

7 Applying Fractions

7-1 Ratios

Objective: To solve problems involving ratios.

Vocabulary

Ratio The ratio of one number to another is the quotient when the first number is divided by a second *nonzero* number.

Example 1 The ratio of 9 to 2 can be written as $9 \div 2$, $\frac{9}{2}$, or 9:2.

Example 2 Write each ratio in simplest form.

a. 18:63

b. 15x:45x

c. $\frac{8c^2d}{12cd^2}$

Solution First rewrite the ratio as a fraction if needed. Then simplify.

a. $18:63 = \frac{18}{63} = \frac{2}{7}$, or 2:7

b. $15x:45x = \frac{15x}{45x} = \frac{1}{3}$, or 1:3

c. $\frac{8c^2d}{12cd^2} = \frac{2c}{3d}$

Write each ratio in simplest form.

1. 12:18 **2:3** 2. 42:35 **6:5** 3. 9:30 **3:10** 4. 15:75 **1:5** 5. 18x:27x **2:3** 6. 9y:48y **3:16**
 7. $6a^2:12a$ **a:2** 8. 30x:6x **5:1** 9. $\frac{a^4}{(2a)^2} : \frac{a^2}{4}$ 10. $\frac{25m^4}{45m^3} : \frac{5m}{9}$ 11. $\frac{56a^2b}{14ab^3} : \frac{4a}{b^2}$ 12. $\frac{36rs^4}{12r^2s^2} : \frac{3s^2}{r}$

Example 3 Write each ratio in simplest form: a. 2 h:10 min b. 8 in.:4 ft

Solution To write the ratio of two quantities of the same kind, first express the measures in the same unit. Then write the ratio.

a. $2 \text{ h}:10 \text{ min} = \frac{2 \text{ h}}{10 \text{ min}} = \frac{120 \text{ min}}{10 \text{ min}} = \frac{12}{1}$, or 12:1

b. $8 \text{ in.}:4 \text{ ft} = \frac{8 \text{ in.}}{4 \text{ ft}} = \frac{8 \text{ in.}}{48 \text{ in.}} = \frac{1}{6}$ or, 1:6

CAUTION Using different units of measure will give an incorrect comparison.

Write each ratio in simplest form.

13. 20 sec:1 min **1:3** 14. 6 days:2 wk **3:7** 15. 2 ft:1 yd **2:3** 16. 12 cm:1.2 m **1:10**
 17. 8 oz:2 lb **1:4** 18. 1 day:30 h **4:5** 19. 5 m:250 cm **2:1** 20. 4 h:30 min **8:1**
 21. 1 lb:6 oz **8:3** 22. 8 wks:1 yr **2:13** 23. 3 kg:150 g **20:1** 24. 6 kg:120 g **50:1**

7-1 Ratios (continued)

Example 4 Write the ratio of wins to losses for a baseball team that played 72 games and won 45 of them.

Solution 45 wins out of 72 games tells you there were $72 - 45$ or 27 losses.
 The number of wins to losses = 45:27, or $\frac{45}{27} = \frac{5}{3}$, or 5:3.

Write each ratio in simplest form.

25. The student-teacher ratio in a school with 2376 students and 132 teachers. **18:1**
 26. The ratio of sunny to cloudy days in a year with 365 days, 275 of them sunny. **55:18**
 27. The ratio of two-door cars to four-door cars in a rental car fleet of 600 cars, 350 of which are four-door cars. **5:7**
 28. Ratio of boys to girls in a school of 1200 students if 660 students are girls. **9:11**
 29. a. The ratio of men to women in an audience if 120 out of 300 are men. **2:3**
 b. The ratio of women to men in part (a). **3:2**
 30. a. The ratio of fiction books to nonfiction books in a library with 1100 fiction books and 1760 nonfiction books. **5:8**
 b. The ratio of nonfiction books to fiction books in the library in part (a). **8:5**

Find the ratio of (a) the perimeters and (b) the areas of each pair of figures.

31. A rectangle with sides 10 cm and 8 cm and one with sides 15 cm and 12 cm. **a. 2:3 b. 4:9**
 32. A rectangle with sides 12 in. and 16 in. and one with sides 15 in. and 20 in. **a. 4:5 b. 16:25**
 33. A rectangle with length 18 cm and perimeter 84 cm and one with length 15 cm and perimeter 70 cm. **a. 6:5 b. 36:25**
 34. A rectangle with length 20 in. and perimeter 100 in. and one with length 30 in. and perimeter 150 in. **a. 2:3 b. 4:9**
 35. A square with sides 75 cm and one with sides 1 m. **a. 3:4 b. 9:16**
 36. A square with sides 18 in. and one with sides 2 yd. **a. 1:4 b. 1:16**

Mixed Review Exercises

Solve.

1. $5x = 21 - 2x$ {3} 2. $3(x - 5) + x = 5$ {5} 3. $4(3 + n) = 3(8 + 2n)$ {-6}
 4. $\frac{x+3}{2} = -9$ {-21} 5. $\frac{18-3y}{3} = 2y$ {2} 6. $-\frac{c}{7} = 3$ {-21}
 7. $(r+3)(r-4) = 0$ {-3, 4} 8. $2x^2 + 10x - 28 = 0$ {-7, 2} 9. $2(x-1) = 3(x-2)$ {4}

Simplify.

10. $\frac{2b+1}{3c} + \frac{b}{c} : \frac{5b+1}{3c}$ 11. $2x + \frac{3}{x} : \frac{2x^2+3}{x}$ 12. $\frac{a}{4} + \frac{3a+4}{4} : a + 1$

7-2 Proportions

Objective: To solve problems using proportions.

Vocabulary

Proportion An equation that states two ratios are equal is called a proportion.

For example, $2:5 = 4:10$, or $\frac{2}{5} = \frac{4}{10}$.

(Both can be read as "2 is to 5 as 4 is to 10.")

Means and extremes In the proportion, $a:b = c:d$, b and c are called the means, and a and d are called the extremes. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

(The product of the means equals the product of the extremes.)

Example 1 Solve: a. $\frac{2}{x} = \frac{6}{4}$ b. $\frac{3}{8} = \frac{-6}{4a}$ c. $\frac{2}{n} = 6$

Solution a. $\frac{2}{x} = \frac{6}{4}$ You can "cross-multiply" to solve a proportion.

$2 \cdot 4 = x \cdot 6$ To do this, multiply the means and the extremes.

$8 = 6x$ Then simplify.

$$\frac{8}{6} = x$$

$$\frac{4}{3} = x$$

The solution set is $\left\{\frac{4}{3}\right\}$.

b. $\frac{3}{8} = \frac{-6}{4a}$

$3 \cdot 4a = 8(-6)$ Cross-multiply.

$12a = -48$ Simplify.

$a = -4$

The solution set is $\{-4\}$.

c. $\frac{2}{n} = 6$

$2 \cdot 1 = 6 \cdot n$ Cross-multiply.

$2 = 6n$ Simplify.

$\frac{1}{3} = n$

The solution set is $\left\{\frac{1}{3}\right\}$.

Solve.

1. $\frac{x}{30} = \frac{3}{5}$ {18}
2. $\frac{x}{24} = \frac{5}{6}$ {20}
3. $\frac{5}{2} = \frac{30}{x}$ {12}
4. $\frac{3}{4} = \frac{x}{8}$ {6}
5. $\frac{x}{16} = \frac{6}{8}$ {12}
6. $\frac{2}{3} = \frac{4}{x}$ {6}
7. $\frac{3}{4} = \frac{x}{32}$ {24}
8. $\frac{x}{12} = \frac{54}{36}$ {18}
9. $\frac{9}{2x} = \frac{6}{4}$ {3}
10. $\frac{4}{3n} = 2$ $\left\{\frac{2}{3}\right\}$
11. $\frac{3}{x} = 4$ $\left\{\frac{3}{4}\right\}$
12. $-8 = \frac{4b}{5}$ $\{-10\}$
13. $\frac{10}{3k} = \frac{2}{5}$ $\left\{\frac{25}{3}\right\}$
14. $\frac{12r}{-7} = \frac{60}{14}$ $\left\{-\frac{5}{2}\right\}$
15. $\frac{3x}{7} = -6$ $\{-14\}$
16. $\frac{4r}{3} = -12$ $\{-9\}$
17. $-2 = \frac{2x}{5}$ $\{-5\}$
18. $\frac{91}{x} = \frac{7}{3}$ {39}
19. $\frac{7x}{45} = \frac{21}{9}$ {15}
20. $\frac{x}{60} = \frac{9}{5}$ {108}
21. $\frac{3}{2y} = \frac{9}{12}$ {2}
22. $\frac{14x}{35} = \frac{8}{5}$ {4}

7-2 Proportions (continued)

Example 2 Solve: a. $\frac{x-3}{8} = \frac{3}{4}$

b. $\frac{2x-1}{3} = \frac{4x-3}{5}$

Solution a. $\frac{x-3}{8} = \frac{3}{4}$
 $4(x-3) = 3 \cdot 8$
 $4x - 12 = 24$
 $4x = 36$
 $x = 9$

The solution set is $\{9\}$.

b. $\frac{2x-1}{3} = \frac{4x-3}{5}$
 $5(2x-1) = 3(4x-3)$
 $10x - 5 = 12x - 9$
 $4 = 2x$
 $2 = x$

The solution set is $\{2\}$.

Solve.

23. $\frac{x-1}{6} = \frac{2}{3}$ {5}
24. $\frac{x-2}{8} = \frac{3}{4}$ {8}
25. $\frac{3+2n}{7} = 5$ {16}
26. $3 = \frac{2-5y}{4}$ $\{-2\}$
27. $\frac{x+1}{6} = \frac{4}{3}$ {7}
28. $\frac{x-4}{6} = \frac{3}{2}$ {13}
29. $\frac{x-3}{8} = \frac{3}{4}$ {9}
30. $\frac{3x+4}{4} = \frac{5}{2}$ {2}
31. $\frac{2x-1}{7} = 5$ {18}
32. $\frac{2n-1}{5} = 9$ {23}
33. $\frac{x+6}{8} = \frac{x-6}{9}$ $\{-102\}$
34. $\frac{x+2}{9} = \frac{x-2}{3}$ {4}
35. $\frac{x+3}{3} = \frac{4x-9}{5}$ {6}
36. $\frac{x-6}{4} = \frac{x-9}{2}$ {12}
37. $\frac{2y-1}{3} = \frac{4y-3}{7}$ $\{-1\}$
38. $\frac{3x+4}{4} = \frac{2x+5}{5}$ {0}
39. $\frac{3x-2}{4} = \frac{x+4}{2}$ {10}
40. $\frac{5x-3}{9} = \frac{3x+3}{7}$ {6}

Mixed Review Exercises

Find the LCD for each group of fractions.

1. $\frac{1}{3xy^2}, \frac{2}{xy}, 3xy^2$
2. $\frac{w+2}{2}, \frac{3w-1}{9}$ 18
3. $\frac{2}{9}, \frac{7}{12}$ 36
4. $\frac{1}{3}, \frac{1}{4}, \frac{5}{24}$ 24
5. $\frac{2}{x-2}, \frac{4}{x+2}$
 $(x-2)(x+2)$
6. $\frac{x}{2y}, \frac{x-1}{3}$ $6y$

Simplify.

7. $\frac{8}{2(x+1)} + \frac{2}{x+1} - \frac{6}{x+1}$
8. $\frac{3r}{4} + \frac{r-1}{12} - \frac{10r-1}{12}$
9. $\frac{3a}{8} + \frac{2a+1}{4} - \frac{7a+2}{8}$
10. $|-6.3| - |2.7|$ 3.6
11. $|-2.7| + |1.2|$ 3.9
12. $6 + 2 \cdot 7$ 20

7-3 Equations with Fractional Coefficients

Objective: To solve equations with fractional coefficients.

Example 1 Solve: a. $\frac{x}{2} + \frac{x}{3} = 5$ b. $\frac{2a}{5} - \frac{a}{4} = \frac{9}{20}$

Solution Multiply both sides of the equation by the LCD of *all* of the fractions in the equations. You will get a new equation with no fractions in it that will be easier to solve than the original equation.

a. $6\left(\frac{x}{2} + \frac{x}{3}\right) = 6(5)$ The LCD of the fractions is 6.

$$6\left(\frac{x}{2}\right) + 6\left(\frac{x}{3}\right) = 30$$

$$3x + 2x = 30 \quad \leftarrow \text{New equation with no fractions}$$

$$5x = 30$$

$$x = 6$$

The solution set is {6}.

b. $20\left(\frac{2a}{5} - \frac{a}{4}\right) = 20\left(\frac{9}{20}\right)$ The LCD of the fractions is 20.

$$20\left(\frac{2a}{5}\right) - 20\left(\frac{a}{4}\right) = 9$$

$$8a - 5a = 9 \quad \leftarrow \text{New equation with no fractions}$$

$$3a = 9$$

$$a = 3$$

The solution set is {3}.

Solve.

1. $\frac{w}{2} + \frac{w}{3} = \frac{5}{3}$ {2}

2. $\frac{x}{3} - \frac{x}{4} = \frac{1}{12}$ {1}

3. $\frac{2y}{3} + \frac{y}{4} = \frac{11}{6}$ {2}

4. $\frac{3a}{4} - \frac{4a}{3} = -\frac{7}{6}$ {2}

5. $\frac{x}{3} + \frac{x}{4} = 7$ {12}

6. $\frac{2x}{3} - \frac{x}{2} = 12$ {72}

7. $\frac{x}{4} + \frac{x}{5} = \frac{9}{5}$ {4}

8. $\frac{3y}{4} - \frac{y}{6} = \frac{7}{3}$ {4}

9. $\frac{a}{3} + \frac{a}{4} = -\frac{7}{4}$ {-3}

10. $\frac{2x}{3} - \frac{5x}{9} = -1$ {-9}

11. $\frac{a}{3} - \frac{a}{9} = 2$ {9}

12. $\frac{2a}{3} - \frac{3a}{2} = \frac{5}{6}$ {-1}

13. $\frac{2a}{5} - \frac{a}{2} = \frac{3}{10}$ {-3}

14. $\frac{6b}{7} - \frac{b}{2} = 5$ {14}

15. $\frac{3n}{8} + \frac{n}{2} = 7$ {8}

16. $\frac{3n}{10} + \frac{n}{5} = \frac{3}{2}$ {3}

17. $\frac{7m}{8} - \frac{m}{4} = -\frac{5}{2}$ {-4}

18. $\frac{5x}{6} - \frac{3x}{8} = \frac{11}{2}$ {12}

7-3 Equations with Fractional Coefficients (continued)

Example 2 Solve: a. $\frac{x}{3} + \frac{x-2}{4} = 0$

b. $3n + \frac{n}{2} = \frac{n}{3} + 19$

Solution a. The LCD of the fractions is 12.

$$12\left(\frac{x}{3} + \frac{x-2}{4}\right) = 12(0)$$

$$12\left(\frac{x}{3}\right) + 12\left(\frac{x-2}{4}\right) = 0$$

$$4x + 3(x-2) = 0$$

$$4x + 3x - 6 = 0$$

$$7x - 6 = 0$$

$$7x = 6$$

$$x = \frac{6}{7}$$

The solution set is $\left\{\frac{6}{7}\right\}$.

b. The LCD of the fractions is 6.

$$6\left(3n + \frac{n}{2}\right) = 6\left(\frac{n}{3} + 19\right)$$

$$6(3n) + 6\left(\frac{n}{2}\right) = 6\left(\frac{n}{3}\right) + 6(19)$$

$$18n + 3n = 2n + 114$$

$$21n = 2n + 114$$

$$19n = 114$$

$$n = 6$$

The solution set is {6}.

Solve.

19. $\frac{x}{2} - \frac{x-1}{3} = 5$ {28}

20. $\frac{x}{8} - \frac{x+3}{5} = \frac{3}{4}$ {-18}

21. $\frac{n}{3} - \frac{n+5}{2} = 0$ {-15}

22. $\frac{x}{2} - \frac{x+3}{5} = 3$ {12}

23. $\frac{x+2}{2} = \frac{2x}{3}$ {6}

24. $x + \frac{x-2}{8} = 20$ {18}

25. $\frac{x+1}{4} = \frac{x-2}{3}$ {11}

26. $x + \frac{x}{2} = 7 - \frac{x}{4}$ {4}

27. $\frac{x-1}{6} + \frac{x+2}{3} = 5$ {9}

28. $0 = 2m - \frac{3m+18}{6}$ {2}

29. $\frac{x+1}{5} - \frac{x-1}{3} = -2$ {19}

30. $\frac{x-3}{5} + \frac{2}{3} = \frac{x+2}{15}$ {1}

31. $\frac{x+1}{5} = \frac{3x-6}{10} + \frac{3}{2}$ {-7}

32. $\frac{x+5}{2} - \frac{x+6}{3} = \frac{x}{4}$ {6}

33. $\frac{3n-1}{7} - \frac{2n-1}{3} = -6$ {26}

34. $\frac{x+6}{6} - \frac{x}{9} = \frac{2}{3}$ {-6}

Mixed Review Exercises

Write each ratio in simplest form.

1. 6 feet : 3 yards 2:3

2. $12x : 72x$ 1:6

3. 15 : 10 3:2

4. $\frac{12m^2n}{30mn}$ $\frac{2m}{5}$

5. $\frac{36xy^2}{24x^2y}$ $\frac{3y}{2x}$

6. $\frac{14a^3}{35ab^2}$ $\frac{2a^2}{5b^2}$

Solve.

7. $\frac{5}{2n} = \frac{3}{6}$ {5}

8. $\frac{x+1}{3} = \frac{5}{2}$ {13}

9. $\frac{3a+2}{4} = \frac{a+9}{3}$ {6}

10. $3x - 1 = 14$ {5}

11. $|x| = 6$ {-6, 6}

12. $6x + 5 = 7x + 3$ {2}

7-4 Fractional Equations

Objective: To solve fractional equations.

Vocabulary

Fractional equation An equation with a variable in the denominator of one or more terms. For example, $\frac{3}{x} - \frac{1}{4} = \frac{1}{12}$. To solve a fractional equation, multiply both sides by the LCD to eliminate fractions.

CAUTION Multiplying both sides of an equation by a variable expression sometimes results in an equation that has an extra root. You must check each root of the transformed equation to see if it satisfies the original equation.

Example 1 Solve: $\frac{2}{x} + \frac{1}{4} = \frac{3}{4}$

Solution $4x\left(\frac{2}{x} + \frac{1}{4}\right) = 4x\left(\frac{3}{4}\right)$ { Multiply both sides of the equation by the LCD, $4x$.
 $4x\left(\frac{2}{x}\right) + 4x\left(\frac{1}{4}\right) = 3x$ { Notice that x cannot equal 0 because $\frac{2}{0}$ has no meaning.
 $8 + x = 3x$
 $8 = 2x$
 $4 = x$

Check: $\frac{2}{4} + \frac{1}{4} \stackrel{?}{=} \frac{3}{4}$ $\frac{3}{4} = \frac{3}{4}$ ✓ The solution set is {4}.

Solve and check. If the equation has no solution, write No solution.

- | | | |
|--|---|---|
| 1. $\frac{1}{3} + \frac{11}{x} = 4$ {3} | 2. $\frac{16}{x} - \frac{3}{5} = 1$ {10} | 3. $\frac{1}{2} + \frac{3}{x} = 2$ {2} |
| 4. $\frac{1}{6} + \frac{2}{x} = \frac{5}{6}$ {3} | 5. $\frac{3}{y} - \frac{1}{4} = \frac{1}{12}$ {9} | 6. $\frac{1}{4} + \frac{2}{x} = \frac{3}{8}$ {16} |
| 7. $\frac{5}{x} + \frac{3}{4} = 2$ {4} | 8. $\frac{1}{x} - \frac{1}{2} = -\frac{1}{3}$ {6} | 9. $\frac{7}{2a} - \frac{3}{a} = -\frac{1}{4}$ {-2} |
| 10. $\frac{3}{n} - \frac{1}{2} = \frac{6}{3n}$ {2} | 11. $\frac{2}{3a} + \frac{5}{6} = \frac{3}{2a}$ {1} | 12. $\frac{2}{a} + \frac{3}{2a} = \frac{7}{6}$ {3} |

Example 2 Solve: $\frac{6-x}{4-x} = \frac{3}{5}$

Solution 1 $5(4-x)\left[\frac{6-x}{4-x}\right] = 5(4-x)\left[\frac{3}{5}\right]$ Multiply both sides by the LCD, $5(4-x)$.
 $5(6-x) = (4-x)(3)$ Notice that x cannot equal 4.
 $30 - 5x = 12 - 3x$
 $18 = 2x$
 $9 = x$ The solution set is {9}.

7-4 Fractional Equations (continued)

Solution 2 $\frac{6-x}{4-x} = \frac{3}{5}$ Solve as a proportion.
 $5(6-x) = 3(4-x)$
 $30 - 5x = 12 - 3x$
 $18 = 2x$
 $9 = x$ The solution set is {9}.

Solve.

- | | | |
|---|---|---|
| 13. $\frac{4-x}{6-x} = \frac{5}{6}$ {-6} | 14. $\frac{x+4}{x-1} = 1$ No solution | 15. $\frac{2}{3} = \frac{x+5}{x+7}$ {-1} |
| 16. $3 = \frac{x-5}{x-3}$ {2} | 17. $\frac{x}{x-1} = \frac{6}{5}$ {6} | 18. $\frac{n}{n-2} = \frac{5}{7}$ {-5} |
| 19. $\frac{x}{x+3} = \frac{2}{5}$ {2} | 20. $\frac{x}{x+5} = \frac{3}{2}$ {-15} | 21. $\frac{x-1}{x+3} = \frac{1}{2}$ {5} |
| 22. $\frac{5x}{x-1} = 4$ {-4} | 23. $\frac{x+1}{3x-1} = \frac{1}{4}$ {-5} | 24. $\frac{x-1}{x+3} = \frac{3}{5}$ {7} |
| 25. $\frac{8}{x+3} = \frac{4}{x}$ {3} | 26. $\frac{5}{x+2} = \frac{3}{x}$ {3} | 27. $\frac{2}{x+3} = \frac{3}{x+1}$ {-7} |
| 28. $\frac{2x-4}{x-2} = 3$ No solution | 29. $\frac{a+1}{2} = \frac{1}{a}$ {-2, 1} | 30. $\frac{3+x}{2x} = \frac{1}{x}$ {-1} |
| 31. $\frac{a+2}{6} = \frac{4}{a}$ {-6, 4} | 32. $\frac{1}{x} + \frac{3x}{x-2} = 0$ {-1, $\frac{2}{3}$ } | 33. $\frac{4}{x+1} - \frac{1}{x} = 1$ {1} |
| 34. $\frac{12}{x+3} = \frac{2}{x-2}$ {3} | 35. $\frac{2}{x+1} - 1 = \frac{1}{1-x}$ {0, 3} | 36. $\frac{2}{y+3} - \frac{1}{y-3} = 1$ {0, 1} |
| 37. $\frac{2}{x-1} + 3 = \frac{4x}{x-1}$ {-1} | 38. $\frac{3m+5}{6} - \frac{m}{2} = \frac{10}{m}$ {12} | 39. $\frac{x-3}{x} + \frac{1}{x} = \frac{x+1}{x+4}$ {8} |
| 40. $\frac{4}{x+1} - 1 = \frac{1}{x}$ {1} | 41. $\frac{3}{1-n} + 2 = \frac{5}{1+n}$ {0, 4} | 42. $\frac{n-2}{n} - \frac{1}{n} = \frac{n-3}{n-6}$ {3} |

Mixed Review Exercises

Solve.

- | | | |
|--|--|--|
| 1. $\frac{3a}{4} + \frac{2a}{5} = 23$ {20} | 2. $\frac{x}{3} - \frac{x}{2} = 6$ {-36} | 3. $\frac{1}{5}(y-1) + \frac{1}{4}(y+2) = 3$ {6} |
| 4. $\frac{8}{3} = \frac{2n}{9}$ {12} | 5. $\frac{-6}{5t} = \frac{3}{10}$ {-4} | 6. $\frac{3z}{4} = \frac{27}{36}$ {1} |

Simplify.

- | | | |
|------------------------------|----------------------------------|-----------------------------|
| 7. $(5-3)^3$ 8 | 8. $6x^4 + 12x^3 - 15x^2$ | 9. $4 \cdot 3^2$ 36 |
| 10. $(3n^2 + n) + (7 + n^2)$ | 11. $(4z^2)(3y^2z^2)$ $12y^2z^4$ | 12. $(2p^2q^3)^2$ $4p^4q^6$ |

7-5 Percents

Objective: To work with percents and decimals.

Vocabulary/Symbol

Percents (%) (Another way of saying hundredths, or divided by 100.)

Example 1 Write each percent as a decimal.

a. 30% b. 2.5% c. 420% d. $\frac{1}{2}$ %

Solution a. $30\% = \frac{30}{100} = 0.30$ b. $2.5\% = \frac{2.5}{100} = \frac{25}{1000} = 0.025$

c. $420\% = \frac{420}{100} = 4\frac{20}{100} = 4.2$ d. $\frac{1}{2}\% = 0.5\% = \frac{0.5}{100} = \frac{5}{1000} = 0.005$

Write each percent as a decimal.

1. 20% **0.2** 2. 35% **0.35** 3. 60% **0.6** 4. 75% **0.75** 5. 4.2% **0.042**

6. 1.5% **0.015** 7. $\frac{1}{4}$ % **0.0025** 8. $\frac{2}{5}$ % **0.004** 9. 260% **2.6** 10. 140% **1.4**

Example 2 Write each number as a percent: a. $\frac{1}{5}$ b. $\frac{2}{3}$ c. 3.5

Solution a. $\frac{1}{5} = \frac{x}{100}$ b. $\frac{2}{3} = \frac{x}{100}$ c. $3.5 = \frac{x}{100}$

$$5x = 100$$

$$x = 20$$

$$\frac{1}{5} = \frac{20}{100} = 20\%$$

$$3x = 200$$

$$x = 66\frac{2}{3}$$

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}\%$$

$$\frac{3.5}{1} = \frac{x}{100}$$

$$x = 350$$

$$3.5 = \frac{350}{100} = 350\%$$

Write each number as a percent.

11. $\frac{3}{5}$ **60%** 12. $\frac{1}{4}$ **25%** 13. $\frac{7}{8}$ **87.5%** 14. 1.25 **125%** 15. 2.7 **270%**

16. $\frac{5}{6}$ **83 $\frac{1}{3}$ %** 17. $1\frac{1}{2}$ **150%** 18. 0.075 **7.5%** 19. 0.002 **0.2%** 20. $\frac{9}{25}$ **36%**

Example 3 25% of 160 is what number?

Solution 1 $\frac{25}{100} \cdot 160 = x$

$$\frac{4000}{100} = x$$

$$40 = x \quad 25\% \text{ of } 160 \text{ is } \underline{40}.$$

Solution 2 $0.25 \cdot 160 = x$

$$40 = x$$

$$25\% \text{ of } 160 \text{ is } \underline{40}.$$

7-5 Percents (continued)

Solve.

21. 24% of 200 is what number? **48**

23. 1% of 1600 is what number? **16**

25. 32% of 300 is what number? **96**

27. 16% of 325 is what number? **52**

29. 7.2% of 250 is what number? **18**

22. 5% of 120 is what number? **6**

24. $3\frac{1}{2}$ % of 400 is what number? **14**

26. 16% of 85 is what number? **13.6**

28. $8\frac{1}{2}$ % of 12,000 is what number? **1020**

30. 120% of 40 is what number? **48**

Example 4 24 is 20% of what number?

Solution $24 = \frac{20}{100} \cdot x$

$$2400 = 20x$$

$$120 = x$$

$$24 \text{ is } 20\% \text{ of } \underline{120}.$$

Example 5 What percent of 60 is 45?

Solution $\frac{x}{100} \cdot 60 = 45$

$$\frac{60x}{100} = 45$$

$$60x = 4500$$

$$x = 75$$

$$\underline{75\%} \text{ of } 60 \text{ is } 45.$$

Solve.

31. 12 is 20% of what number? **60**

33. 21 is 3% of what number? **700**

35. 18 is 15% of what number? **120**

37. 48 is 32% of what number? **150**

39. 4.2 is 50% of what number? **8.4**

41. What percent of 25 is 16? **64%**

43. What percent of 56 is 7? **12.5%**

45. What percent of 90 is 225? **250%**

47. What percent of 120 is 24? **20%**

32. 15 is 40% of what number? **37.5**

34. 18 is 6% of what number? **300**

36. 27 is 75% of what number? **36**

38. 225 is $33\frac{1}{3}$ % of what number? **675**

40. 32.4 is 25% of what number? **129.6**

42. What percent of 72 is 18? **25%**

44. What percent of 150 is 60? **40%**

46. What percent of 220 is 132? **60%**

48. What percent of 36 is 45? **125%**

Mixed Review Exercises

Factor completely.

1. $2ab^2 + 10b$

$$2b(ab + 5)$$

2. $2y^3 + 2y^2 - 4y$

$$2y(y + 2)(y - 1)$$

3. $m^2 + 6m + 8$

$$(m + 2)(m + 4)$$

4. $x^2 - 4y^2$

$$(x + 2y)(x - 2y)$$

5. $4m^2 + 8mn - 12n^2$

$$4(m + 3n)(m - n)$$

6. $4a^2 + 7a - 2$

$$(4a - 1)(a + 2)$$

7. $a(a - c) + 3(a - c)$

$$(a - c)(a + 3)$$

8. $x^2 + 20x + 100$

$$(x + 10)^2$$

NAME _____ DATE _____

7-6 Percent Problems**Objective:** To solve problems involving percents.

- Example 1** Find the change in price.
 a. The original price of the suit Carmen wants was \$275. It is now on sale for \$198.
 b. Calvin originally paid \$90 for an old coin. It is now worth \$145.

Solution To find the change in price, you calculate the difference between the original price and the new price.
 a. The price decreased by $\$275 - \198 , or $\$77$.
 b. The price increased by $\$145 - \90 , or $\$55$.

- Example 2** The price of a salad bar increased from \$3.00 to \$3.45. What was the percent increase?

Solution

Step 1 The problem asks for the percent of increase.

Step 2 Let n = the percent of increase.

Step 3 $\frac{\text{percent of change}}{100} = \frac{\text{change in price}}{\text{original price}}$

Step 4
$$\frac{n}{100} = \frac{45}{300}$$

$$300n = 4500$$

$$n = 15$$

Step 5 The check is left to you. There was a 15% increase.

Complete the table.

	Item	Original price	New price	% of increase	
1.	Shirt	\$20.00	\$22.00	?	10%
2.	Sweater	\$48.00	\$60.00	?	25%
3.	Tennis racket	\$32.00	\$36.00	?	12.5%
4.	Movie ticket	\$4.00	\$5.00	?	25%
5.	Bus ticket	\$40.00	?	5%	\$42.00
6.	Newspaper	\$.25	?	60%	\$4.00
7.	Books	\$80.00	?	20%	\$96.00
8.	Magazine subscription	?	\$15.00	25%	\$12.00
9.	Taxi fare	?	\$14.00	$33\frac{1}{3}\%$	\$10.50
10.	Airplane ticket	?	\$168.00	5%	\$160.00

NAME _____ DATE _____

7-6 Percent Problems (continued)

- Example 3** The price of a video camera decreased from \$800 to \$760. What was the percent decrease?

Solution

Step 1 The problem asks for the percent of decrease.

Step 2 Let x = the percent of decrease.

Step 3 Use the formula $\frac{\text{percent of change}}{100} = \frac{\text{change in price}}{\text{original price}}$

Step 4
$$\frac{x}{100} = \frac{40}{800}$$

$$800x = 4000$$

$$x = 5$$

Step 5 The check is left to you. There was a 5% decrease.

Complete the table.

	Item	Original price	New price	% of decrease	
11.	Video tape	\$6.00	\$4.50	?	25%
12.	Baseball glove	\$32.00	\$28.00	?	12.5%
13.	Skates	\$60.00	\$51.00	?	15%
14.	T-shirt	\$7.50	\$6.00	?	20%
15.	Watch	\$80.00	?	30%	\$56.00
16.	Video rental	\$2.00	?	25%	\$1.50
17.	Audio cassette	\$6.00	?	20%	\$4.80
18.	Camera	?	\$210.00	12.5%	\$240.00
19.	Film	?	\$4.00	20%	\$5.00
20.	Suit	?	\$136.00	15%	\$160.00

Mixed Review ExercisesSolve. If the equation has no solution, write *No solution*.

1. $\frac{2x+3}{x-3} = 5$ {6} 2. $\frac{a+2}{3} = \frac{1}{a}$ {-3, 1} 3. $\frac{4}{n} = \frac{7}{3n}$ No solution
 4. $1.3x = 52$ {40} 5. $m^2 + 4m + 3 = 0$ {-3, -1} 6. $0.4x + 3.2 = 1.2x$ {4}

Simplify.

7. $(x+2)8$ $8x + 16$ 8. $-3(x-2y)$ $-3x + 6y$ 9. $8a + 5 - 3a + c$ $5a + c + 5$

7-7 Mixture Problems

Objective: To solve mixture problems.

Example 1 A health food store sells a mixture of raisins and roasted nuts. Raisins sell for \$4.00/kg and nuts sell for \$6.00/kg. How many kilograms of each should be mixed to make 40 kg of this snack worth \$4.75/kg?

Solution

Step 1 The problem asks for the number of kilograms of raisins and the number of kilograms of nuts.

Step 2 Let x = the number of kilograms of raisins.
Then $40 - x$ = the number of kilograms of nuts.

	Number of kg	× Price per kg	= Cost
Raisins	x	\$4.00	$4x$
Nuts	$40 - x$	\$6.00	$6(40 - x)$
Mixture	40	\$4.75	190

Step 3 The value of a mixture is equal to the value of the individual ingredients.

Cost of raisins + Cost of nuts = Total cost of mixture
 $4x + 6(40 - x) = 190$

Step 4

$$\begin{aligned} 4x + 6(40 - x) &= 190 \\ 4x + 240 - 6x &= 190 \\ 240 - 2x &= 190 \\ -2x &= -50 \\ x &= 25 \\ 40 - x &= 15 \end{aligned}$$

Step 5

$$\begin{aligned} 4(25) + 6(15) &\stackrel{?}{=} 190 \\ 100 + 90 &\stackrel{?}{=} 190 \\ 190 &= 190 \checkmark \quad 25 \text{ kg of raisins and } 15 \text{ kg of nuts should be mixed.} \end{aligned}$$

Solve.

- The owner of a specialty food store wants to mix cashews selling at \$8.00/kg and pecans selling at \$6.00/kg. How many kilograms of each should be mixed to get 12 kg of nuts worth \$7.50/kg? **9 kg of cashews; 3 kg of pecans**
- A grocer mixed 12 lb of egg noodles costing 80¢/lb with 3 lb of spinach noodles costing \$1.20/lb. What will the cost per pound of the mixture be? **88¢/lb**
- A special tea blend is made from two varieties of herbal tea, one that costs \$4.00/kg and another that costs \$2.00/kg. How many kilograms of each type are needed to make 20 kg of a blend worth \$2.50/kg? **5 kg of \$4.00 tea; 15 kg of \$2.00 tea**
- A grocer has two kinds of nuts. One costs \$5/kg and another costs \$4.20/kg. How many kilograms of each type of nut should be mixed in order to get 60 kg of a mixture worth \$4.80/kg? **45 kg of \$5.00 nuts; 15 kg of \$4.20 nuts**

7-7 Mixture Problems (continued)

Example 2 A chemist has 60 mL of a solution that is 70% acid. How much water should be added to make a solution that is 40% acid?

Solution

Step 1 The problem asks for the number of milliliters of water to be added.

Step 2 Let x = the number of milliliters of water to be added.

	Total amount	× % acid	= Amount of acid
Original solution	60	70%	$0.70(60)$
Water	x	0%	0
New solution	$60 + x$	40%	$0.40(60 + x)$

Step 3 Original amount of acid + Added acid = New amount of acid
 $0.70(60) + 0 = 0.40(60 + x)$

Step 4

$$\begin{aligned} 70(60) &= 40(60 + x) & \left\{ \begin{array}{l} \text{Multiply both sides by} \\ 100 \text{ to clear decimals.} \end{array} \right. \\ 4200 &= 2400 + 40x \\ 1800 &= 40x \\ 45 &= x \end{aligned}$$

Step 5 The check is left to you. 45 mL of water should be added.

Solve.

- A chemist has 80 mL of a solution that is 70% salt. How much water should he add to make a solution that is 40% salt? **60 mL**
- If 800 mL of a juice drink is 10% grape juice, how much grape juice should be added to make a drink that is 20% grape juice? **100 mL**
- How many liters of water must be added to 70 L of a 40% acid solution in order to produce a 28% acid solution? **30 L**
- How many mL of pure water must be added to 60 mL of a 20% salt solution to make a 12% salt solution? **40 mL**
- A nurse has 100 mL of a solution that is 10% salt. How much sterile water must be added to make an 8% salt solution? **25 mL**

Mixed Review Exercises

Evaluate.

- 8% of 50 + 0.2% of 120 **4.24**
- What percent of 60 is 18? **30%**
- What percent of 120 is 30? **25%**
- 12 is 25% of what number? **48**

Evaluate if $a = 1$, $b = 2$, $x = 3$, and $y = 6$.

- $|-3| + y$ **9**
- $\frac{5 - 2a}{x - b}$ **3**
- $\frac{1}{8}(6x + y)$ **38**
- x^2y **54**
- $2a + 3b$ **8**
- $(x - a)^2$ **4**

7-8 Work Problems

Objective: To solve work problems.

Vocabulary

Work rate The fractional part of a job done in a given unit of time. For example, if you can mow a lawn in 2 h, your work rate is $\frac{1}{2}$ job per hour. A whole job is done when the sum of the fractional parts is 1.

Example 1 Ted can paint a wall in 20 min. Vern can paint the same wall in 30 min. How long would it take them to paint the wall working together?

Solution

Step 1 The problem asks for the number of minutes needed to do the job.

Step 2 Let x = the number of minutes needed to do the job together. Ted and Vern will each work x min. Since Ted can do the whole job in 20 min, his work rate is $\frac{1}{20}$ job per min. Vern's work rate is $\frac{1}{30}$ job per min.

	Work rate	× Time	= Work done
Ted	$\frac{1}{20}$	x	$\frac{x}{20}$
Vern	$\frac{1}{30}$	x	$\frac{x}{30}$

Step 3 Ted's part of the job + Vern's part of the job = whole job

$$\frac{x}{20} + \frac{x}{30} = 1$$

Step 4

$$60\left(\frac{x}{20} + \frac{x}{30}\right) = 60(1) \quad \text{Multiply by the LCD, 60.}$$

$$3x + 2x = 60$$

$$5x = 60$$

$$x = 12$$

Step 5 The check is left to you. It would take 12 min for them to do the job together.

Solve.

- A file clerk needs 6 h to file an average day's paperwork. It takes a trainee 12 h to do the same job. How long will it take if they work together? **4 h**
- Luis can load his truck in 24 min. It takes his brother Ramon 40 min to load the truck. How long would it take them to do the job together? **15 min**
- Ross can do a job in 8 h. Brock can do the same job in 12 h. How long would it take them working together? **$4\frac{4}{5}$ h**
- Bernice can wallpaper a room in 4 h. Annie can wallpaper the room in 8 h. How long would it take them working together? **$2\frac{2}{3}$ h**

7-8 Work Problems (continued)

Example 2 Robot A takes 6 min to weld a frame. With the help of Robot B, the job can be done in 4 min. How long would it take Robot B working alone?

Solution

Step 1 The problem asks for the amount of time it would take Robot B to weld the frame.

Step 2 Let x = the number of minutes needed for Robot B to weld the frame. Then Robot B does $\frac{1}{x}$ of the job per min.

	Work rate	× Time	= Work done
Robot A	$\frac{1}{6}$	4	$\frac{4}{6}$
Robot B	$\frac{1}{x}$	4	$\frac{4}{x}$

Step 3 A's part of the job + B's part of the job = whole job.

$$\frac{4}{6} + \frac{4}{x} = 1$$

Step 4

$$6x\left(\frac{4}{6} + \frac{4}{x}\right) = 6x(1)$$

$$4x + 24 = 6x$$

$$24 = 2x$$

$$12 = x$$

Step 5 The check is left to you. It will take Robot B 12 min to weld the frame.

Solve.

- Sherry can do a job in 60 min. If her sister helps her, it takes them 36 min. How long does it take her sister alone? **90 min, or $1\frac{1}{2}$ h**
- A roofer can shingle a house in 20 h. If an apprentice helps, they can do the job in 12 h. How long does it take the apprentice alone? **30 h**
- It takes Cabin A 18 min to set the tables in the camp dining hall. If Cabin B helps them, the job can be done in 10 min. How long would it take Cabin B to set the tables by themselves? **$22\frac{1}{2}$ min**
- One machine can print a magazine in 30 min. If a second machine works with the first machine, the magazine can be printed in 18 min. How long does it take the second machine to do the job alone? **45 min**

Mixed Review Exercises

Solve.

- $\left\{-1, -\frac{7}{3}\right\}$
- $\frac{1}{x+2} + \frac{4}{x+3} = 3$
- $\frac{6}{n} = \frac{12}{5}$ {2.5}
- $\frac{x+2}{5} = \frac{x+3}{10}$ {-1}
- $5a - 3 = 2(a + 6)$ {5}
- $-0.4 + k = 0.6$ {1}
- $-5k = 0$ {0}

7-9 Negative Exponents

Objective: To use negative exponents.

Definitions

a^{-n} If a is a nonzero real number and n is a positive integer, $a^{-n} = \frac{1}{a^n}$.

For example, $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$. (Notice that 2^{-3} is not a negative number.)

a^0 If a is a real number not equal to zero, $a^0 = 1$. For example, $2^0 = 1$, $3^0 = 1$, $25^0 = 1$, and so on. The expression 0^0 has no meaning.

Example 1 a. $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$ b. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ c. $8^{-1} = \frac{1}{8^1} = \frac{1}{8}$

Simplify. Give your answers using positive exponents.

- | | | | |
|---------------------------|--------------------------|--------------------------------|----------------------------|
| 1. $3^{-1} \frac{1}{3}$ | 2. $6^{-1} \frac{1}{6}$ | 3. $4^{-3} \frac{1}{64}$ | 4. $3^{-4} \frac{1}{81}$ |
| 5. $5^{-2} \frac{1}{25}$ | 6. $7^{-2} \frac{1}{49}$ | 7. $2^{-5} \frac{1}{32}$ | 8. $11^{-2} \frac{1}{121}$ |
| 9. $9^0 1$ | 10. $-7^0 -1$ | 11. $1^{-3} 1$ | 12. $5^{-1} \frac{1}{5}$ |
| 13. $6^{-2} \frac{1}{36}$ | 14. $5^0 1$ | 15. $10^{-4} \frac{1}{10,000}$ | 16. $7^{-3} \frac{1}{343}$ |

All of the rules for positive exponents also hold for zero and negative exponents:

Summary of Rules for Exponents	Examples
Let m and n be any integers. Let a and b be any nonzero integers.	
1. Products of Powers: $b^m b^n = b^{m+n}$	$2^2 \cdot 2^{-4} = 2^{2+(-4)} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
2. Quotients of Powers: $b^m \div b^n = b^{m-n}$	$3^2 \div 3^5 = 3^{2-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$
3. Power of a Power: $(b^m)^n = b^{mn}$	$(2^2)^{-3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$
4. Power of a Product: $(ab)^m = a^m b^m$	$(2x)^{-3} = 2^{-3} \cdot x^{-3} = \frac{1}{2^3} \cdot \frac{1}{x^3} = \frac{1}{8x^3}$
5. Power of a Quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{2^2}{3^2} = \frac{1}{2^2} \cdot \frac{3^2}{1} = \frac{3^2}{2^2} = \frac{9}{4}$

CAUTION Remember that in $3x^2$, the exponent is applied to x but not to 3. However, in $(3x)^2$, the exponent is applied both to 3 and to x .

7-9 Negative Exponents (continued)

Example 2 Simplify. Give your answers using positive exponents.

a. $\frac{3}{3^{-2}}$ b. $(x^{-1})^{-2}$ c. $(2x^{-1})^3$

Solution a. $\frac{3}{3^{-2}} = 3^{1-(-2)} = 3^3 = 27$ Use the rule for quotients of powers.

b. $(x^{-1})^{-2} = x^{(-1)(-2)} = x^2$ Use the rule for a power of a power.

c. $(2x^{-1})^3 = 2^3 \cdot x^{(-1)(3)} = 8x^{-3} = \frac{8}{x^3}$ Use the rule for a power of a power and the rule for a power of a product. Use the rule for negative exponents.

Simplify. Give your answers using positive exponents.

- | | | | | |
|---|---|---------------------------------|---|---|
| 17. $\frac{2}{2^{-3}} 16$ | 18. $\frac{4^{-2}}{4^{-3}} 4$ | 19. $3^{-3} \cdot 3^5 9$ | 20. $(5^{-1})^2 \frac{1}{25}$ | 21. $\left(\frac{2^{-1}}{1}\right)^2 \frac{1}{4}$ |
| 22. $\left(\frac{4^3}{4^{-2}}\right)^0 1$ | 23. $(5^{-1})^{-2} 25$ | 24. $(3^{-2})^{-1} 9$ | 25. $\left(\frac{3}{2}\right)^{-2} \frac{4}{9}$ | 26. $\left(\frac{3^4}{3^{-2}}\right)^0 1$ |
| 27. $\frac{5^{-2} \cdot 5}{5^{-1}} 1$ | 28. $\frac{3^{-4} \cdot 3^2}{3^{-2}} 1$ | 29. $2x^{-2} \frac{2}{x^2}$ | 30. $3x^{-3} \frac{3}{x^3}$ | 31. $(2x)^{-2} \frac{1}{4x^2}$ |
| 32. $(3x)^{-3} \frac{1}{27x^3}$ | 33. $x^{-2}y \frac{y}{x^2}$ | 34. $a^{-2}b^3 \frac{b^3}{a^2}$ | 35. $a^5 \cdot a^{-3} a^2$ | 36. $n^3 \cdot n^{-4} \frac{1}{n}$ |
| 37. $(m^{-2})^3 \frac{1}{m^6}$ | 38. $(x^{-3})^2 \frac{1}{x^6}$ | 39. $(2x^{-2})^2 \frac{4}{x^4}$ | 40. $(3x^{-1})^2 \frac{9}{x^2}$ | 41. $\frac{y^2}{y^{-3}} y^5$ |
| 42. $\frac{u^{-3}}{u^7} \frac{1}{u^{10}}$ | 43. $\frac{c^{-5}}{c^3} \frac{1}{c^8}$ | 44. $\frac{d^3}{d^{-3}} d^6$ | 45. $\frac{x^{-4}}{x^{-2}} \frac{1}{x^2}$ | 46. $\frac{m^{-6}}{m^{-8}} m^2$ |

Mixed Review Exercises

Simplify. Give restrictions on the variables. $\frac{y-2}{y-5}; y \neq 5$

- | | | |
|--|---|--|
| 1. $\frac{24x^2y}{16xy^2} \frac{3x}{2y}; x \neq 0, y \neq 0$ | 2. $\frac{y^2-7y+10}{y^2-10y+25} \frac{-2-2a}{a-1}; a \neq 1$ | 3. $\left(\frac{-2a}{b}\right)^3 - \frac{8a^3}{b^3}; b \neq 0$ |
| 4. $\frac{6}{5mn} - \frac{2}{n} \frac{6-10m}{5mn}; m \neq 0, n \neq 0$ | 5. $2 - \frac{4a}{a-1} \frac{-2-2a}{a-1}; a \neq 1$ | 6. $\frac{x^2-4}{x^2+3x+2}$ |

Divide. Write your answer as a polynomial or as a mixed expression. $\frac{x-2}{x+1}; x \neq -1, x \neq -2$

- | | | |
|----------------------------------|---------------------------------|---|
| 7. $\frac{3x^2+10x+3}{x+3} 3x+1$ | 8. $\frac{a^3+8}{a+2} a^2-2a+4$ | 9. $\frac{32}{x^2-25} \div \frac{24}{x^2+10x+25} \frac{4(x+5)}{3(x-5)}$ |
|----------------------------------|---------------------------------|---|

7-10 Scientific Notation

Objective: To use scientific notation.

Vocabulary

Scientific notation A positive number in scientific notation is expressed as the product of a number greater than or equal to 1 but less than 10, and an integral power of 10. For example, 2.6×10^3 and 5.02×10^{-4} are written in scientific notation, but 0.4×10^{-5} and 10.3×10^2 are not.

Expanded notation A way of writing numbers using powers of 10 to show place value.

Example 1 Write each number in scientific notation: a. 34,610,000 b. 0.0000027

Solution a. Move the decimal point left 7 places to get a number between 1 and 10.

$$\underbrace{34,610,000}_{7} = 3.461 \times 10,000,000 = 3.461 \times 10^7$$

b. Move the decimal point right 6 places to get a number between 1 and 10.

$$\underbrace{0.0000027}_{6} = \frac{2.7}{1,000,000} = \frac{2.7}{10^6} = 2.7 \times 10^{-6}$$

When a number greater than 1 is written in scientific notation, the power of 10 used is positive. When the number is less than 1, the power of 10 used is negative.

Write each number in scientific notation.

1. 27,300 2.73×10^4 2. 3,060,000 3.06×10^6 3. 25,010,000 2.501×10^7
 4. 0.00305 3.05×10^{-3} 5. 0.0000017 1.7×10^{-6} 6. 0.000000804 8.04×10^{-7}

Example 2 Write each number in decimal form: a. 3.16×10^6 b. 6.74×10^{-4}

Solution a. Move the decimal point 6 places. b. Move the decimal point 4 places.

$$3.16 \times 10^6 = \underbrace{3,160,000}_{6} \qquad 6.74 \times 10^{-4} = \underbrace{0.000674}_{4}$$

Rewrite each number in decimal form.

7. 3.0×10^7 **30,000,000** 8. 2.27×10^8 **227,000,000** 9. 4.6×10^{11} **460,000,000,000**
 10. 1.8×10^5 **180,000** 11. 5.29×10^{-5} **0.0000529** 12. 6.0×10^{-8} **0.00000006**

Example 3 Write each number in expanded notation using powers of 10.

a. 7341 b. 0.2865 c. 48.09

Solution

$$\begin{aligned} \text{a. } 7341 &= 7000 + 300 + 40 + 1 \\ &= 7 \cdot 10^3 + 3 \cdot 10^2 + 4 \cdot 10^1 + 1 \cdot 10^0 \end{aligned}$$

$$\begin{aligned} \text{b. } 0.2865 &= 0.2 + 0.08 + 0.006 + 0.0005 \\ &= 2 \cdot 10^{-1} + 8 \cdot 10^{-2} + 6 \cdot 10^{-3} + 5 \cdot 10^{-4} \end{aligned}$$

$$\begin{aligned} \text{c. } 48.09 &= 40 + 8 + 0.0 + 0.09 \\ &= 4 \cdot 10^1 + 8 \cdot 10^0 + 0 \cdot 10^{-1} + 9 \cdot 10^{-2} \end{aligned}$$

7-10 Scientific Notation (continued)

Write each number in expanded notation. Answers given at the back of this Answer Key.

13. 1700 14. 4812 15. 0.143 16. 0.1756 17. 36.07 18. 175.1
 19. 10,396 20. 0.0061 21. 64,000 22. 0.00032 23. 0.000015 24. 85,020,000

Example 4 Simplify. Write your answers in scientific notation.

a. $\frac{4.8 \times 10^6}{3.0 \times 10^2}$ b. $(1.5 \times 10^2)(8.0 \times 10^4)$ c. 0.3×10^5

Solution a. Subtract exponents when you divide.

$$\begin{aligned} \frac{4.8 \times 10^6}{3.0 \times 10^2} &= \frac{4.8}{3.0} \times \frac{10^6}{10^2} \\ &= 1.6 \times 10^{6-2} \\ &= 1.6 \times 10^4 \end{aligned}$$

b. Add exponents when you multiply.

$$\begin{aligned} (1.5 \times 10^2)(8.0 \times 10^4) &= (1.5 \times 8.0)(10^2 \times 10^4) \\ &= (12)(10^{2+4}) \\ &= (12)(10^6) \\ &= 12 \times 10^6 \\ &= (1.2 \times 10) \times 10^6 \\ &= 1.2 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{c. } 0.3 \times 10^5 &= (3 \times 10^{-1}) \times 10^5 \\ &= 3 \times 10^4 \end{aligned}$$

Simplify. Write your answers in scientific notation.

25. $\frac{6.0 \times 10^5}{1.5 \times 10^2}$ 4×10^3 26. $(4.0 \times 10^{-6})(1.6 \times 10^8)$ 6.4×10^2 27. $(6 \times 10^6)(7 \times 10^{-2})$ 4.2×10^5
 28. $(1.25 \times 10^4)(12 \times 10^3)$ 1.5×10^8 29. $\frac{2 \times 10^4}{(4 \times 10^{-2})(5 \times 10)}$ 1×10^4 30. $(4 \times 10^4)(7 \times 10^{-2})$ 2.8×10^3

Mixed Review Exercises

Simplify. Give your answers using positive exponents.

1. $-[x + (-9)] + 3y + 4$ 2. $(x^{-3}y^4)^2 \frac{y^8}{x^6}$ 3. $(11 - 23) - (5 - 15) - 2$
 $-x + 3y + 13$
 4. 25:15 **5:3** 5. $[3 + (-5)] + 6 + 4$ 6. $2t + [(-4) + (-1) + 7]$ $2t + 2$
 7. $\left(\frac{c^{-2}}{2}\right)^2 \frac{1}{4c^4}$ 8. $-\frac{3}{4} + 4 + \left(-\frac{1}{4}\right) + 3$ 9. $\frac{20b^2c^3}{16bc^2} - \frac{5bc}{4}$