > 2$ (x is greater than 2.) $x \geq 2$ (x is greater than *or* equal to 2.)
$x < 2$ (x is less than 2.) $x \leq 2$ (x is less than *or* equal to 2.)

$-3 < x < 1$ (x is greater than -3 *and* less than 1.)

Example 1 Translate the statements into symbols.

a. -2 is greater than -6 **b.** x is less than or equal to 5.

Solution **a.** $-2 > -6$ **b.** $x \leq 5$

Translate the statements into symbols.

5. $\mathbf{1 < 4 < 5.5}$ **6.** $\mathbf{-2 < 0 < 3}$
9. $\mathbf{0 < 3 < 3.5}$ **10.** $\mathbf{-3.5 < -3 < 0}$

1. -2 is less than 5. $\mathbf{-2 < 5}$
2. -3 is greater than -4. $\mathbf{-3 > -4}$
3. -6 is less than or equal to -2. $\mathbf{-6 \leq -2}$
4. 4 is greater than or equal to 1. $\mathbf{4 \geq 1}$
5. 4 is greater than 1 and less than 5.5.
6. 0 is greater than -2 and less than 3.
7. -5 is between 1 and -7. $\mathbf{-7 < -5 < 1}$
8. 3 is between -5 and 5. $\mathbf{-5 < 3 < 5}$
9. 3.5 is greater than 3 and 3 is greater than 0.
10. -3.5 is less than -3 and -3 is less than 0.
11. The number n is greater than 6. $\mathbf{n > 6}$
12. The number n is less than 12. $\mathbf{n < 12}$

Example 2 Classify each statement as true or false.

a. $-2 < 2 < 4$ **b.** $-1 < 5 < 3$

Solution **a.** $-2 < 2 < 4$ is true since *both* $-2 < 2$ and $2 < 4$ are true.

b. $-1 < 5 < 3$ is false because $-1 < 5$ is true *but* $5 < 3$ is false.

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10–1 Order of Real Numbers *(continued)*

Example 3 Classify each statement as true or false.

a. $5 \leq 8$ **b.** $2 \leq 2$

Solution **a.** For $5 \leq 8$ to be true, *either* $5 < 8$ or $5 = 8$ must be true. $5 \leq 8$ is true since $5 < 8$.

b. $2 \leq 2$ is true since $2 = 2$ is true.

Classify each statement as true or false.

13. $-5 < 1 < 6$ **True** **14.** $-8 < 2 < 5$ **True** **15.** $3 > 0 > 1$ **False** **16.** $-3 < -2 < 3$ **True**
17. $8 \geq 4$ **True** **18.** $13 \leq 22$ **True** **19.** $-9 \leq -16$ **False** **20.** $-2 \geq -3$ **True**
21. $|-2| \geq -2$ **True** **22.** $|-2| \leq 0$ **False** **23.** $|0.5| < -0.3$ **False** **24.** $|-5| \leq |-10|$ **True**

Example 4 Solve $y + 2 \leq 3$ if $y \in \{-2, -1, 0, 1, 2, 3\}$.

Solution Find all the values in the domain that make the inequality true.

Replace y with each of its values in turn:

$-2 + 2 \leq 3$	True
$-1 + 2 \leq 3$	True
$0 + 2 \leq 3$	True
$1 + 2 \leq 3$	True
$2 + 2 \leq 3$	False
$3 + 2 \leq 3$	False

The solution set is $\{-2, -1, 0, 1\}$

25. $\{-3, -2, -1, 0, 1\}$
26. $\{-3, -2, -1, 0\}$
27. $\{-3, -2, -1, 0, 1, 2, 3\}$
28. $\{-3, -2, -1, 0, 1\}$
29. $\{-3, -2, -1, 0, 1, 2\}$
30. $\{-3, -2, -1, 0, 1\}$
31. $\{-3, 3\}$
32. $\{-3, -2, -1, 0, 1, 2, 3\}$

Solve each inequality if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

25. $2x < 4$ **26.** $3x < 3$ **27.** $-2x \leq 6$ **28.** $x + 1 < 3$
29. $-2 + x \leq 0$ **30.** $1 - x \geq 0$ **31.** $x^2 \geq 8$ **32.** $x^2 \leq 9$

Mixed Review Exercises

Solve.

1. $x - 4 = 11$ $\{\mathbf{15}\}$ **2.** $12 = 3(c - 1)$ $\{\mathbf{5}\}$ **3.** $3 - 2a = 15$ $\{\mathbf{-6}\}$
4. $\frac{x}{2} = -15$ $\{\mathbf{-30}\}$ **5.** $\frac{24}{y} = \frac{8}{3}$ $\{\mathbf{9}\}$ **6.** $\frac{2x + 4}{4} = \frac{x + 8}{3}$ $\{\mathbf{10}\}$
7. $3(4 + n) = 2(n - 5)$ $\{\mathbf{-22}\}$ **8.** $(x + 2)(x + 5) = (x + 3)^2$ $\{\mathbf{-1}\}$ **9.** $\frac{n}{3} + 6 = n$ $\{\mathbf{9}\}$
10. $\frac{x}{3} = \frac{x - 5}{4}$ $\{\mathbf{-15}\}$ **11.** $\frac{3x}{8} + \frac{x}{4} = \frac{5}{4}$ $\{\mathbf{2}\}$ **12.** $\frac{1}{2}(x + 6) = 8$ $\{\mathbf{10}\}$

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10–2 *Solving Inequalities*

Objective: To transform inequalities in order to solve them.

Properties

Property of Comparison For all real numbers a and b, one and only one of the following statements is true: $a < b$, $a = b$, $a > b$.

Transitive Property of Order For all real numbers a, b, and c,
1. If $a < b$ and $b < c$, then $a < c$;
2. If $c > b$ and $b > a$, then $c > a$.

Addition Property of Order For all real numbers a, b, and c,
1. If $a < b$, then $a + c < b + c$;
2. If $a > b$, then $a + c > b + c$.

Multiplication Property of Order
For all real numbers a, b, and c, such that $c > 0$:
1. If $a < b$, then $ac < bc$;
2. If $a > b$, then $ac > bc$.

For all real numbers a, b, and c, such that $c < 0$:
1. If $a < b$, then $ac > bc$;
2. If $a > b$, then $ac < bc$.

Vocabulary

Equivalent inequality An inequality with the same solution set as another inequality.

Transformations That Produce an Equivalent Inequality

1. **Substituting** for either side of the inequality an expression equivalent to that side.
2. **Adding to (or subtracting from)** each side of the inequality the same real number.
3. **Multiplying (or dividing)** each side of the inequality by the same *positive* number.
4. **Multiplying (or dividing)** each side of the inequality by the same *negative* number and *reversing the direction of the inequality.*

CAUTION Multiplying both sides of an inequality by zero does not produce an inequality; the result is the identity $0 = 0$.

Example 1 Tell how to transform the first inequality into the second one.

a. $m - 6 > 2$
$m > 8$

b. $-6k \geq 18$
$k \leq -3$

Solution
a. Add 6 to each side.
b. Divide each side by -6 and reverse the direction of the inequality.

Tell how to transform the first inequality into the second one.

1. $t + 2 < 6$, $t < 4$ **Subtract 2 from each side.**
2. $x - 3 > 7$, $x > 10$ **Add 3 to each side.**
3. $x + 5 < 0$, $x < -5$ **Subtract 5 from each side.**

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10–2 *Solving Inequalities* (continued) Answers given at the back of this Answer Key.

Tell how to transform the first inequality into the second one.

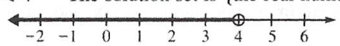
4. $4p < 28$, $p < 7$
5. $2m < -12$, $m < -6$
6. $-7a < 21$, $a > -3$
7. $3 < \frac{x}{5}$, $15 < x$
8. $\frac{x}{-2} \leq -4$, $x \geq 8$
9. $-\frac{t}{3} \geq 0$, $t \leq 0$

Example 2 Solve $4x - 1 < 7 + 2x$ and graph its solution set.

Solution

$4x - 1 + 1 < 7 + 2x + 1$ Add 1 to each side.
$4x < 8 + 2x$
$4x - 2x < 8 + 2x - 2x$ Subtract $2x$ from each side.
$2x < 8$
$\frac{2x}{2} < \frac{8}{2}$ Divide each side by 2.
$x < 4$ The solution set is {the real numbers less than 4}.

The graph is



Example 3 Solve $2(w - 6) \geq 3(1 - w)$ and graph its solution set.

Solution

$2w - 12 \geq 3 - 3w$ Use the distributive property.
$5w \geq 15$ Add $3w$ to each side and add 12 to each side.
$w \geq 3$ Divide each side by 5.

The solution set is {the real numbers greater than or equal to 3}.

The graph is

Solve each inequality. Graph the solution set. Graphs given at the back of this Answer Key.

10. $x - 2 \geq 3$ **$x \geq 5$**
11. $8 < z + 2$ **$6 < z$**
12. $4p < 20$ **$p < 5$**
13. $15 \leq 5w$ **$3 \leq w$**
14. $-24 > -6m$ **$m > 4$**
15. $\frac{d}{2} > -3$ **$d > -6$**
16. $3 - g > 0$ **$3 > g$**
17. $2v + 1 > 9$ **$v > 4$**
18. $6 \geq 2k - 6$ **$6 \geq k$**
19. $3 + \frac{x}{2} \leq 4$ **$x \leq 2$**
20. $6 - \frac{2}{3}c > 0$ **$9 > c$**
21. $3r - 4 < 4r + 1$ **$-5 < r$**
22. $4y < 3y + 6$ **$y < 6$**
23. $3f - 2 < 2f + 3$ **$f < 5$**
24. $2r - 3 < 3r + 1$ **$-4 < r$**
25. $6 - 2b > 3 - b$ **$b < 3$**
26. $2(x - 3) \leq 4$ **$x \leq 5$**
27. $6 < 3(2 - m)$ **$m < 0$**
28. $3(x + 2) \leq 3x + 2$ **No solution**
29. $4(k - 3) \geq 6(1 - k)$ **$k \geq \frac{9}{5}$**

Mixed Review Exercises

Classify each statement as true or false.

1. $|-2| > -(-1)$ **True**
2. $|-4| \leq |4|$ **True**
3. $|-7| > |-8|$ **False**

Solve.

4. $5f - 3 = f + 17$ {5}
5. $0 = 3x + 12$ {−4} {2}
6. $3y - 2(y - 1) = -4$ {−6}
7. $x - 2(8 - x) = -x$ {4}
8. $a(a + 4) = (a - 6)(a - 5)$
9. $3x + 2(x - 1) = x + 22$ {6}

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10–3 Solving Problems Involving Inequalities

Objective: To solve problems involving inequalities.

Example 1 The sum of two consecutive integers is less than 80. Find the pair of such integers with the greatest sum.

Solution

Step 1 The problem asks for the largest pair of consecutive integers whose sum is less than 80.

Step 2 Let n = the smaller of the two consecutive integers. Then $n + 1$ = the larger of the two consecutive integers.

Step 3 Use the variables to write an inequality based on the given information. The sum of two consecutive integers is less than 80.

$$n + (n + 1) < 80$$

Step 4 Solve the open sentence:

$$n + n + 1 < 80$$
$$2n + 1 < 80$$
$$2n < 79$$
$$n < 39\frac{1}{2}$$

The largest integer less than $39\frac{1}{2}$ is 39. Thus, $n = 39$ and $n + 1 = 40$.

Step 5 *Check:* Is the sum 39 + 40 less than 80?

$$39 + 40 \stackrel{?}{<} 80$$
$$79 < 80 \checkmark$$

39 and 40 form the largest pair of consecutive integers whose sum is less than 80.

For each of the following: Answers may vary.
a. Choose a variable to represent the number in bold face type.
b. Use the variable to write an inequality based on the given information. (Do not solve.)

1. Harry, who is not yet 16 years old, is three years younger than Lena. **(Lena's age)** **a.** l **b.** $l - 3 < 16$
2. After driving 125 miles, Barry still has more than 75 miles to travel. **(the total number of miles Barry will drive)** **a.** d **b.** $d - 125 > 75$

Example 2 Translate each phrase into mathematical terms.

	Solution
a. The age of the house *is at least* 75 years	a. $a \geq 75$
b. The distance *is no less than* 250 km	b. $d \geq 250$
c. The price of the ticket *is at most* \$190	c. $p \leq 190$
d. Her driving time to school *is no more than* 30 min	d. $t \leq 30$

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10–3 Solving Problems Involving Inequalities *(continued)*

For each of the following: Answers may vary.
a. Choose a variable to represent the number in bold face type.
b. Use the variable to write an inequality based on the given information. (Do not solve.)

3. Katrina's balance in her checking account is \$160. She must deposit at least enough money in her account to be able to pay her car payment of \$295. **(the amount of deposit)** **a.** d **b.** $160 + d \geq 295$
4. Dan bicycled 12 more kilometers than one third **the number of kilometers Manuel bicycled.** Dan bicycled at most 24 km. **a.** m **b.** $12 + \frac{1}{3}m \leq 24$
5. The length of a rectangle is 7 cm longer than **the width.** The perimeter is no more than 38 cm. **a.** w **b.** $2w + 2(w + 7) \leq 38$
6. The sum of two consecutive odd integers is at most 185. **(the greater integer)** **a.** n **b.** $n + (n - 2) \leq 185$
7. The product of two consecutive integers is no less than 75. **(the smaller integer)** **a.** n **b.** $n(n + 1) \geq 75$

Solve.

8. The sum of two consecutive integers is less than 100. Find the pair of integers with the greatest sum. **49 and 50**
9. The sum of two consecutive even integers is at most 180. Find the pair of integers with the greatest sum. **88 and 90**
10. After selling 160 copies of the program to a school play, an usher had fewer than 40 copies left. How many copies of the program did the usher have originally? **less than 200 copies**
11. A house and a lot together cost more than \$86,000. The house costs \$2000 more than six times the cost of the lot. How much does the lot cost by itself? **more than \$12,000**
12. Andrew's salary is \$1200 a month plus a 4% commission on all his sales. What must the amount of his sales be to earn at least \$1600 each month? **at least \$10,000**

Mixed Review Exercises

Solve.

1. $|x| = 5$ **{−5, 5}**
2. $|1 - 5| = k$ **{4}**
3. $|y| - 2 = 6$ **{−8, 8}**
4. $2|b| = 16$ **{−8, 8}**
5. $x = |-1 - (-3)|$ **{2}**
6. $n = -|5 - 8|$ **{−3}**

Factor completely.

7. $x^2 + 12x + 35$ **$(x + 5)(x + 7)$**
8. $x^3 - 3x^2 - 18x$ **$x(x - 6)(x + 3)$**
9. $36x^2 - 25$ **$(6x + 5)(6x - 5)$**
10. $2y^2 - 5y - 3$ **$(2y + 1)(y - 3)$**
11. $x^2 + 8xy + 16y^2$ **$(x + 4y)^2$**
12. $12x^3 - 3x$ **$3x(2x + 1)(2x - 1)$**

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10-4 Solving Combined Inequalities

Objective: To find the solution sets of combined inequalities.

Vocabulary

Conjunction A sentence formed by joining two open sentences by the word *and.* For example, $-1 < x$ *and* $x < 4$, which can also be written as $-1 < x < 4$.

Solve a conjunction To find the values of the variables for which *both* open sentences in the conjunction are true.

Disjunction A sentence formed by joining two open sentences by the word *or.* For example, $y > 1$ *or* $y = 1$.

Solve a disjunction To find the values of the variables for which *at least one* of the open sentences in the disjunction is true.

Example 1 Draw the graph of each open sentence.

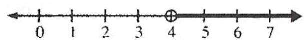
a. $4 < x$

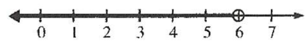
b. $x < 6$

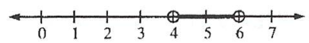
c. conjunction: $4 < x$ *and* $x < 6$

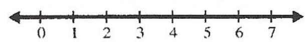
d. disjunction: $4 < x$ *or* $x < 6$

Solution

a. 

b. 

c. 

d. 

Draw the graph of each open sentence. Graphs given at the back of this Answer Key.

1. $-2 < t$ and $t \le 1$ 2. $r > 2$ or $r \le -1$ 3. $2 \le n$ and $n \le 6$ 4. $x < -1$ or $x \ge 1$

Example 2 Describe the graph of each open sentence.

a. conjunction: $t < 3$ *and* $t \ge 3$ b. disjunction: $t < 3$ *or* $t \ge 3$.

Solution

a. No real number can be less than 3 and also greater than or equal to 3. The solution set is the empty set. It has no graph.

b. Every real number is either less than 3 or greater than or equal to 3. The solution set is {the real numbers}. Its graph is the entire number line.

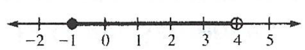
Example 3 Solve the conjunction $-2 \le x - 1 < 3$ and graph its solution set.

Solution 1 Solve the conjunction:

$-2 \le x - 1$	and	$x - 1 < 3$
$-2 + 1 \le x - 1 + 1$	|	$x - 1 + 1 < 3 + 1$
$-1 \le x$	and	$x < 4$

$-1 \le x < 4$

The solution set is $\{-1$, and all the real numbers *between* -1 and $4\}$.

The graph is 

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10-4 Solving Combined Inequalities *(continued)*

Solution 2

$-2 \le x - 1 < 3$

$-2 + \mathbf{1} \le x - 1 + \mathbf{1} < 3 + \mathbf{1}$ Add 1 to each part of the inequality.

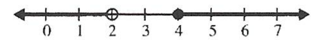
$-1 \le x < 4$

Example 4 Solve the disjunction $2x + 1 < 5$ or $3x \ge x + 8$ and graph its solution set.

Solution

$2x + 1 < 5$	or	$3x \ge x + 8$
$2x + 1 - 1 < 5 - 1$	|	$3x - x \ge x + 8 - x$
$2x < 4$	|	$2x \ge 8$
$x < 2$	or	$x \ge 4$

The solution set is {4, and the real numbers greater than 4 *or* less than 2}.

The graph is 

Solve each open sentence. Graph the solution set, if there is one. Solutions and graphs given at the back of this Answer Key.

5. $-1 < a - 1 < 4$
6. $-3 < y + 1 \le 2$
7. $-2 < -3 + d \le 1$
8. $-4 \le 2 + r < 2$
9. $-4 \le 2a + 6 < 10$
10. $-3 < 2b + 1 \le 5$
11. $-8 \le 3m + 1 < 7$
12. $-4 < 3n + 5 \le 8$
13. $x - 1 < -4$ or $x - 1 > 5$
14. $h + 3 \le -1$ or $h + 3 \ge 1$
15. $2x - 1 \le -5$ or $2x - 1 > 5$
16. $3 + 2y < -5$ or $3 + 2y > 5$
17. $-5x > 20$ or $10 + 5x \ge 0$
18. $2d - 3 < -5$ or $5 < 2d - 3$
19. $-3m < 6$ and $18 + 3m < 0$
20. $-3 \le 1 - t$ and $1 - t < 2$

Mixed Review Exercises

Choose a variable and use the variable to write an inequality. Answers may vary.

1. The finish line is at least 20 yd away. $\boldsymbol{f \ge 20}$
2. The temperature cannot exceed 25°C. $\boldsymbol{t \le 25}$
3. The weight is at most 105 lb. $\boldsymbol{w \le 105}$
4. The flight takes at least 2 h. $\boldsymbol{f \ge 2}$
5. The cost is not more than $75. $\boldsymbol{c \le 75}$
6. The tolerance is smaller than 1 cm. $\boldsymbol{t < 1}$
7. Ray averages at most 15 points per game. $\boldsymbol{a \le 15}$
8. Joy won at least 12 tennis matches. $\boldsymbol{w \ge 12}$

Evaluate each expression if $k = -3$, $m = 9$, and $x = 3$.

9. $|x - k|$ **6**
10. $|m - k|$ **12**
11. $|x + k|$ **0**
12. $|k - x|$ **6**
13. $|k - m|$ **12**
14. $|k + m|$ **6**

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10–5 Absolute Value in Open Sentences

Objective: To solve equations and inequalities involving absolute value.

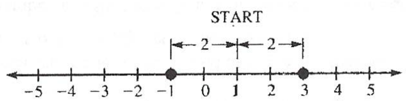
Symbols

$|a - b| = |b - a|$ (The distance between a and b on a number line.)

$|a + b| = |a - (-b)|$ (The distance between a and the opposite of b on a number line.)

Example 1 Solve $|x - 1| = 2$.

Solution 1 To satisfy $|x - 1| = 2$, x must be a number whose distance from 1 is 2. To arrive at x, start at 1 and move 2 units in either direction on the number line.



You arrive at 3 and -1 as the values of x. The solution set is $\{-1, 3\}$.

Solution 2 Note that $|x - 1| = 2$ is equivalent to the disjunction:

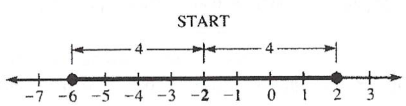
$x - 1 = -2$ or $x - 1 = 2$
$x = -1$ or $x = 3$ The solution set is $\{-1, 3\}$.

Solve.

1. $|m - 3| = 5$ **{−2, 8}**
2. $|k + 4| = 1$ **{−3, −5}**
3. $|2 + x| = 4$ **{−6, 2}**
4. $|7 - x| = 3$ **{4, 10}**
5. $|x - 5| = 2$ **{3, 7}**
6. $|6 - x| = 7$ **{−1, 13}**

Example 2 Solve $|x + 2| \le 4$ and graph its solution set.

Solution 1 $|x + 2| \le 4$ is equivalent to $|x - (-2)| \le 4$.
The distance between x and -2 must be no more than 4.



Starting at -2, numbers within 4 units in either direction will satisfy $|x + 2| \le 4$. Thus, $|x + 2| \le 4$ is equivalent to $-6 \le x \le 2$.

The solution set is $\{-6, 2$, and the real numbers between -6 and $2\}$.
The graph is shown above.

Solution 2 $|x + 2| \le 4$ is equivalent to the conjunction:

$-4 \le x + 2 \le 4$
$-4 - 2 \le x + 2 - 2 \le 4 - 2$
$-6 \le x \le 2$

The solution set and graph are as in Solution 1.

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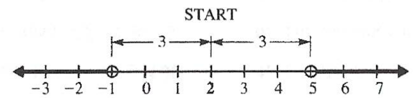
Example 3 Solve $|t - 2| > 3$ and graph its solution set.

Solution 1 The distance between t and 2 must be greater than 3, as shown below:



Therefore, $|t - 2| > 3$ is equivalent to the disjunction

$t < -1$ or $t > 5$.

The solution set is {the real numbers less than -1 or greater than 5}.
The graph is shown above.

Solution 2 $|t - 2| > 3$ is equivalent to the disjunction:

$t - 2 < -3$ or $t - 2 > 3$
$t < -1$ or $t > 5$

The solution set and graph are as in Solution 1.

Solve each open sentence and graph its solution set. **Solutions and graphs given at the back of this Answer Key.**

7. $|x| > 2$
8. $|x| \le 2$
9. $|x| \ge 1$
10. $|x - 2| < 1$
11. $|x - 2| > 2$
12. $|x + 2| \ge 1$
13. $|x - 1| \le 1$
14. $|x - 1| \ge 1$
15. $|x + 3| \le 1$
16. $|x + 1| > 1$
17. $|x - 3| \ge 4$
18. $|x - 4| < 2$
19. $|3 - y| \ge 5$
20. $|2 - x| \ge 1$
21. $|-2 - x| \le 4$

Mixed Review Exercises

Solve each inequality and graph its solution set. **Solutions and graphs given at the back of this Answer Key.**

1. $x - 3 < 5$
2. $\frac{x}{3} + 6 < 2$
3. $8 < 4(3 + m)$
4. $-1 < x + 4 < 1$
5. $h + 2 \le 8$ or $h - 3 > 2$
6. $2 \le -x \le 8$

Simplify.

7. $\frac{15x}{4y^2} \div 3xy$ $\mathbf{\frac{5}{4y^3}}$
8. $\left(\frac{4a}{b}\right)\cdot\left(\frac{5b}{2a}\right)^2$ $\mathbf{\frac{25b}{a}}$
9. $\frac{x+2}{3} - \frac{2x}{6}$ $\mathbf{\frac{2}{3}}$
10. $2x + \frac{x}{5}$ $\mathbf{\frac{11x}{5}}$

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10–6 Absolute Values of Products in Open Sentences

Objective: To extend your skill in solving open sentences that involve absolute value.

Property

The absolute value of a product of numbers equals the product of their absolute values.

$$|ab| = |a| \cdot |b|$$

Examples: $|-3 \cdot 5| = |-15| = 15 = 3 \cdot 5 = |-3| \cdot |5|$

$|-6 \cdot (-2)| = |12| = 12 = 6 \cdot 2 = |-6| \cdot |-2|$

Example 1 Solve $|2x + 1| = 5$.

Solution 1 $|2x + 1| = 5$ is equivalent to the disjunction:

$2x + 1 = -5$	or	$2x + 1 = 5$
$2x + 1 - 1 = -5 - 1$	|	$2x + 1 - 1 = 5 - 1$
$2x = -6$	|	$2x = 4$
$x = -3$	or	$x = 2$

The solution set is $\{-3, 2\}$.

Solution 2

$|2x + 1| = 5$

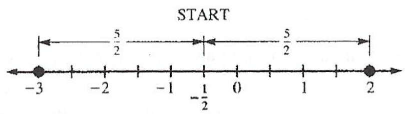
$\left|2\left(x + \frac{1}{2}\right)\right| = 5$

$|2| \cdot \left|x + \frac{1}{2}\right| = 5$

$2\left|x + \frac{1}{2}\right| = 5$

$\left|x + \frac{1}{2}\right| = \frac{5}{2}$

$\left|x - \left(-\frac{1}{2}\right)\right| = \frac{5}{2}$ Thus the distance between x and $-\frac{1}{2}$ is $\frac{5}{2}$.



Starting at $-\frac{1}{2}$ the numbers -3 and 2 are exactly $\frac{5}{2}$ units away in either direction.

The solution set is $\{-3, 2\}$.

Solve each open sentence and graph its solution set. **Graphs given at the back of this Answer Key.**

1. $\lvert 2y\rvert = 6$ **{−3, 3}**	2. $\lvert 6y\rvert = 24$ **{−4, 4}**	3. $\lvert 5x\rvert = 10$ **{−2, 2}**
4. $\left\lvert\frac{x}{3}\right\rvert = 2$ **{−6, 6}**	5. $\left\lvert\frac{x}{2}\right\rvert = 4$ **{−8, 8}**	6. $\lvert 2a - 1\rvert = 5$ **{−2, 3}**
7. $\lvert 2x + 1\rvert = 7$ **{−4, 3}**	8. $\lvert 3x - 1\rvert = 5$ $\left\{-\frac{4}{3}, 2\right\}$	9. $\left\lvert\frac{x}{2} - 1\right\rvert = 3$ **{−4, 8}**

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Example 2 Solve $|8 - 2k| \geq 8$ and graph its solution set.

Solution 1

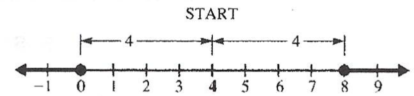
$|8 - 2k| \geq 8$

$|-2k + 8| \geq 8$

$|(-2)(k - 4)| \geq 8$ Factor.

$|-2| \cdot |k - 4| \geq 8$ Use the property about the absolute value of a product.

$|k - 4| \geq 4$



The distance between k and 4 must be 4 or more, as shown above.

Thus the given inequality is equivalent to the disjunction

$k \leq 0$ or $k \geq 8$

The solution set is {0, 8, and the real numbers less than 0 or greater than 8}.

The graph is shown above.

Solution 2 $|8 - 2k| \geq 8$ is equivalent to the disjunction

$8 - 2k \leq -8$	or	$8 - 2k \geq 8$
$-2k \leq -16$	|	$-2k \geq 0$
$k \geq 8$	or	$k \leq 0$

The solution set and graph are as given in Solution 1.

Solve each open sentence and graph its solution set. **Solutions and graphs given at the back of this Answer Key.**

10. $\lvert 2y - 1\rvert \leq 5$	11. $\lvert 2x + 1\rvert \geq 1$	12. $\lvert 2x - 3\rvert < 7$
13. $\lvert 2n - 1\rvert \geq 3$	14. $\lvert 4x - 13\rvert > 7$	15. $\lvert 6 - 3k\rvert \geq 9$
16. $\lvert 4 - 2k\rvert \leq 4$	17. $\left\lvert\frac{x}{2} - 1\right\rvert \geq 3$	18. $\left\lvert\frac{x}{3} - 2\right\rvert \leq 2$

Mixed Review Exercises

Give the slope and y-intercept of each line.

1. $y = 3x + 1$ **$m = 3; b = 1$**	2. $3y = 12x - 6$ **$m = 4; b = -2$**	3. $3y - 2x + 6 = 0$ **$m = \frac{2}{3}; b = -2$**
4. $y = 6$ **$m = 0; b = 6$**	5. $2x - y = 5$ **$m = 2; b = -5$**	6. $x = -2y + 4$ **$m = -\frac{1}{2}; b = 2$**

Graph each equation. **Graphs given at the back of this Answer Key.**

7. $y = -x + 2$	8. $y = 2x - 3$	9. $x = -2$
10. $y = 3$	11. $y = \frac{2}{3}x + 1$	12. $y = -\frac{1}{2}x - 2$

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10–7 Graphing Linear Inequalities

Objective: To graph linear inequalities in two variables.

Vocabulary

Boundary A line that separates the coordinate plane into three sets of points: the points *on* the line, the points *above* the line, the points *below* the line.
If the boundary line is part of a graph, it is drawn as a *solid* line.
If the boundary line is *not* part of the graph, it is drawn as a *dashed* line.

Open half-plane Either of the two regions into which a boundary line separates the coordinate plane.

Closed half-plane The graph of an open half-plane and its boundary.

To graph a linear inequality in the variables x and y, when the coefficient of y is not zero:

1. **Transform** the given inequality into an equivalent inequality that has y alone as one side.
2. **Graph** the equation of the boundary. Use a solid line if the symbol $\geq$ or $\leq$ is used; use a dashed line if $>$ or $<$ is used.
3. **Shade** the appropriate region.

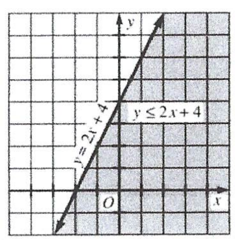
Example 1 Graph $2x - y \geq -4$.

Solution

1. Transform the inequality.

$$2x - y \geq -4$$
$$-y \geq -4 - 2x$$
$$y \leq 4 + 2x$$
$$y \leq 2x + 4$$

2. Draw the boundary line $y = 2x + 4$ as a *solid* line, since the symbol $\leq$ includes the equals sign.
3. Shade the region *below* the line since the symbol $\leq$ indicates the less than sign.



Check: Choose a point on the graph not on the boundary, say (0, 0). See whether it satisfies the given inequality:

$$2x - y \geq -4$$
$$2(0) - 0 \geq -4$$
$$0 \geq -4 \quad \checkmark$$

Thus, (0, 0) is in the solution set, and the correct region has been shaded.

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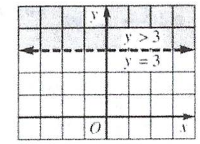
10–7 Graphing Linear Inequalities *(continued)*

Example 2 Graph $y > 3$.

Solution Graph $y = 3$ as a dashed horizontal line.

Any point above that line has a y-coordinate that satisfies $y > 3$.

Therefore, the graph of $y > 3$ is the open half-plane *above* the graph of $y = 3$.

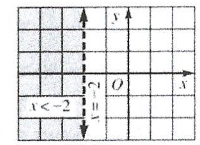


Example 3 Graph $x < -2$.

Solution Graph $x = -2$ as a dashed vertical line.

Any point to the left of that vertical line has an x-coordinate that satisfies $x < -2$.

Therefore, the graph of $x < -2$ is the open half-plane *to the left* of the graph of $x = -2$.



Graph each inequality. **Graphs given at the back of this Answer Key.**

1. $y \geq 2$
2. $y > 2$
3. $x < 2$
4. $x \leq 2$
5. $x > 1$
6. $y < 3$
7. $y \leq -1$
8. $y \leq 3$
9. $y < x + 3$
10. $y > -x + 2$
11. $y \leq 4 - x$
12. $y \geq 1 - 2x$

Transform each inequality into an equivalent inequality with y as one side. Then graph the inequality. **Graphs given at the back of this Answer Key.**

13. $x + y > 1$ **$y > -x + 1$**
14. $x - y \geq 2$ **$y \leq x - 2$**
15. $x - 2y \leq -4$ **$y \geq \frac{1}{2}x + 2$**
16. $2x + y > -2$ **$y > -2x - 2$**
17. $3x - y > 6$ **$y < 3x - 6$**
18. $y - 2x \leq -6$ **$y \leq 2x - 6$**
19. $2x - 3y \geq 6$ **$y \leq \frac{2}{3}x - 2$**
20. $2y - 3x < 0$ **$y < \frac{3}{2}x$**
21. $3y - 5 > 2(x + 2y)$ **$y < -2x - 5$**
22. $2y - 1 > 3x - 5$ **$y > \frac{3}{2}x - 2$**
23. $3(x - y) \geq 2x + 1$ **$y \leq \frac{1}{3}x - \frac{1}{3}$**
24. $4y - 6 < 2(x + y)$ **$y < x + 3$**

Mixed Review Exercises

Solve each system by whatever method you prefer.

1. $y = 2x$; $x - y = 1$ **(−1, −2)**
2. $m + n = 7$; $m - n = 3$ **(5, 2)**
3. $8p + q = -6$; $8p - 6q = -20$ **(−1, 2)**

Solve each open sentence and graph its solution set. **Graphs given at the back of this Answer Key.**

4. $|3p| = 12$ **{−4, 4}**
5. $|2p + 2| = 10$ **{−6, 4}**
6. $|2x| < 12$
7. $|2x + 3| \geq 7$
8. $-5 \leq x + 1 < 4$
9. $2x + 3 > 5$ or $3 - x \geq 1$ **{all real numbers}**

6. {the real numbers between −6 and 6}
7. {−5, 2, and the real numbers less than −5 or greater than 2}
8. {−6 and the real numbers between −6 and 3}

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10–8 Systems of Linear Inequalities

Objective: To graph the solution set of a system of two linear inequalities in two variables.

Example Graph the solution set of the system:

$$y - x - 2 \le 0$$
$$3x + 2y > -6$$

Solution

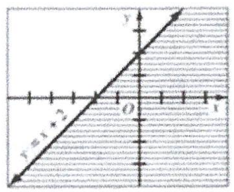
1. **Transform** each inequality into an equivalent one with y as one side.

$$y - x - 2 \le 0 \longrightarrow y \le x + 2$$
$$3x + 2y > -6 \longrightarrow y > -\frac{3}{2}x - 3$$

2. a. **Draw** the graph of $y = x + 2$, the boundary for $y \le x + 2$.

Use a solid line because the inequality has a $\le$.

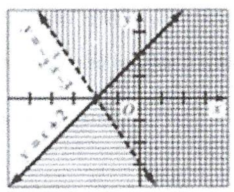
b. **Shade** the region below this line to show the graph of $y \le x + 2$ (horizontal lines).



3. a. In the same coordinate system, **draw** the graph of $y = -\frac{3}{2}x - 3$, the boundary for $y > -\frac{3}{2}x - 3$.

Use a dashed line because the inequality has a $>$.

b. **Shade** the region above this line to show the graph of $y > -\frac{3}{2}x - 3$ (vertical lines).



4. The doubly shaded region (the intersection of the vertical and horizontal lines) is the graph of the solution set of the given system.

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Graph each pair of inequalities and indicate the solution set of the system with crosshatching. **Graphs given at the back of this Answer Key.**

1. $y > 0$, $x \ge 0$
2. $y \le 3$, $x \ge 2$
3. $y > 2$, $x < -1$
4. $y < -2$, $x > 2$
5. $x < y$, $y > 1$
6. $y > 2x$, $x < 2$
7. $x \le 2$, $y > 3 - x$
8. $x > -1$, $y \le 2x + 5$
9. $y \le x + 2$, $y > 1 - x$
10. $y < 2x + 2$, $y > -2x + 2$
11. $y > 2x - 1$, $y < 2x + 2$
12. $y < 3x + 4$, $y > 3 - 3x$
13. $x - y \ge 2$, $x + y \le 4$
14. $x + y \ge 3$, $x - 2y > 4$
15. $2x - y > -1$, $x - y > -2$
16. $x - y < 5$, $x - 2y > 6$
17. $2x - 3y < -6$, $2x + 3y \le 0$
18. $2x - y > 0$, $x - 2y \le -6$

Mixed Review Exercises

Rewrite each group of fractions with their LCD.

1. $\frac{1}{3}, \frac{8}{15}, \frac{2}{5}$ **$\frac{5}{15}, \frac{8}{15}, \frac{6}{15}$**
2. $\frac{a}{2}, \frac{3}{8}, \frac{a+1}{12}$ **$\frac{12a}{24}, \frac{9}{24}, \frac{2(a+1)}{24}$**
3. $\frac{k}{k+3}, \frac{2k}{k^2+6k+9}$ **$\frac{k(k+3)}{(k+3)^2}, \frac{2k}{(k+3)^2}$**
4. $\frac{n+2}{n-4}, \frac{2}{n}, \frac{n}{4}$ **$\frac{4n(n+2)}{4n(n-4)}, \frac{8(n-4)}{4n(n-4)}, \frac{n^2(n-4)}{4n(n-4)}$**
5. $\frac{6}{x+1}, \frac{x}{x-1}$ **$\frac{6(x-1)}{(x+1)(x-1)}, \frac{x(x+1)}{(x+1)(x-1)}$**
6. $\frac{1}{x^2-4}, \frac{3}{2-x}, \frac{x}{2+x}$ **$\frac{1}{(x+2)(x-2)}, \frac{-3(x+2)}{(x+2)(x-2)}, \frac{x(x-2)}{(x+2)(x-2)}$**

Evaluate each expression if $a = \frac{2}{5}$, $b = \frac{1}{2}$, and $c = \frac{3}{10}$.

7. $a + b + c$ **$\frac{6}{5}$**
8. $c(b - a)$ **$\frac{3}{100}$**
9. $a - (b + c)$ **$-\frac{2}{5}$**
10. $\frac{1}{3}(a + b + c)$ **$\frac{2}{5}$**
11. $a + \frac{1}{2}(b - c)$ **$\frac{1}{2}$**
12. $a - \frac{1}{2}(b - c)$ **$\frac{3}{10}$**

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