Algebra

Structure and Method
Book 1

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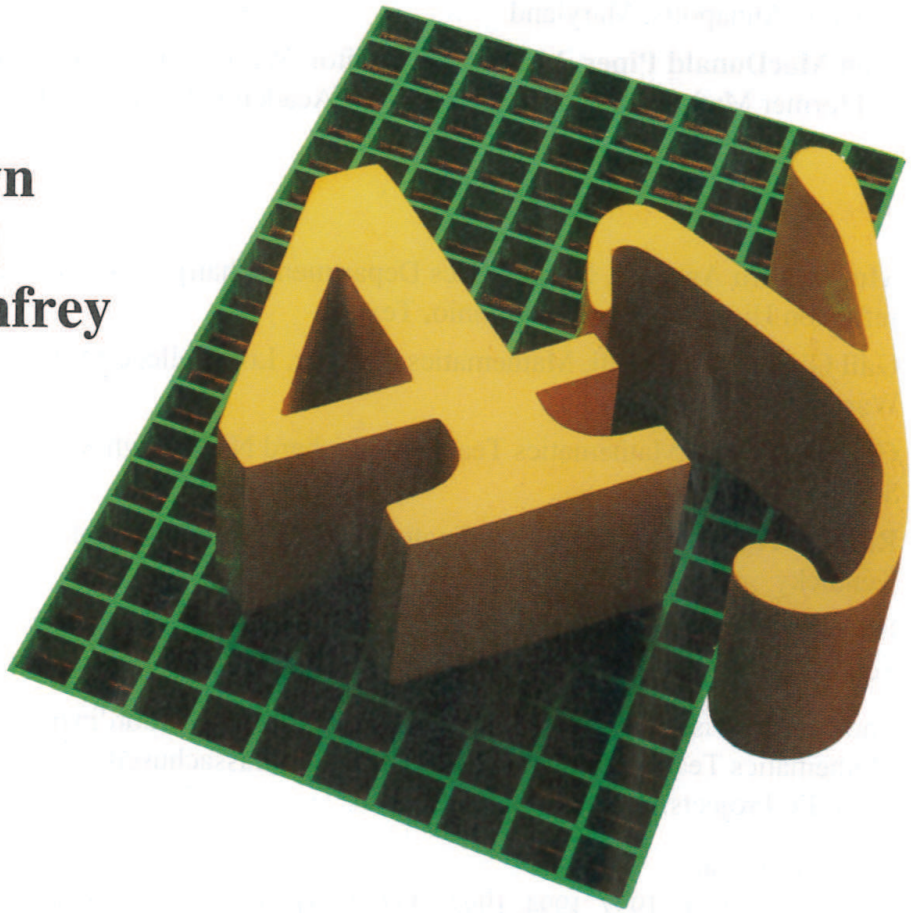

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Answers to Selected Exercises

Using Technology with This Course

There are three types of optional technology material in this text: Computer Key-In features, Computer Exercises, and suggestions for using graphing calculators and software to explore concepts and confirm results.

The Computer Key-In features can be used by students without previous programming experience. They include a program that students can run to explore an algebra topic covered in the chapter. Some writing of programs may be required in some of these features.

The optional Computer Exercises are designed for students who have some familiarity with programming in BASIC. Students are usually asked to write one or more programs related to the lesson just presented.

The suggestions for applying computer graphing techniques are appropriate for use with a graphing calculator or with graphing software such as *Algebra Plotter Plus* or *McDougal Littell Mathpack.*

Calculator Key-In features and certain exercise sets also suggest appropriate use of scientific and graphing calculators with this course.

Reading Your Algebra Book

An algebra book requires a different type of reading than a novel or a short story. Every sentence in a math book is full of information and logically linked to the surrounding sentences. You should read the sentences carefully and think about their meaning. As you read, remember that algebra builds upon itself; for example, the method of multiplying binomials that you'll study on page 200 will be useful to you on page 544. Be sure to read with a pencil and paper: Do calculations, draw sketches, and take notes.

Vocabulary

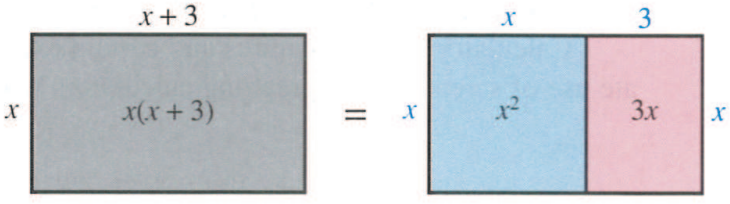
You'll learn many new words in algebra. Some, such as *polynomial* and *parabola,* are mathematical in nature. Others, such as *power* and *proof,* are used in everyday speech but have different meanings when used in algebra. Important words whose meanings you'll learn are printed in **heavy type.** They are also listed at the beginning of each Self-Test. If you don't recall the meaning of a word, you can look it up in the Glossary or the Index at the back of the book. The Glossary will give you a definition, and the Index will give you page references for more information.

Symbols

Algebra, and mathematics in general, has its own symbolic language. You must be able to read these symbols in order to understand algebra. For example, $|x| > 2$ means "the absolute value of x is greater than 2." If you aren't sure what a symbol means, check the list of symbols on page xvi.

Diagrams

Throughout this book you'll find many diagrams. They contain information that will help you understand the concepts under discussion. Study the diagrams carefully as you read the text that accompanies them.



Displayed Material

Throughout this book important information is displayed in gray boxes. This information includes properties, definitions, methods, and summaries. Be sure to read and understand the material in these boxes. You should find these boxes useful when reviewing for tests and exams.

If a is a real number and m and n are positive integers, then $a^m \cdot a^n = a^{m+n}$.

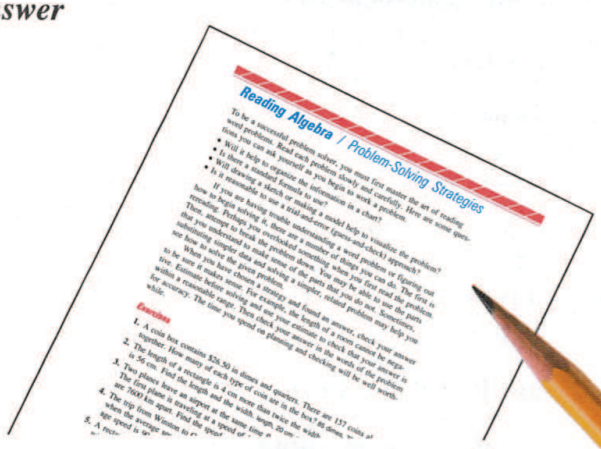
This book also contains worked-out examples. They will help you in doing many of the exercises and problems.

Example Simplify $x^3 \cdot x^5$.

Solution $x^3 \cdot x^5 = x^{3+5} = x^8$ ***Answer***

Reading Aids

Throughout this book you will find sections called Reading Algebra. These sections deal with such topics as independent study and problem solving strategies. They will help you become a more effective reader and problem solver.

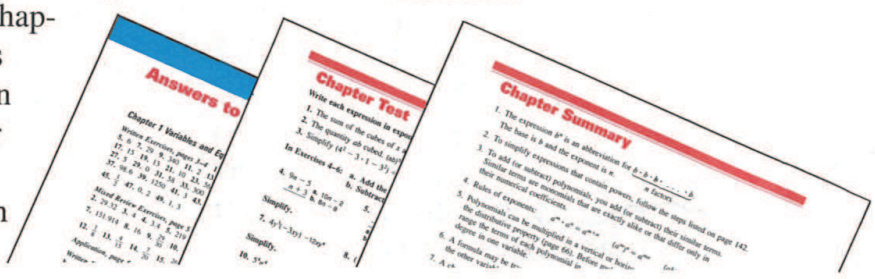


Exercises, Tests, and Reviews

Each lesson in this book is followed by Oral, Written, and Mixed Review Exercises. Lessons may also include Problems and optional Computer Exercises. Answers for all Mixed Review Exercises and for selected Written Exercises, Problems, and Computer Exercises are given at the back of this book.

Within each chapter you will find Self-Tests that you can use to check your progress. Answers for all Self-Tests are also given at the back of this book.

Each chapter concludes with a Chapter Summary that lists important ideas from the chapter, a Chapter Review in multiple-choice format, and a Chapter Test. Lesson numbers in the margins of the Review and Test indicate which lesson a group of questions covers.



Reading Algebra/*Symbols*

		Page
$\cdot$	$\times$ (times)	1
$=$	equals, is equal to	2
$\neq$	is not equal to	2
( )	parentheses—a grouping symbol	2
[ ]	brackets—a grouping symbol	6
π	pi, a number approximately equal to $\frac{22}{7}$	8
$\in$	is a member of, belongs to	10
$\therefore$	therefore	11
$\stackrel{?}{=}$	is this statement true?	27
$-$	negative	31
$+$	positive	31
$<$	is less than	32
$>$	is greater than	32
$-a$	opposite or additive inverse of a	36
$\lvert a \rvert$	absolute value of a	37
$\frac{1}{b}$	reciprocal or multiplicative inverse of b	79
$\emptyset$	empty set, null set	117
$a:b$	ratio of a to b	287

		Page
(a, b)	ordered pair whose first component is a and second component is b	349
$f(x)$	f of x, the value of f at x	379
$\geq$	is greater than or equal to	457
$\leq$	is less than or equal to	457
$\cap$	the intersection of	476
$\cup$	the union of	476
$\approx$	is approximately equal to	514
$\sqrt{\ }$	principal square root	517
$P(A)$	probability of event A	603
$\overleftrightarrow{AB}$	line AB	616
$\overline{AB}$	segment AB	616
AB	the length of $\overline{AB}$	616
$\overrightarrow{AB}$	ray AB	616
$\angle$	angle	616
$^\circ$	degree(s)	617
$\triangle$	triangle	621
$\sim$	is similar to	624
$\cos A$	cosine of A	627
$\sin A$	sine of A	627
$\tan A$	tangent of A	627

Metric Units

Length	10 millimeters (mm)	=	1 centimeter (cm)
	100 centimeters 1000 millimeters	=	1 meter (m)
	1000 meters	=	1 kilometer (km)
Area	100 square millimeters (mm^2)	=	1 square centimeter (cm^2)
	10,000 square centimeters	=	1 square meter (m^2)
Volume	1000 cubic millimeters (mm^3)	=	1 cubic centimeter (cm^3)
	1,000,000 cubic centimeters	=	1 cubic meter (m^3)
Liquid Capacity	1000 milliliters (mL)	=	1 liter (L)
	1000 cubic centimeters	=	1 liter
Mass	1000 milligrams (mg)	=	1 gram (g)
	1000 grams	=	1 kilogram (kg)
Temperature in degrees Celsius (°C)	0°C	=	freezing point of water
	100°C	=	boiling point of water

United States Customary Units

Length	12 inches (in.)	=	1 foot (ft)
	36 inches 3 feet	=	1 yard (yd)
	5280 feet 1760 yards	=	1 mile (mi)
Area	144 square inches ($in.^2$)	=	1 square foot (ft^2)
	9 square feet	=	1 square yard (yd^2)
Volume	1728 cubic inches ($in.^3$)	=	1 cubic foot (ft^3)
	27 cubic feet	=	1 cubic yard (yd^3)
Liquid Capacity	16 fluid ounces (fl oz)	=	1 pint (pt)
	2 pints	=	1 quart (qt)
	4 quarts	=	1 gallon (gal)
Weight	16 ounces (oz)	=	1 pound (lb)
Temperature in degrees Fahrenheit (°F)	32°F	=	freezing point of water
	212°F	=	boiling point of water

Time

	60 seconds (s)	=	1 minute (min)
	60 minutes	=	1 hour (h)