

10-2 Solving Inequalities

Objective: To transform inequalities in order to solve them.

Properties

Property of Comparison For all real numbers a and b , one and only one of the following statements is true: $a < b$, $a = b$, $a > b$.

Transitive Property of Order For all real numbers a , b , and c ,

1. If $a < b$ and $b < c$, then $a < c$;
2. If $c > b$ and $b > a$, then $c > a$.

Addition Property of Order For all real numbers a , b , and c ,

1. If $a < b$, then $a + c < b + c$;
2. If $a > b$, then $a + c > b + c$.

Multiplication Property of Order

For all real numbers a , b , and c , such that $c > 0$:

1. If $a < b$, then $ac < bc$;
2. If $a > b$, then $ac > bc$.

For all real numbers a , b , and c , such that $c < 0$:

1. If $a < b$, then $ac > bc$;
2. If $a > b$, then $ac < bc$.

Vocabulary

Equivalent inequality An inequality with the same solution set as another inequality.

Transformations That Produce an Equivalent Inequality

1. **Substituting** for either side of the inequality an expression equivalent to that side.
2. **Adding to (or subtracting from)** each side of the inequality the same real number.
3. **Multiplying (or dividing)** each side of the inequality by the same *positive* number.
4. **Multiplying (or dividing)** each side of the inequality by the same *negative* number and *reversing the direction of the inequality*.

CAUTION Multiplying both sides of an inequality by zero does not produce an inequality; the result is the identity $0 = 0$.

Example 1 Tell how to transform the first inequality into the second one.

a. $m - 6 > 2$
 $m > 8$

b. $-6k \geq 18$
 $k \leq -3$

Solution a. Add 6 to each side.

b. Divide each side by -6 and reverse the direction of the inequality.

Tell how to transform the first inequality into the second one.

1. $t + 2 < 6$
 $t < 4$

2. $x - 3 > 7$
 $x > 10$

3. $x + 5 < 0$
 $x < -5$

10-2 Solving Inequalities (continued)

Tell how to transform the first inequality into the second one.

4. $4p < 28$
 $p < 7$

5. $2m < -12$
 $m < -6$

6. $-7a < 21$
 $a > -3$

7. $3 < \frac{x}{5}$
 $15 < x$

8. $\frac{x}{-2} \leq -4$
 $x \geq 8$

9. $-\frac{t}{3} \geq 0$
 $t \leq 0$

Example 2 Solve $4x - 1 < 7 + 2x$ and graph its solution set.**Solution** $4x - 1 + 1 < 7 + 2x + 1$ Add 1 to each side.

$$4x < 8 + 2x$$

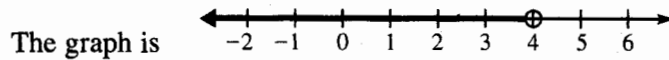
 $4x - 2x < 8 + 2x - 2x$ Subtract 2x from each side.

$$2x < 8$$

$$\frac{2x}{2} < \frac{8}{2}$$

Divide each side by 2.

$$x < 4$$
 The solution set is {the real numbers less than 4}.

**Example 3** Solve $2(w - 6) \geq 3(1 - w)$ and graph its solution set.**Solution** $2w - 12 \geq 3 - 3w$ Use the distributive property.

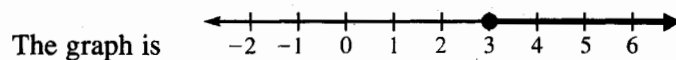
$$5w \geq 15$$

Add 3w to each side and add 12 to each side.

$$w \geq 3$$

Divide each side by 5.

The solution set is {the real numbers greater than or equal to 3}.



Solve each inequality. Graph the solution set.

10. $x - 2 \geq 3$

11. $8 < z + 2$

12. $4p < 20$

13. $15 \leq 5w$

14. $-24 > -6m$

15. $\frac{d}{2} > -3$

16. $3 - g > 0$

17. $2v + 1 > 9$

18. $6 \geq 2k - 6$

19. $3 + \frac{x}{2} \leq 4$

20. $6 - \frac{2}{3}c > 0$

21. $3r - 4 < 4r + 1$

22. $4y < 3y + 6$

23. $3f - 2 < 2f + 3$

24. $2r - 3 < 3r + 1$

25. $6 - 2b > 3 - b$

26. $2(x - 3) \leq 4$

27. $6 < 3(2 - m)$

28. $3(x + 2) \leq 3x + 2$

29. $4(k - 3) \geq 6(1 - k)$

Mixed Review Exercises

Classify each statement as true or false.

1. $|-2| > -(-1)$

2. $|-4| \leq |4|$

3. $|-7| > |-8|$

Solve.

4. $5f - 3 = f + 17$

5. $0 = 3x + 12$

6. $3y - 2(y - 1) = -4$

7. $x - 2(8 - x) = -x$

8. $a(a + 4) = (a - 6)(a - 5)$

9. $3x + 2(x - 1) = x + 22$