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11 Rational Numbers

11–1 Properties of Rational Numbers

4. $-\frac{41}{10}, -3.9, -\frac{31}{8}$
5. $-\frac{5}{7}, -\frac{3}{5}, -\frac{6}{11}, -\frac{4}{9}$
6. $\frac{5}{24}, \frac{1}{4}, \frac{4}{15}, \frac{5}{12}$

Objective: To learn and apply some properties of rational numbers.

Vocabulary

Rational number A real number that can be expressed as the quotient of two integers.

Examples: $\frac{2}{3}$, $6 = \frac{6}{1}$, $0 = \frac{0}{1}$, $4\frac{2}{3} = \frac{14}{3}$, $0.57 = \frac{57}{100}$, $-\frac{3}{5} = \frac{-3}{5}$

Average of two numbers The number halfway between two numbers.

Properties

Density Property of Rational Numbers Between every pair of different rational numbers there is another rational number.

Order Property of Rational Numbers For all integers a and b and all positive integers c and d:

1. $\frac{a}{c} > \frac{b}{d}$ if and only if $ad > bc$.
2. $\frac{a}{c} < \frac{b}{d}$ if and only if $ad < bc$.

Example 1 Which rational number is greater, $\frac{7}{3}$ or $\frac{9}{4}$?

Solution 1 The LCD is 12.

$\frac{7}{3} = \frac{28}{12}$ and $\frac{9}{4} = \frac{27}{12}$

Compare $\frac{28}{12}$ and $\frac{27}{12}$.

Since $28 > 27$, $\frac{28}{12} > \frac{27}{12}$. So $\frac{7}{3} > \frac{9}{4}$.

Solution 2 $\frac{7}{3} \,\underline{?}\, \frac{9}{4}$

$(7)(4) \,\underline{?}\, (3)(9)$

$28 > 27$

So $\frac{7}{3} > \frac{9}{4}$.

Arrange each group of numbers in order from least to greatest. 4–6. See above.

1. $\frac{3}{4}, \frac{5}{8}, \frac{4}{5}$ **$\frac{5}{8}, \frac{3}{4}, \frac{4}{5}$**
2. $\frac{2}{3}, \frac{11}{15}, \frac{5}{7}$ **$\frac{2}{3}, \frac{5}{7}, \frac{11}{15}$**
3. $4.6, \frac{105}{22}, -4$ **$-4, 4.6, \frac{105}{22}$**
4. $-\frac{31}{8}, -3.9, -\frac{41}{10}$
5. $-\frac{4}{9}, -\frac{6}{11}, -\frac{5}{7}, -\frac{3}{5}$
6. $\frac{5}{24}, \frac{4}{15}, \frac{5}{12}, \frac{1}{4}$

Example 2 Replace the $\underline{?}$ with <, =, or > to make a true statement.

a. $-\frac{3}{8} \,\underline{?}\, -\frac{4}{11}$ b. $5\frac{3}{7} \,\underline{?}\, \frac{49}{9}$

Solution

a. $\frac{-3}{8} \,\underline{?}\, \frac{-4}{11}$

$(-3)(11) \,\underline{?}\, (8)(-4)$ {Use the order property of rational numbers.

$-33 < -32$

So $-\frac{3}{8} < -\frac{4}{11}$.

b. $\frac{38}{7} \,\underline{?}\, \frac{49}{9}$

$(38)(9) \,\underline{?}\, (7)(49)$

$342 < 343$

So $5\frac{3}{7} < \frac{49}{9}$.

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11–1 Properties of Rational Numbers *(continued)*

Replace the $\underline{?}$ with <, =, or > to make a true statement.

7. $\frac{5}{6} \,\underline{?}\, \frac{13}{18}$ >
8. $\frac{3}{4} \,\underline{?}\, \frac{11}{16}$ >
9. $\frac{7}{8} \,\underline{?}\, \frac{28}{32}$ =
10. $\frac{2}{9} \,\underline{?}\, \frac{13}{54}$ <
11. $\frac{1}{6} \,\underline{?}\, \frac{5}{32}$ >
12. $\frac{3}{4} \,\underline{?}\, \frac{11}{15}$ >
13. $-\frac{3}{5} \,\underline{?}\, -\frac{7}{12}$ <
14. $-\frac{7}{8} \,\underline{?}\, -\frac{5}{6}$ <
15. $\frac{2}{5} \,\underline{?}\, \frac{21}{50}$ <
16. $-\frac{3}{4} \,\underline{?}\, -\frac{8}{10}$ >
17. $-12\frac{1}{5} \,\underline{?}\, -\frac{86}{7}$ >
18. $\frac{-107}{7} \,\underline{?}\, -15\frac{5}{8}$ >

Example 3 Find the number halfway between $\frac{3}{8}$ and $\frac{2}{5}$.

Solution

$$\frac{3}{8} + \frac{1}{2}\left(\frac{2}{5} - \frac{3}{8}\right) = \frac{3}{8} + \frac{1}{2}\left(\frac{16}{40} - \frac{15}{40}\right)$$
$$= \frac{3}{8} + \frac{1}{2}\left(\frac{1}{40}\right)$$
$$= \frac{3}{8} + \frac{1}{80}$$
$$= \frac{30}{80} + \frac{1}{80}$$
$$= \frac{31}{80}$$

Check: Is $\frac{3}{8} < \frac{31}{80}$?
Is $(3)(80) < (8)(31)$?
$240 < 248$ ✓
Is $\frac{31}{80} < \frac{2}{5}$?
Is $(31)(5) < (80)(2)$?
$155 < 160$ ✓

$\frac{31}{80}$ is a rational number halfway between $\frac{3}{8}$ and $\frac{2}{5}$.

Find the number halfway between the given numbers.

19. $\frac{3}{8}, \frac{5}{9}$ **$\frac{67}{144}$**
20. $\frac{7}{12}, \frac{5}{6}$ **$\frac{17}{24}$**
21. $\frac{6}{11}, \frac{2}{3}$ **$\frac{20}{33}$**
22. $-\frac{11}{15}, -\frac{7}{12}$ **$-\frac{79}{120}$**
23. $-1\frac{2}{5}, -2\frac{5}{6}$ **$-2\frac{7}{60}$**
24. $-2\frac{3}{5}, 4\frac{1}{3}$ **$\frac{13}{15}$**

Example 4 If $x \in \{0, 1, 2, 3\}$, state whether $\frac{x}{2}$ increases or decreases in value as x takes on its values in increasing order.

Solution Replace x with 0, 1, 2, and 3 in order. $\frac{x}{2}$ becomes $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}$. So $\frac{x}{2}$ increases.

If $x \in \{0, 1, 2, 3\}$, state whether each fraction increases or decreases in value as x takes on its value in increasing order.

25. $\frac{x}{6}$ **Increases**
26. $-\frac{x}{8}$ **Decreases**
27. $\frac{4}{x+1}$ **Decreases**
28. $\frac{x+1}{4}$ **Increases**
29. $\frac{7-2x}{5}$ **Decreases**
30. $\frac{12}{10-3x}$ **Increases**

Mixed Review Exercises

Solve each inequality and graph its solution. **Solutions and graphs given at the back of this Answer Key.**

1. $3y + 1 \le 7$
2. $|0.2 + x| < 8$
3. $6 + 4|2 - k| \ge 14$
4. $|x + 1| \ge 3$
5. $6 \le 3x + 6 < 9$
6. $5 \le 3 - 2m$

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11–2 Decimal Forms of Rational Numbers

Objective: To express rational numbers as decimals or fractions.

Vocabulary

Terminating decimal The result when a common fraction is written as a decimal by dividing the numerator by the denominator and the remainder is zero. Also called *ending decimal* or *finite decimal.* For example, $\frac{3}{8} = 0.375$.

Nonterminating decimal The result when a common fraction is written as a decimal by dividing the numerator by the denominator and a digit or a block of digits repeat endlessly as the remainder. Also called *unending, infinite, repeating,* or *periodic decimals.* For example, $\frac{7}{11} = 0.6363\ldots = 0.\overline{63}$. Dots or an overbar are used to indicate the repeating block of digits.

Example 1 Express $\frac{5}{8}$ as a decimal.

Solution The division at the right shows that $\frac{5}{8}$ can be expressed as the terminating decimal 0.625.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Example 2 Express each rational number as a decimal: **a.** $\frac{1}{6}$ **b.** $\frac{2}{11}$ **c.** $2\frac{1}{7}$

Solution If you don't reach a remainder of zero when dividing the numerator by the denominator, continue to divide until the remainders begin to repeat.

a. $\frac{1}{6} \longrightarrow \begin{array}{r} 0.166 \\ 6\overline{)1.000} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$

$\frac{1}{6} = 0.166\ldots = 0.1\overline{6}$

b. $\frac{2}{11} \longrightarrow \begin{array}{r} 0.1818 \\ 11\overline{)2.0000} \\ \underline{1\,1} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$

$\frac{2}{11} = 0.1818\ldots = 0.\overline{18}$

c. $2\frac{1}{7} = \frac{15}{7} \longrightarrow \begin{array}{r} 2.142857 \\ 7\overline{)15.000000} \\ \underline{14} \\ 1\,0 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 1 \end{array}$

$2\frac{1}{7} = 2.142857\ldots = 2.\overline{142857}$

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11–2 Decimal Forms of Rational Numbers *(continued)*

Express each rational number as a terminating or repeating decimal.

1.a. $0.\overline{3}$ b. $0.0\overline{3}$
2.a. 2.5 b. 0.025
3.a. $-0.\overline{2}$ b. $-0.000\overline{2}$
4.a. −0.4 b. −0.04

1. a. $\frac{1}{3}$ **b.** $\frac{1}{30}$ **2. a.** $\frac{5}{2}$ **b.** $\frac{5}{200}$ **3. a.** $-\frac{2}{9}$ **b.** $-\frac{2}{9000}$ **4. a.** $-\frac{2}{5}$ **b.** $-\frac{2}{50}$

5. $\frac{13}{8}$ **1.625** **6.** $\frac{5}{12}$ $\mathbf{0.41\overline{6}}$ **7.** $\frac{7}{27}$ $\mathbf{0.\overline{259}}$ **8.** $-\frac{5}{18}$ $\mathbf{-0.2\overline{7}}$ **9.** $3\frac{3}{20}$ **3.15** **10.** $2\frac{4}{11}$ $\mathbf{2.\overline{36}}$ **11.** $-5\frac{3}{4}$ **−5.75** **12.** $\frac{11}{27}$ $\mathbf{0.\overline{407}}$

Example 3 Express each terminating decimal as a fraction in simplest form.

a. 0.24 **b.** 0.325

Solution **a.** $0.24 = \frac{24}{100} = \frac{6}{25}$ **b.** $0.325 = \frac{325}{1000} = \frac{13}{40}$

Example 4 Express $0.5\overline{21}$ as a fraction in simplest form.

Solution Let N = the number $0.5\overline{21}$ and n = the number of digits in the block of repeating digits.

Multiply N by 10^n. Since $0.5\overline{21}$ has 2 digits in the repeating block, $n = 2$. Therefore, multiply both sides of the equation $N = 0.5\overline{21}$ by 10^2 or 100.

$$100N = 100(0.5\overline{21}).$$

Since $0.5\overline{21} = 0.52121\ldots$, $0.5\overline{21}$ can also be written as $0.521\overline{21}$.

Then $100(0.5\overline{21}) = 100(0.521\overline{21})$
$= 52.1\overline{21}$

Subtract N from $100N$. $100N = 52.1\overline{21}$
$\underline{N = 0.5\overline{21}}$

Solve for N. $99N = 51.6$

$N = \frac{51.6}{99} = \frac{516}{990} = \frac{86}{165}$ So $0.5\overline{21} = \frac{86}{165}$.

Express each rational number as a fraction in simplest terms.

13. 0.3 $\frac{3}{10}$ **14.** 0.88 $\frac{22}{25}$ **15.** 0.225 $\frac{9}{40}$ **16.** 2.6 $\frac{13}{5}$ **17.** 4.26 $\frac{213}{50}$

18. $0.\overline{2}$ $\frac{2}{9}$ **19.** $1.8\overline{3}$ $\frac{11}{6}$ **20.** $0.\overline{074}$ $\frac{2}{27}$ **21.** $2.\overline{21}$ $\frac{73}{33}$ **22.** $0.\overline{09}$ $\frac{1}{11}$

23. $0.\overline{37}$ $\frac{37}{99}$ **24.** $0.\overline{63}$ $\frac{7}{11}$ **25.** $0.08\overline{3}$ $\frac{1}{12}$ **26.** $0.\overline{08}$ $\frac{8}{99}$ **27.** $-2.\overline{18}$ $-\frac{24}{11}$

Mixed Review Exercises

Find the prime factorization of each number.

1. 120 $2^3 \cdot 3 \cdot 5$ **2.** 50 $2 \cdot 5^2$ **3.** 484 $2^2 \cdot 11^2$ **4.** 1125 $3^2 \cdot 5^3$ **5.** 196 $2^2 \cdot 7^2$ **6.** 288 $2^5 \cdot 3^2$

Solve.

7. $(y + 2)(y - 3) = 0$ **{−2, 3}** **8.** $(a + 2)^2 = 16$ **{−6, 2}** **9.** $x^2 = -9$ **No solution**

10. $k^3 - 25k = 0$ **{0, −5, 5}** **11.** $|x + 2| = 6$ **{−8, 4}** **12.** $k + 3 < 12$ **{the real numbers less than 9}**

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11-3 Rational Square Roots

Objective: To find the square roots of numbers that have rational square roots.

Vocabulary

Square root If $a^2 = b$, then a is a square root of b. Positive numbers have two square roots that are opposites. Example: Since $5^2 = 25$, 5 is a square root of 25. Since $(-5)^2 = 25$, -5 is also a square root of 25.

Radicand The symbol written beneath a radical sign.

Principal square root The positive square root of a positive number.

Symbols

$\sqrt{\ }$ (radical sign)

$\sqrt{9}$ (the positive square root of 9)

$-\sqrt{9}$ (the negative square root of 9)

$\pm\sqrt{9}$ (the positive or negative square root of 9)

Properties	Examples
Product Property of Square Roots For any nonnegative real numbers a and b: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.	$\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9}$
Quotient Property of Square Roots For any nonnegative real number a and any positive real number b: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.	$\sqrt{\frac{36}{4}} = \frac{\sqrt{36}}{\sqrt{4}}$

CAUTION Negative numbers do not have square roots in the set of real numbers. The square root of zero is zero.

Example 1 Find $\sqrt{256}$.

Solution
$$\sqrt{256} = \sqrt{4 \cdot 64} = \sqrt{4} \cdot \sqrt{64} = 2 \cdot 8 = 16$$

Example 2 Find $\sqrt{1764}$.

Solution
$$\sqrt{1764} = \sqrt{2^2 \cdot 3^2 \cdot 7^2} = \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{7^2} = 2 \cdot 3 \cdot 7 = 42$$

{ If you cannot see any squares that divide into the radicand, begin by factoring the radicand.

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11-3 Rational Square Roots *(continued)*

Find the indicated square root.

1. $\sqrt{49}$ **7**
2. $\sqrt{81}$ **9**
3. $\sqrt{144}$ **12**
4. $\sqrt{196}$ **14**
5. $-\sqrt{225}$ **−15**
6. $-\sqrt{121}$ **−11**
7. $\sqrt{576}$ **24**
8. $\sqrt{400}$ **20**
9. $\pm\sqrt{1600}$ **±40**
10. $\pm\sqrt{2025}$ **±45**
11. $\sqrt{900}$ **30**
12. $\sqrt{784}$ **28**
13. $\pm\sqrt{676}$ **±26**
14. $\pm\sqrt{529}$ **±23**
15. $-\sqrt{441}$ **−21**
16. $\sqrt{484}$ **22**

Example 3 Find the indicated square root: **a.** $\sqrt{\frac{25}{81}}$ **b.** $\pm\sqrt{\frac{121}{289}}$

Solution **a.** $\sqrt{\frac{25}{81}} = \frac{\sqrt{25}}{\sqrt{81}} = \frac{5}{9}$ **b.** $\pm\sqrt{\frac{121}{289}} = \pm\frac{\sqrt{121}}{\sqrt{289}} = \pm\frac{11}{17}$

Find the indicated square root.

17. $\sqrt{\frac{81}{400}}$ $\mathbf{\frac{9}{20}}$
18. $-\sqrt{\frac{225}{64}}$ $\mathbf{-\frac{15}{8}}$
19. $\pm\sqrt{\frac{121}{36}}$ $\mathbf{\pm\frac{11}{6}}$
20. $\sqrt{\frac{144}{625}}$ $\mathbf{\frac{12}{25}}$
21. $\pm\sqrt{\frac{484}{529}}$ $\mathbf{\pm\frac{22}{23}}$
22. $-\sqrt{\frac{324}{361}}$ $\mathbf{-\frac{18}{19}}$
23. $\sqrt{\frac{225}{484}}$ $\mathbf{\frac{15}{22}}$
24. $-\sqrt{\frac{289}{400}}$ $\mathbf{-\frac{17}{20}}$
25. $\pm\sqrt{\frac{64}{2025}}$ $\mathbf{\pm\frac{8}{45}}$
26. $\sqrt{\frac{256}{1225}}$ $\mathbf{\frac{16}{35}}$
27. $\pm\sqrt{\frac{441}{1024}}$ $\mathbf{\pm\frac{21}{32}}$
28. $\sqrt{\frac{529}{256}}$ $\mathbf{\frac{23}{16}}$
29. $-\sqrt{\frac{169}{100}}$ $\mathbf{-\frac{13}{10}}$
30. $-\sqrt{\frac{289}{729}}$ $\mathbf{-\frac{17}{27}}$
31. $\sqrt{\frac{361}{2500}}$ $\mathbf{\frac{19}{50}}$
32. $\pm\sqrt{\frac{1156}{225}}$ $\mathbf{\pm\frac{34}{15}}$

Example 4 $\sqrt{0.64} = \sqrt{\frac{64}{100}} = \frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = 0.8$

Find the indicated square root.

33. $\sqrt{0.16}$ **0.4**
34. $\pm\sqrt{0.49}$ **±0.7**
35. $-\sqrt{1.44}$ **−1.2**
36. $\sqrt{2.56}$ **1.6**
37. $-\sqrt{2.89}$ **−1.7**
38. $\sqrt{3.24}$ **1.8**
39. $\pm\sqrt{4.84}$ **±2.2**
40. $\sqrt{6.25}$ **2.5**

Mixed Review Exercises

Express as a fraction in simplest form.

1. 0.375 $\mathbf{\frac{3}{8}}$
2. -3.2 $\mathbf{-\frac{16}{5}}$
3. $0.\overline{2}$ $\mathbf{\frac{2}{9}}$
4. $1.\overline{08}$ $\mathbf{\frac{107}{99}}$
5. $\frac{1}{2}\left(\frac{3}{4} - \frac{2}{3}\right)$ $\mathbf{\frac{1}{24}}$
6. $\frac{3}{4}\left(\frac{x}{2} - \frac{2x}{5}\right)$ $\mathbf{\frac{3x}{40}}$

Factor completely.

7. $2n^2 - 10n - 48$ $\mathbf{2(n - 8)(n + 3)}$
8. $6(x - 1) + y(x - 1)$ $\mathbf{(x - 1)(6 + y)}$
9. $9k^3 - k$ $\mathbf{k(3k + 1)(3k - 1)}$
10. $4w^2 - 20w + 25$ $\mathbf{(2w - 5)^2}$
11. $2x^2 + 5xy - 3y^2$ $\mathbf{(2x - y)(x + 3y)}$
12. $2 - 7ab + 3a^2b^2$ $\mathbf{(2 - ab)(1 - 3ab)}$

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11–4 Irrational Square Roots

Objective: To simplify radicals and to find decimal approximations of irrational square roots.

Vocabulary

Irrational numbers Real numbers that can't be expressed in the form $\frac{a}{b}$, where a and b are integers. Their exact values can't be expressed as either terminating or repeating decimals.

Property

Property of Completeness Every decimal represents a real number, and every real number can be represented by a decimal.

Example 1 Simplify: **a.** $\sqrt{256}$ **b.** $\sqrt{50}$ **c.** $2\sqrt{80}$ **d.** $\sqrt{704}$

Solution

a. $\sqrt{256} = \sqrt{4 \cdot 64}$ Factor within the radical sign.

$= \sqrt{4} \cdot \sqrt{64}$ Use the product property of square roots.

$= 2 \cdot 8$ Simplify.

$= 16$

b. $\sqrt{50} = \sqrt{25 \cdot 2}$

$= \sqrt{25} \cdot \sqrt{2}$

$= 5\sqrt{2}$

c. $2\sqrt{80} = 2\sqrt{16 \cdot 5}$

$= 2 \cdot 4\sqrt{5}$

$= 8\sqrt{5}$

d. $\sqrt{704} = \sqrt{64 \cdot 11}$

$= 8\sqrt{11}$

Simplify.

1. $\sqrt{27}$ **$3\sqrt{3}$**
2. $\sqrt{20}$ **$2\sqrt{5}$**
3. $\sqrt{72}$ **$6\sqrt{2}$**
4. $\sqrt{32}$ **$4\sqrt{2}$**
5. $\sqrt{48}$ **$4\sqrt{3}$**
6. $\sqrt{45}$ **$3\sqrt{5}$**
7. $\sqrt{196}$ **14**
8. $\sqrt{80}$ **$4\sqrt{5}$**
9. $2\sqrt{63}$ **$6\sqrt{7}$**
10. $4\sqrt{98}$ **$28\sqrt{2}$**
11. $7\sqrt{28}$ **$14\sqrt{7}$**
12. $4\sqrt{40}$ **$8\sqrt{10}$**
13. $\sqrt{441}$ **21**
14. $\sqrt{289}$ **17**
15. $3\sqrt{50}$ **$15\sqrt{2}$**
16. $12\sqrt{50}$ **$60\sqrt{2}$**
17. $\sqrt{729}$ **27**
18. $\sqrt{432}$ **$12\sqrt{3}$**
19. $8\sqrt{75}$ **$40\sqrt{3}$**
20. $2\sqrt{90}$ **$6\sqrt{10}$**
21. $\sqrt{147}$ **$7\sqrt{3}$**
22. $\sqrt{288}$ **$12\sqrt{2}$**
23. $\sqrt{4225}$ **65**
24. $5\sqrt{800}$ **$100\sqrt{2}$**
25. $5\sqrt{1025}$ **$25\sqrt{41}$**

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Example 2 Approximate $\sqrt{396}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$\sqrt{396} = \sqrt{2^2 \cdot 3^2 \cdot 11}$

$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{11}$

$= 6\sqrt{11}$

From the table: $\sqrt{11} \approx 3.317$

$6\sqrt{11} \approx 6(3.317) \approx 19.902$

Therefore $\sqrt{396} \approx 19.90$.

Example 3 Approximate $\sqrt{0.6}$ to the nearest hundredth. Use your calculator or the table at the back of your textbook.

Solution

$\sqrt{0.6} = \frac{\sqrt{60}}{\sqrt{100}} = \frac{\sqrt{60}}{10} \approx \frac{7.746}{10} = 0.7746$

Therefore $\sqrt{0.6} \approx 0.77$.

In Exercises 26–37, use your calculator or the table at the back of the book.

Approximate each square root to the nearest tenth.

26. $\sqrt{600}$ **24.5**
27. $\sqrt{200}$ **14.1**
28. $-\sqrt{800}$ **−28.3**
29. $-\sqrt{500}$ **−22.4**
30. $-\sqrt{2700}$ **−52.0**
31. $-\sqrt{2200}$ **−46.9**
32. $\pm\sqrt{6600}$ **±81.2**
33. $\pm\sqrt{4800}$ **±69.3**

Approximate each square root to the nearest hundredth.

34. $\sqrt{56}$ **7.48**
35. $\sqrt{32}$ **5.66**
36. $-\sqrt{0.7}$ **−0.84**
37. $-\sqrt{0.2}$ **−0.45**

Mixed Review Exercises

Find the indicated square roots.

1. $\sqrt{100}$ **10**
2. $-\sqrt{144}$ **−12**
3. $\sqrt{\frac{9}{25}}$ **$\frac{3}{5}$**
4. $-\sqrt{\frac{36}{121}}$ **$-\frac{6}{11}$**
5. $\sqrt{154^2}$ **154**
6. $\sqrt{\left(\frac{2}{5}\right)^2}$ **$\frac{2}{5}$**

Simplify.

7. $(13x)^2$ **$169x^2$**
8. $(2y^3z^6)^2$ **$4y^6z^{12}$**
9. $(x + 2y)^2$ **$x^2 + 4xy + 4y^2$**
10. $[10(a + 1)]^2$ **$100a^2 + 200a + 100$**
11. $(9a^3b^7c)^2$ **$81a^6b^{14}c^2$**
12. $(4z^2 + 3y^3)(4z^2 - 3y^3)$ **$16z^4 - 9y^6$**

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11-5 Square Roots of Variable Expressions

Objective: To find square roots of variable expressions and to use them to solve equations and problems.

Property

Property of Square Roots of Equal Numbers For any real numbers r and s:
$r^2 = s^2$ if and only if $r = s$ or $r = -s$.

CAUTION When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example, $\sqrt{x^2} = |x|$, not x.

Example 1 Simplify: a. $\sqrt{144x^2}$ b. $\sqrt{25n^8}$ c. $\sqrt{12a^3}$

Solution

a. $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$
$= 12|x|$

b. $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$
$= \sqrt{25} \cdot \sqrt{(n^4)^2}$
$= 5n^4$ (n^4 is always nonnegative)

c. $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$
$= \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$
$= 2|a|\sqrt{3a}$

Simplify.

1. $\sqrt{81x^2}$ **$9|x|$**
2. $\sqrt{121x^2}$ **$11|x|$**
3. $\sqrt{20x^2}$ **$2\sqrt{5}|x|$**
4. $\sqrt{45x^4}$ **$3\sqrt{5}x^2$**
5. $-\sqrt{25x^2}$ **$-5|x|$**
6. $-\sqrt{16c^4}$ **$-4c^2$**
7. $-\sqrt{64d^8}$ **$-8d^4$**
8. $-\sqrt{98n^6}$ **$7\sqrt{2}|n^3|$**
9. $\sqrt{225y^4}$ **$15y^2$**
10. $\sqrt{400a^6b^4}$ **$20|a^3|b^2$**
11. $\sqrt{81m^{12}}$ **$9m^6$**
12. $\sqrt{441n^6}$ **$21|n^3|$**
13. $\pm\sqrt{75x^2y^3}$ **$\pm 5|xy|\sqrt{3y}$**
14. $\pm\sqrt{60x^6y^4}$ **$\pm 2|x^3|y^2\sqrt{15}$**
15. $-\sqrt{121x^2y^2}$ **$-11|xy|$**
16. $-\sqrt{900a^4b^6}$ **$-30a^2|b^3|$**
17. $\pm\sqrt{\frac{81x^8}{100}}$ **$\pm\frac{9}{10}x^4$**
18. $\pm\sqrt{\frac{121}{225x^{10}}}$ **$\pm\frac{11}{15|x^5|}$**
19. $\sqrt{\frac{x^4y^8}{9z^2}}$ **$\frac{x^2y^4}{3|z|}$**
20. $\sqrt{\frac{32m^3n^2}{2mn^2}}$ **$4|m|$**
21. $\sqrt{\frac{16x^{18}}{3600y^{20}}}$ **$\frac{|x^9|}{15y^{10}}$**
22. $\sqrt{\frac{256x^{40}}{25}}$ **$\frac{16x^{20}}{5}$**
23. $\sqrt{2.25x^4}$ **$1.5x^2$**
24. $-\sqrt{2.56k^2}$ **$-1.6|k|$**

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11-5 Square Roots of Variable Expressions *(continued)*

Example 2 Simplify $\sqrt{m^2 - 8m + 16}$.

Solution $\sqrt{m^2 - 8m + 16} = \sqrt{(m-4)^2} = |m - 4|$

Simplify.

25. $\sqrt{x^2 + 4x + 4}$ **$|x + 2|$**
26. $\sqrt{n^2 - 14n + 49}$ **$|n - 7|$**
27. $\sqrt{x^2 - 6x + 9}$ **$|x - 3|$**
28. $\sqrt{m^2 - 10m + 25}$ **$|m - 5|$**

Example 3 Solve $4x^2 = 25$.

Solution 1

$4x^2 = 25$
$4x^2 - 25 = 0$
$(2x + 5)(2x - 5) = 0$
$2x = -5$ or $2x = 5$
$x = -\frac{5}{2}$ or $x = \frac{5}{2}$

Check: $4\left(\frac{5}{2}\right)^2 \stackrel{?}{=} 25$ and $4\left(-\frac{5}{2}\right)^2 \stackrel{?}{=} 25$

$25 = 25 \checkmark$ and $25 = 25 \checkmark$

The solution set is $\left\{\frac{5}{2}, -\frac{5}{2}\right\}$.

Solution 2

$4x^2 = 25$
$x^2 = \frac{25}{4}$
$x = \pm\sqrt{\frac{25}{4}}$
$x = \pm\frac{5}{2}$

31. $\{-3, 3\}$ 32. $\{-5, 5\}$ 33. $\{-7, 7\}$
34. $\{-8, 8\}$ 35. $\{-3, 3\}$ 36. $\{-2, 2\}$

Solve.

29. $x^2 = 16$ **$\{-4, 4\}$**
30. $n^2 = 36$ **$\{-6, 6\}$**
31. $x^2 - 9 = 0$
32. $d^2 - 25 = 0$
33. $0 = a^2 - 49$
34. $0 = m^2 - 64$
35. $2m^2 - 18 = 0$
36. $40b^2 - 160 = 0$
37. $36y^2 - 16 = 0$ **$\left\{-\frac{2}{3}, \frac{2}{3}\right\}$**
38. $4c^2 - 25 = 0$ **$\left\{-\frac{5}{2}, \frac{5}{2}\right\}$**
39. $0 = 49z^2 - 9$ **$\left\{-\frac{3}{7}, \frac{3}{7}\right\}$**
40. $0 = 45x^2 - 125$ **$\left\{-\frac{5}{3}, \frac{5}{3}\right\}$**

Mixed Review Exercises

Simplify.

1. $\pm\sqrt{80}$ **$\pm 4\sqrt{5}$**
2. $-4\sqrt{75}$ **$-20\sqrt{3}$**
3. $3\sqrt{256}$ **48**
4. $2^{-3} - 3^{-2}$ **$\frac{1}{72}$**
5. $4^3 \cdot 2^{-5}$ **2**
6. $(3x^2)^3(-2x^4)^2$ **$108x^{14}$**

Evaluate if $x = 9$, $y = 16$, and $n = 1$.

7. $x^2 + y^2$ **337**
8. x^2n^2 **81**
9. $y^2 - x^2$ **175**
10. $\sqrt{\frac{y}{n}}$ **4**
11. $\sqrt{\frac{x}{y}}$ **$\frac{3}{4}$**
12. $\left(\sqrt{y}\right)^2$ **16**

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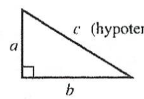
11-6 The Pythagorean Theorem

Objective: To use the Pythagorean theorem and its converse to solve geometric problems.

Theorems

Pythagorean Theorem In any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

$$a^2 + b^2 = c^2$$

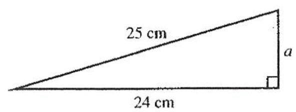


Converse of the Pythagorean Theorem If the sum of the squares of the lengths of the two shorter sides of a triangle is equal to the square of the length of the longest, then the triangle is a right triangle. The right angle is opposite the longest side.

Example 1 The length of one side of a right triangle is 24 cm. The length of the hypotenuse is 25 cm. Write and solve an equation to find the unknown side.

Solution

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$a = \sqrt{c^2 - b^2}$$
$$= \sqrt{25^2 - 24^2}$$
$$= \sqrt{625 - 576}$$
$$= \sqrt{49}$$
$$= 7$$

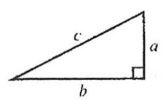


Check: $7^2 + 24^2 = 25^2$

$49 + 576 = 625$ ✓

The length of the third side of the right triangle is 7 cm.

In Exercises 1–10, refer to the triangle at the right. Find the missing length correct to the nearest hundredth. A calculator may be helpful.



1. $a = 12, b = 16, c =$?. **20.00**
2. $a = 14, b = 48, c =$?. **50.00**
3. $a = 7, b = 4, c =$?. **8.06**
4. $a = 12, b = 8, c =$?. **14.42**
5. $a = 10, b = 10, c =$?. **14.14**
6. $a = 14, b = 7, c =$?. **15.65**
7. $a =$?, $b = 10, c = 16$. **12.49**
8. $a =$?, $b = 35, c = 37$. **12.00**
9. $a = 10, b =$?, $c = 26$. **24.00**
10. $a = 40, b =$?, $c = 41$. **9.00**

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Example 2 State whether or not the three given numbers represent the lengths of the sides of a right triangle.

a. 5, 12, 13 **b.** 12, 18, 22

Solution

a. $a^2 + b^2 = c^2$

$5^2 + 12^2 \stackrel{?}{=} 13^2$

$25 + 144 \stackrel{?}{=} 169$

$169 = 169$ ✓

5, 12, and 13 form a right triangle.

b. $a^2 + b^2 = c^2$

$12^2 + 18^2 \stackrel{?}{=} 22^2$

$144 + 324 \stackrel{?}{=} 484$

$468 \neq 484$

12, 18, and 22 do not form a right triangle.

State whether or not the three given numbers represent the lengths of the sides of a right triangle.

11. 9, 16, 20 **no**
12. 9, 40, 41 **yes**
13. 9, 12, 15 **yes**
14. 5, 6, 7 **no**
15. 6, 8, 10 **yes**
16. 11, 12, 16 **no**
17. 18, 24, 30 **yes**
18. 45, 60, 75 **yes**
19. 17, 18, 33 **no**
20. 20, 21, 29 **yes**
21. 21, 28, 35 **yes**
22. 15, 36, 38 **no**

In Exercises 23–30, refer to the diagram for Exercises 1–10. Find the missing length correct to the nearest hundredth.

23. $a = b = 16, c =$?. **22.63**
24. $a = b = 9, c =$?. **12.73**
25. $a = 16, b = \frac{1}{2}a, c =$?. **17.89**
26. $a = \frac{1}{2}b, b = 10, c =$?. **11.18**
27. $a = 12, b = \frac{1}{3}a, c =$?. **12.65**
28. $a = 16, b = \frac{1}{4}a, c =$?. **16.49**
29. $a = b = 10, c =$?. **14.14**
30. $a = \frac{1}{2}b, b = 8, c =$?. **8.94**

Mixed Review Exercises

Simplify.

1. $\sqrt{25x^{12}y^2}$ $\mathbf{5x^6|y|}$
2. $\sqrt{c^2 - 8c + 16}$ $\mathbf{|c - 4|}$
3. $\sqrt{48a^5(b + 2)^2}$ $\mathbf{4a^2|b + 2|\sqrt{3a}}$

Write a fraction in simplest form.

4. $(2 \cdot 10^{-2})^3$ $\mathbf{\frac{1}{125{,}000}}$
5. $(3x^{-2}y^{-3})^2$ $\mathbf{\frac{9}{x^4y^6}}$
6. $\frac{5}{6} \div \frac{6}{5}$ $\mathbf{\frac{25}{36}}$
7. $\frac{a-2}{4} + \frac{2-3a}{6}$ $\mathbf{\frac{-3a-2}{12}}$
8. $4y + \frac{y-1}{y-2}$ $\mathbf{\frac{4y^2 - 7y - 1}{y - 2}}$
9. $\frac{3r^2}{5x} \div 15rx$ $\mathbf{\frac{r}{25x^2}}$
10. $\left(-\frac{k^2}{4}\right)^3$ $\mathbf{-\frac{k^6}{64}}$
11. $\frac{2x^2 - 16x - 40}{2x - 20}$ $\mathbf{x + 2}$
12. $\frac{2x^2 + 5x - 3}{3 + x}$ $\mathbf{2x - 1}$

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11-7 Multiplying, Dividing, and Simplifying Radicals

Objective: To simplify products and quotients of radicals.

Vocabulary

Rationalize the denominator The process of eliminating a radical from the denominator of a fraction. Remember that $(\sqrt{a})^2 = a$.

Simplest form of a square-root radical

When all of the following are true:	Simplest form	*Not* in simplest form
1. No integral radicand has a perfect-square factor other than 1.	$2\sqrt{5}$	$\sqrt{20}$
2. No fractions are under a radical sign.	$\frac{\sqrt{3}}{3}$	$\sqrt{\frac{1}{3}}$
3. No radicals are in a denominator.	$\frac{5\sqrt{2}}{2}$	$\frac{5}{\sqrt{2}}$

Example 1 Simplify $2\sqrt{3} \cdot 3\sqrt{48}$.

Solution
$$2\sqrt{3} \cdot 3\sqrt{48} = (2 \cdot 3)(\sqrt{3} \cdot \sqrt{48})$$
$$= 6\sqrt{144}$$
$$= 6 \cdot 12$$
$$= 72$$

Simplify. Assume that all variables represent positive real numbers.

1. $6\sqrt{2} \cdot 3\sqrt{2}$ **36**
2. $3\sqrt{5} \cdot 2\sqrt{5}$ **30**
3. $\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{9}$ **6**
4. $\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{16}$ **12**
5. $2\sqrt{3} \cdot \sqrt{5}$ **$2\sqrt{15}$**
6. $4\sqrt{2} \cdot \sqrt{3}$ **$4\sqrt{6}$**
7. $\sqrt{2} \cdot \sqrt{32}$ **8**
8. $\sqrt{3} \cdot \sqrt{27}$ **9**
9. $\sqrt{11} \cdot \sqrt{99}$ **33**
10. $\sqrt{8} \cdot \sqrt{18}$ **12**
11. $4\sqrt{108}$ **$24\sqrt{3}$**
12. $7\sqrt{80}$ **$28\sqrt{5}$**

Example 2 Simplify $\sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}}$.

Solution
$$\sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}} = \sqrt{\frac{7}{6} \cdot \frac{54}{28}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Simplify. Assume that all variables represent positive real numbers.

13. $\sqrt{\frac{7}{10}} \cdot \sqrt{\frac{10}{7}}$ **1**
14. $\sqrt{\frac{5}{3}} \cdot \sqrt{\frac{3}{20}}$ **$\frac{1}{2}$**
15. $\sqrt{\frac{24}{11}} \cdot \sqrt{\frac{33}{2}}$ **6**
16. $\sqrt{\frac{7}{5}} \cdot \sqrt{\frac{5}{28}}$ **$\frac{1}{2}$**
17. $\sqrt{\frac{3}{8}} \cdot \sqrt{\frac{8}{27}}$ **$\frac{1}{3}$**
18. $\sqrt{\frac{3}{5}} \cdot \sqrt{\frac{125}{3}}$ **5**
19. $\sqrt{\frac{7}{3}} \cdot \sqrt{\frac{3}{112}}$ **$\frac{1}{4}$**
20. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{10}{8}}$ **$\sqrt{\frac{1}{2}}$, or $\frac{\sqrt{2}}{2}$**

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11-7 Multiplying, Dividing, and Simplifying Radicals *(continued)*

Example 3 Simplify: **a.** $\frac{2}{\sqrt{3}}$ **b.** $\sqrt{\frac{5}{8}}$ **c.** $\frac{5\sqrt{2}}{\sqrt{12}}$ **d.** $\sqrt{4\frac{4}{5}} \cdot \sqrt{3\frac{1}{3}}$

Solution

a. $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{(\sqrt{3})^2} = \frac{2\sqrt{3}}{3}$

b. $\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{5 \cdot 2}}{2(\sqrt{2})^2} = \frac{\sqrt{10}}{4}$

c. $\frac{5\sqrt{2}}{\sqrt{12}} = \frac{5\sqrt{2}}{\sqrt{2^2 \cdot 3}} = \frac{5\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{6}}{2(\sqrt{3})^2} = \frac{5\sqrt{6}}{6}$

d. $\sqrt{4\frac{4}{5}} \cdot \sqrt{3\frac{1}{3}} = \sqrt{\frac{24}{5}} \cdot \sqrt{\frac{10}{3}} = \sqrt{\frac{24}{5} \cdot \frac{10}{3}} = \sqrt{16} = 4$

Simplify.

21. $\frac{3}{\sqrt{5}}$ **$\frac{3\sqrt{5}}{5}$**
22. $\frac{4}{\sqrt{6}}$ **$\frac{2\sqrt{6}}{3}$**
23. $\sqrt{\frac{1}{6}}$ **$\frac{\sqrt{6}}{6}$**
24. $\sqrt{\frac{3}{8}}$ **$\frac{\sqrt{6}}{4}$**
25. $\frac{6\sqrt{5}}{\sqrt{80}}$ **$\frac{3}{2}$**
26. $\frac{2\sqrt{3}}{\sqrt{48}}$ **$\frac{1}{2}$**
27. $\sqrt{3\frac{3}{4}} \cdot \sqrt{2\frac{2}{3}}$ **$\sqrt{10}$**
28. $\sqrt{1\frac{1}{6}} \cdot \sqrt{4\frac{2}{3}}$ **$\frac{7}{3}$**

Example 4 Simplify $\sqrt{3}(\sqrt{3} - 4)$.

Solution
$$\sqrt{3}(\sqrt{3} - 4) = \sqrt{3} \cdot \sqrt{3} - \sqrt{3} \cdot 4$$
$$= 3 - 4\sqrt{3}$$

Simplify.

29. $\sqrt{2}(\sqrt{2} - 1)$ **$2 - \sqrt{2}$**
30. $\sqrt{6}(5 - \sqrt{6})$ **$5\sqrt{6} - 6$**
31. $2\sqrt{3}(\sqrt{27} - \sqrt{3})$ **12**
32. $3\sqrt{5}(2\sqrt{5} - \sqrt{125})$ **-45**

Mixed Review Exercises

Solve.

1. $x^2 = 121$ **$\{-11, 11\}$**
2. $2x^2 - 128 = 0$ **$\{-8, 8\}$**
3. $16x^2 - 1 = 24$ **$\left\{-\frac{5}{4}, \frac{5}{4}\right\}$**
4. $\frac{1}{c} + \frac{1}{3} = \frac{1}{2}$ **$\{6\}$**
5. $\frac{3}{5} = \frac{15}{y}$ **$\{25\}$**
6. $\frac{6b - 1}{3b - 1} = \frac{5}{2}$ **$\{1\}$**

Simplify.

7. $15x + 2(3x - 5) + 2$ **$21x - 8$**
8. $10a + 6 - (6a - 12)$ **$4a + 18$**
9. $3(2a - 5) - 4(a - 3)$ **$2a - 3$**
10. $(-4cd^2)(-3c^2d)$ **$12c^3d^3$**
11. $-3m + 2 + 9m - 5$ **$6m - 3$**
12. $x(x - 1) + (x - 3)(2x - 1)$ **$3x^2 - 8x + 3$**

11-8 Adding and Subtracting Radicals

Objective: To simplify sums and differences of radicals.

Example 1 Simplify: **a.** $3\sqrt{5} + 4\sqrt{5}$ **b.** $2\sqrt{7} + 3\sqrt{5}$

Solution **a.** Terms with common factors can be combined.

$3\sqrt{5} + 4\sqrt{5} = (3 + 4)\sqrt{5} = 7\sqrt{5}$

b. Terms that have unlike radicals cannot be combined.

Therefore $2\sqrt{7} + 3\sqrt{5}$ is in simplest form.

Simplify.

1. $6\sqrt{2} - 3\sqrt{2}$ $\mathbf{3\sqrt{2}}$
2. $8\sqrt{3} + 4\sqrt{3}$ $\mathbf{12\sqrt{3}}$
3. $2\sqrt{15} - \sqrt{13}$ $\mathbf{2\sqrt{15} - \sqrt{13}}$
4. $-5\sqrt{2} + 8\sqrt{2}$ $\mathbf{3\sqrt{2}}$
5. $4\sqrt{5} + 3\sqrt{7}$ $\mathbf{4\sqrt{5} + 3\sqrt{7}}$
6. $9\sqrt{3} - 4\sqrt{3}$ $\mathbf{5\sqrt{3}}$
7. $7\sqrt{5} + 6\sqrt{5}$ $\mathbf{13\sqrt{5}}$
8. $8\sqrt{3} - \sqrt{3}$ $\mathbf{7\sqrt{3}}$
9. $-3\sqrt{10} + 7\sqrt{6}$ $\mathbf{-3\sqrt{10} + 7\sqrt{6}}$

Example 2 Simplify $4\sqrt{3} - 2\sqrt{7} + 8\sqrt{3}$.

Solution Use the distributive property to regroup.

$$4\sqrt{3} - 2\sqrt{7} + 8\sqrt{3} = (4 + 8)\sqrt{3} - 2\sqrt{7}$$
$$= 12\sqrt{3} - 2\sqrt{7}$$

Simplify.

10. $10\sqrt{6} - 3\sqrt{6} + \sqrt{6}$ $\mathbf{8\sqrt{6}}$
11. $7\sqrt{2} + 6\sqrt{2} - 3\sqrt{2}$ $\mathbf{10\sqrt{2}}$
12. $15\sqrt{7} + 2\sqrt{7} - 10\sqrt{7}$ $\mathbf{7\sqrt{7}}$
13. $2\sqrt{3} + 5\sqrt{2} + 8\sqrt{3}$ $\mathbf{10\sqrt{3} + 5\sqrt{2}}$
14. $8\sqrt{6} - \sqrt{3} + \sqrt{6} - 3\sqrt{3}$ $\mathbf{9\sqrt{6} - 4\sqrt{3}}$
15. $2\sqrt{5} - 5\sqrt{2} + 9\sqrt{2} - 6\sqrt{5}$ $\mathbf{4\sqrt{2} - 4\sqrt{5}}$
16. $2\sqrt{6} - \sqrt{10} + 4\sqrt{6}$ $\mathbf{6\sqrt{6} - \sqrt{10}}$
17. $3\sqrt{5} + \sqrt{11} - 4\sqrt{11} + \sqrt{5}$ $\mathbf{4\sqrt{5} - 3\sqrt{11}}$
18. $6\sqrt{3} - \sqrt{7} + 8\sqrt{3} - 6\sqrt{7}$ $\mathbf{14\sqrt{3} - 7\sqrt{7}}$
19. $5\sqrt{13} - 3\sqrt{2} + 2\sqrt{13} - 8\sqrt{2}$ $\mathbf{7\sqrt{13} - 11\sqrt{2}}$

Example 3 Simplify $5\sqrt{2} - 3\sqrt{6} + 2\sqrt{8} - 5\sqrt{54}$.

Solution First express each radical in simplest form.

$$5\sqrt{2} - 3\sqrt{6} + 2\sqrt{8} - 5\sqrt{54} = 5\sqrt{2} - 3\sqrt{6} + 2\sqrt{4 \cdot 2} - 5\sqrt{9 \cdot 6}$$
$$= 5\sqrt{2} - 3\sqrt{6} + 2(2\sqrt{2}) - 5(3\sqrt{6})$$
$$= 5\sqrt{2} - 3\sqrt{6} + 4\sqrt{2} - 15\sqrt{6}$$
$$= 9\sqrt{2} - 18\sqrt{6}$$

Simplify.

20. $3\sqrt{2} + 3\sqrt{50}$ $\mathbf{18\sqrt{2}}$
21. $3\sqrt{24} - 2\sqrt{6}$ $\mathbf{4\sqrt{6}}$
22. $5\sqrt{63} - 2\sqrt{28}$ $\mathbf{11\sqrt{7}}$
23. $5\sqrt{18} - 7\sqrt{72}$ $\mathbf{-27\sqrt{2}}$
24. $2\sqrt{80} - 6\sqrt{45}$ $\mathbf{-10\sqrt{5}}$
25. $5\sqrt{48} - 6\sqrt{27}$ $\mathbf{2\sqrt{3}}$
26. $4\sqrt{2} + \sqrt{72}$ $\mathbf{10\sqrt{2}}$
27. $\sqrt{108} - \sqrt{27}$ $\mathbf{3\sqrt{3}}$
28. $3\sqrt{8} - 2\sqrt{50}$ $\mathbf{-4\sqrt{2}}$
29. $5\sqrt{12} - 3\sqrt{48}$ $\mathbf{-2\sqrt{3}}$
30. $\sqrt{32} - \sqrt{50}$ $\mathbf{-\sqrt{2}}$
31. $\sqrt{98} - \sqrt{72}$ $\mathbf{\sqrt{2}}$
32. $4\sqrt{72} - 6\sqrt{32}$ $\mathbf{0}$
33. $2\sqrt{75} + 3\sqrt{108}$ $\mathbf{28\sqrt{3}}$
34. $10\sqrt{18} - 5\sqrt{32}$ $\mathbf{10\sqrt{2}}$
35. $2\sqrt{2} + 3\sqrt{8} + \sqrt{32}$ $\mathbf{12\sqrt{2}}$
36. $5\sqrt{28} + 3\sqrt{112}$ $\mathbf{22\sqrt{7}}$
37. $4\sqrt{54} - 2\sqrt{6}$ $\mathbf{10\sqrt{6}}$
38. $8\sqrt{8} - 4\sqrt{32} + 3\sqrt{2}$ $\mathbf{3\sqrt{2}}$
39. $2\sqrt{27} - \sqrt{75} - \sqrt{3}$ $\mathbf{0}$
40. $4\sqrt{75} - 3\sqrt{48} - \sqrt{27}$ $\mathbf{5\sqrt{3}}$
41. $2\sqrt{45} - 5\sqrt{20} + 2\sqrt{5}$ $\mathbf{-2\sqrt{5}}$

Mixed Review Exercises

Write each equation in slope-intercept form.

1. $2y = 4x + 6$ $\mathbf{y = 2x + 3}$
2. $3y - x + 9 = 0$ $\mathbf{y = \frac{1}{3}x - 3}$
3. $2x - y = 2$ $\mathbf{y = 2x - 2}$
4. $3x + 3y = 2$ $\mathbf{y = -x + \frac{2}{3}}$
5. $x = 2y + 10$ $\mathbf{y = \frac{1}{2}x - 5}$
6. $2x - 5y = 0$ $\mathbf{y = \frac{2}{5}x}$
7. $x = -y + 7$ $\mathbf{y = -x + 7}$
8. $4 - x + 2y = 0$ $\mathbf{y = \frac{1}{2}x - 2}$
9. $6x - 9 = 3y$ $\mathbf{y = 2x - 3}$
10. $8y + 12 = 4x$ $\mathbf{y = \frac{1}{2}x - \frac{3}{2}}$

For each parabola whose equation is given, find the coordinates of the vertex and the equation of the axis of symmetry.

11. $y = -2x^2$ $\mathbf{(0, 0); x = 0}$
12. $y = x^2 - 4x + 4$ $\mathbf{(2, 0); x = 2}$
13. $y = 3 - 2x^2$ $\mathbf{(0, 3); x = 0}$
14. $y = -x^2 - 4x$ $\mathbf{(-2, 4); x = -2}$

Solve. Assume that no denominator equals zero.

15. $\frac{x - 3}{8} = \frac{x}{5}$ $\mathbf{\{-5\}}$
16. $\frac{2}{x + 4} = \frac{-1}{2x + 3}$ $\mathbf{\{-2\}}$
17. $9 - \frac{8}{x} = x$ $\mathbf{\{1, 8\}}$
18. $\frac{1}{x} = \frac{2x}{x + 1}$ $\mathbf{\left\{-\frac{1}{2}, 1\right\}}$
19. $\frac{2}{x + 1} + \frac{1}{x - 1} = 1$ $\mathbf{\{0, 3\}}$
20. $\frac{x + 1}{x + 4} - \frac{1}{8} = \frac{x - 3}{x}$ $\mathbf{\{-12, 8\}}$

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11–9 Multiplication of Binomials Containing Radicals

Objective: To multiply binomials containing square-root radicals and to rationalize binomial denominators that contain square-root radicals.

Vocabulary

Conjugates If b and d are both nonnegative, then the binomials

$$a\sqrt{b} + c\sqrt{d} \quad \text{and} \quad a\sqrt{b} - c\sqrt{d}$$

are called conjugates of one another.

Example 1 Simplify $(4 + \sqrt{5})(4 - \sqrt{5})$.

Solution The pattern is $(a + b)(a - b) = a^2 - b^2$.

$$(4 + \sqrt{5})(4 - \sqrt{5}) = 4^2 - (\sqrt{5})^2 = 16 - 5 = 11$$

Example 2 Simplify $(2 + \sqrt{3})^2$.

Solution The pattern is $(a + b)^2 = a^2 + 2ab + b^2$.

$$(2 + \sqrt{3})^2 = 2^2 + 2(2)(\sqrt{3}) + (\sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$$

Simplify.

1. $(5 + \sqrt{3})(5 - \sqrt{3})$ **22**
2. $(2 - \sqrt{3})(2 + \sqrt{3})$ **1**
3. $(3 + \sqrt{7})(3 - \sqrt{7})$ **2**
4. $(\sqrt{15} - 2)(\sqrt{15} + 2)$ **11**
5. $(\sqrt{11} - 5)(\sqrt{11} + 5)$ **−14**
6. $(\sqrt{7} + 6)(\sqrt{7} - 6)$ **−29**
7. $(\sqrt{3} + 5)(\sqrt{3} - 5)$ **−22**
8. $(6 - \sqrt{5})(6 + \sqrt{5})$ **31**
9. $(2\sqrt{3} - 1)^2$ $\mathbf{13 - 4\sqrt{3}}$
10. $(4\sqrt{6} + 3)^2$ $\mathbf{105 + 24\sqrt{6}}$
11. $(8 + \sqrt{3})^2$ $\mathbf{67 + 16\sqrt{3}}$
12. $(\sqrt{5} + 3)^2$ $\mathbf{14 + 6\sqrt{5}}$

Example 3 Simplify $(3\sqrt{2} - 7\sqrt{5})^2$.

Solution The pattern is $(a - b)^2 = a^2 - 2ab + b^2$.

$$\begin{aligned}(3\sqrt{2} - 7\sqrt{5})^2 &= (3\sqrt{2})^2 - 2(3\sqrt{2})(7\sqrt{5}) + (7\sqrt{5})^2 \\ &= (3\sqrt{2})^2 - 2(3)(7)(\sqrt{2})(\sqrt{5}) + (7\sqrt{5})^2 \\ &= 9(2) - 42\sqrt{10} + 49(5) \\ &= 18 - 42\sqrt{10} + 245 \\ &= 263 - 42\sqrt{10}\end{aligned}$$

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11–9 Multiplication of Binomials Containing Radicals *(continued)*

Simplify.

13. $(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})$ **17**
14. $(3\sqrt{7} - \sqrt{5})(3\sqrt{7} + \sqrt{5})$ **58**
15. $(4\sqrt{5} - \sqrt{17})(4\sqrt{5} + \sqrt{17})$ **63**
16. $(6\sqrt{10} + 2\sqrt{6})(6\sqrt{10} - 2\sqrt{6})$ **336**
17. $(2\sqrt{3} - 3)(3\sqrt{3} + 2)$ $\mathbf{12 - 5\sqrt{3}}$
18. $(5\sqrt{2} + 2)(3\sqrt{2} - 2)$ $\mathbf{26 - 4\sqrt{2}}$
19. $(2\sqrt{5} - 5\sqrt{7})(3\sqrt{5} + \sqrt{7})$ $\mathbf{-5 - 13\sqrt{35}}$
20. $(6\sqrt{11} + \sqrt{6})(2\sqrt{11} + 3\sqrt{6})$ $\mathbf{150 + 20\sqrt{66}}$
21. $(5\sqrt{11} - 2\sqrt{3})(4\sqrt{11} + 3\sqrt{3})$ $\mathbf{202 + 7\sqrt{33}}$
22. $(4\sqrt{6} - 3\sqrt{2})(6\sqrt{6} - 2\sqrt{2})$ $\mathbf{156 - 52\sqrt{3}}$

Example 4 Rationalize the denominator of $\frac{2}{3 + \sqrt{5}}$.

Solution

$$\frac{2}{3 + \sqrt{5}} = \frac{2}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}}$$ Use the conjugate of the denominator.

$$= \frac{2(3 - \sqrt{5})}{9 - (\sqrt{5})^2} = \frac{6 - 2\sqrt{5}}{9 - 5} = \frac{6 - 2\sqrt{5}}{4}$$

Rationalize the denominator of each fraction.

23. $\frac{1}{1 + \sqrt{3}}$ $\mathbf{\frac{\sqrt{3} - 1}{2}}$
24. $\frac{1}{3 + \sqrt{2}}$ $\mathbf{\frac{3 - \sqrt{2}}{7}}$
25. $\frac{2}{\sqrt{5} - 1}$ $\mathbf{\frac{\sqrt{5} + 1}{2}}$
26. $\frac{1}{\sqrt{7} - 2}$ $\mathbf{\frac{\sqrt{7} + 2}{3}}$
27. $\frac{2 + \sqrt{5}}{1 - \sqrt{5}}$ $\mathbf{\frac{-7 - 3\sqrt{5}}{4}}$
28. $\frac{\sqrt{5} - 1}{\sqrt{5} + 2}$ $\mathbf{7 - 3\sqrt{5}}$
29. $\frac{6}{2\sqrt{3} - 1}$ $\mathbf{\frac{12\sqrt{3} + 6}{11}}$
30. $\frac{7}{3\sqrt{7} + 2}$ $\mathbf{\frac{21\sqrt{7} - 14}{59}}$

Mixed Review Exercises

Simplify. Assume that all variables represent positive real numbers.

1. $\sqrt{72x^6}$ $\mathbf{6x^3\sqrt{2}}$
2. $3\sqrt{12x} \cdot 4\sqrt{3}$ $\mathbf{72\sqrt{x}}$
3. $4\sqrt{20} - 2\sqrt{5}$ $\mathbf{6\sqrt{5}}$
4. $3\sqrt{27} + 5\sqrt{48}$ $\mathbf{29\sqrt{3}}$
5. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{3}{32}}$ $\mathbf{\frac{1}{4}}$
6. $\sqrt{1\frac{2}{3}} \cdot \sqrt{3\frac{1}{3}}$ $\mathbf{\frac{5\sqrt{2}}{3}}$
7. $(2 - 3k^2)^2$ $\mathbf{4 - 12k^2 + 9k^4}$
8. $(3p + 5z)^2$ $\mathbf{9p^2 + 30pz + 25z^2}$
9. $(4ab + x)(4ab - x)$ $\mathbf{16a^2b^2 - x^2}$

Solve.

10. $5p - 1 = 4(p + 3)$ $\{\mathbf{13}\}$
11. $x^2 - 11x + 28 = 0$ $\{\mathbf{4, 7}\}$
12. $4g^2 = 100$ $\{\mathbf{-5, 5}\}$

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11–10 Simple Radical Equations

Objective: To solve simple radical equations.

Vocabulary

Radical equation An equation containing a radical with a variable in the radicand. For example, $\sqrt{5x+9} = 12$.

CAUTION When you square both sides of an equation, the new equation may not be equivalent to the original equation. Therefore, you must *check every possible root in the original equation* to see whether it is indeed a root.

Example 1 Solve $\sqrt{3x+1} = 5$.

Solution

$\sqrt{3x+1} = 5$

$(\sqrt{3x+1})^2 = 5^2$ — Square both sides of the equation.

$3x + 1 = 25$

$3x = 24$

$x = 8$

Check: $\sqrt{3(8)+1} \stackrel{?}{=} 5$

$\sqrt{24+1} \stackrel{?}{=} 5$

$\sqrt{25} \stackrel{?}{=} 5$

$5 = 5 \ \checkmark$

The solution set is $\{8\}$.

Solve.

1. $\sqrt{x} = 4$ **{16}**
2. $\sqrt{y} = 9$ **{81}**
3. $\sqrt{x} = 12$ **{144}**
4. $\sqrt{x} = 15$ **{225}**
5. $\sqrt{d} = 20$ **{400}**
6. $\sqrt{e} = 25$ **{625}**
7. $\sqrt{2x} = 8$ **{32}**
8. $12 = \sqrt{3a}$ **{48}**
9. $4 = \sqrt{\frac{x}{3}}$ **{48}**
10. $6 = \sqrt{\frac{x}{2}}$ **{72}**
11. $\sqrt{x-1} = 7$ **{50}**
12. $\sqrt{x+2} = 6$ **{34}**
13. $\sqrt{x+1} = 4$ **{15}**
14. $\sqrt{m+4} = 1$ **{−3}**
15. $15 = 5\sqrt{3x}$ **{3}**
16. $\sqrt{3x-2} = 7$ **{17}**
17. $\sqrt{7x+1} = 8$ **{9}**
18. $\sqrt{2x+5} = 7$ **{22}**
19. $\sqrt{x} = 2\sqrt{5}$ **{20}**
20. $\sqrt{y} = 3\sqrt{2}$ **{18}**
21. $\sqrt{x} = 6\sqrt{3}$ **{108}**

Example 2 Solve $\sqrt{4x+1} + 3 = 8$.

Solution

$\sqrt{4x+1} + 3 = 8$

$\sqrt{4x+1} = 5$ — First set apart the radical on one side of the equals sign.

$4x + 1 = 25$ — Then square each side.

$4x = 24$

$x = 6$

The solution set is $\{6\}$.

Check:

$\sqrt{4(6)+1} + 3 \stackrel{?}{=} 8$

$\sqrt{24+1} + 3 \stackrel{?}{=} 8$

$\sqrt{25} + 3 \stackrel{?}{=} 8$

$5 + 3 \stackrel{?}{=} 8$

$8 = 8 \ \checkmark$

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Solve.

22. $1 = \sqrt{m} - 2$ **{9}**
23. $6 = \sqrt{x} - 2$ **{64}**
24. $\sqrt{a} - 5 = 4$ **{81}**
25. $\sqrt{2m-1} + 2 = 9$ **{25}**
26. $\sqrt{4x+1} - 1 = 2$ **{2}**
27. $\sqrt{2x+3} + 5 = 8$ **{3}**
28. $\sqrt{3m-2} - 4 = 1$ **{9}**
29. $10 = 7 + \sqrt{2m-1}$ **{5}**
30. $\sqrt{5y+1} + 5 = 11$ **{7}**

Example 3 Solve $\sqrt{5m^2 - 36} = 2m$.

Solution

$\sqrt{5m^2 - 36} = 2m$

$5m^2 - 36 = 4m^2$ — Square each side of the equation.

$m^2 = 36$

$m = 6$ or $m = -6$

Check:

$\sqrt{5(6)^2 - 36} \stackrel{?}{=} 2(6)$	$\sqrt{5(-6)^2 - 36} \stackrel{?}{=} 2(-6)$
$\sqrt{5(36) - 36} \stackrel{?}{=} 12$	$\sqrt{5(36) - 36} \stackrel{?}{=} -12$
$\sqrt{180 - 36} \stackrel{?}{=} 12$	$\sqrt{180 - 36} \stackrel{?}{=} -12$
$\sqrt{144} \stackrel{?}{=} 12$	$\sqrt{144} \stackrel{?}{=} -12$
$12 = 12 \ \checkmark$	$12 \neq -12$

The solution set is $\{6\}$.

Solve.

31. $\sqrt{3a^2 - 2} = 5$ **{−3, 3}**
32. $\sqrt{2a^2 + 1} = 3$ **{−2, 2}**
33. $\sqrt{c^2 + 15} = 4c$ **{1}**
34. $\sqrt{5x^2 - 1} = x$ $\left\{\frac{1}{2}\right\}$
35. $\sqrt{\frac{2x+5}{3}} = 5$ **{35}**
36. $\sqrt{\frac{2n-4}{6}} = 2$ **{14}**
37. $4 = \sqrt{\frac{5c-3}{7}}$ **{23}**
38. $4 = \sqrt{\frac{3k+8}{2}}$ **{8}**
39. $3 = \sqrt{\frac{4x-1}{7}}$ **{16}**

Mixed Review Exercises

Express in simplest form.

1. $(3 + \sqrt{6})(3 - \sqrt{6})$ **3**
2. $(2 + \sqrt{3})^2$ **$7 + 4\sqrt{3}$**
3. $\frac{2}{2 + \sqrt{5}}$ **$-4 + 2\sqrt{5}$**
4. $\frac{2 + \sqrt{3}}{1 - \sqrt{3}}$ **$-\frac{5 + 3\sqrt{3}}{2}$**
5. $3\sqrt{2}(\sqrt{18} - 2\sqrt{2})$ **6**
6. $(2\sqrt{5} + 1)(\sqrt{5} - 3)$ **$7 - 5\sqrt{5}$**

Factor completely.

7. $5a^2 - 10a + 5$ **$5(a - 1)^2$**
8. $t^3 - t^2 - 20t$ **$t(t - 5)(t + 4)$**
9. $3x(x + 1) + 2(x + 1)$ **$(x + 1)(3x + 2)$**
10. $x^3 + x^2 - 3x - 3$ **$(x^2 - 3)(x + 1)$**
11. $3x^5 - 48x$ **$3x(x^2 + 4)(x + 2)(x - 2)$**
12. $2x^2 - 12xy + 18y^2$ **$2(x - 3y)^2$**