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DATE _____

6 Fractions

6-1 Simplifying Fractions

Objective: To simplify algebraic fractions.

Vocabulary

Simplest form of an algebraic fraction A form of the fraction in which the numerator and denominator have no common factor other than 1 and -1 .

CAUTION In a fraction, you cannot cancel terms. You must factor to find common factors to cancel. For example, $\frac{x+y}{x+2} \neq \frac{y}{2}$.

Example 1 Simplify: a. $\frac{21x - 14y}{7}$ b. $\frac{3c - 24}{c - 8}$ c. $\frac{2a + 6}{4a - 12}$

Solution Factor. Then look for common factors to cancel.

$$\text{a. } \frac{21x - 14y}{7} = \frac{7(3x - 2y)}{7} = 3x - 2y$$

$$\text{b. } \frac{3c - 24}{c - 8} = \frac{3(c - 8)}{c - 8} = 3 \quad (c \neq 8) \quad \left\{ \begin{array}{l} \text{The denominator can't equal 0.} \\ \text{So } c - 8 \neq 0, \text{ or } c \neq 8. \end{array} \right.$$

$$\begin{aligned} \text{c. } \frac{2a + 6}{4a - 12} &= \frac{2(a + 3)}{2(2a - 3)} \\ &= \frac{1(a + 3)}{2(a - 3)} \\ &= \frac{a + 3}{2(a - 3)}, \quad (a \neq 3) \end{aligned} \quad \left\{ \begin{array}{l} \text{If } a = 3, a - 3 = 0. \\ \text{You must restrict the variable} \\ \text{in the denominator.} \end{array} \right.$$

Simplify. Give any restrictions on the variables.

1. $\frac{3x - 3y}{9}$
2. $\frac{10m - 15n}{5}$
3. $\frac{4a - 20}{a - 5}$
4. $\frac{3n + 12}{n + 4}$
5. $\frac{4n + 24}{n + 6}$
6. $\frac{2n - 18}{n - 9}$
7. $\frac{2m + 3}{6m + 9}$
8. $\frac{6x + 6y}{6x - 6y}$
9. $\frac{3w + 5}{9w + 15}$
10. $\frac{4m - 4n}{4m + 4n}$

Example 2 Simplify $\frac{x^2 - 4}{2x^2 + 3x - 2}$.

Solution $\frac{x^2 - 4}{2x^2 + 3x - 2} = \frac{(x - 2)(x + 2)}{(2x - 1)(x + 2)}$ Factor. $x + 2$ is a common factor.

$$= \frac{x - 2}{2x - 1}, \quad (x \neq -2, x \neq \frac{1}{2})$$

To see which values of x to exclude, look at the denominator of the original fraction.

Since $2x - 1 \neq 0$ and $x + 2 \neq 0$, $x \neq \frac{1}{2}$ and $x \neq -2$.

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6-1 Simplifying Fractions (continued)

Simplify. Give any restrictions on the variables. **Answers given at the back of this Answer Key.**

11. $\frac{3x - 9}{x^2 - 9}$
12. $\frac{5y + 30}{y^2 - 36}$
13. $\frac{b^2 - 4}{b + 2}$
14. $\frac{x^2 - 49}{x + 7}$
15. $\frac{8n^2 - 72}{4n - 12}$
16. $\frac{15c + 25d}{90c^2 - 250d^2}$
17. $\frac{4xy}{x^2y - xy^2}$
18. $\frac{3x^2 - 6x}{3x^3}$
19. $\frac{a^2 - 3a - 10}{a^2 - 4}$
20. $\frac{a^2 - 5a - 36}{a^2 - 81}$
21. $\frac{2w^2 - w - 6}{2w - 4}$
22. $\frac{2x^2 + 5x - 3}{x^2 + 2x - 3}$

Example 3 Simplify: $\frac{2x^2 - 3x - 2}{4 - x^2}$

Solution $\frac{2x^2 - 3x - 2}{4 - x^2} = \frac{(2x + 1)(x - 2)}{(2 + x)(2 - x)}$ { Factor. Since $(x - 2)$ and $(2 - x)$ are opposites, $(2 - x) = -(x - 2)$.

$$= \frac{(2x + 1)(\cancel{x - 2})}{-(2 + x)(\cancel{x - 2})}$$

$$= \frac{2x + 1}{-(2 + x)}, \text{ or } -\frac{2x + 1}{x + 2}, \quad (x \neq 2, x \neq -2)$$

Simplify. Give any restrictions on the variables. **Answers given at the back of this Answer Key.**

23. $\frac{(3n + 2)(n - 3)}{(3 + n)(3 - n)}$
24. $\frac{(x - 4)(3x + 4)}{(4 - x)(5x + 2)}$
25. $\frac{(x - 5)(2x - 7)}{(5 - x)(3x + 2)}$
26. $\frac{(x - 7)(x - 4)}{(7 - x)(x + 2)}$
27. $\frac{x^2 - 10x + 25}{25 - x^2}$
28. $\frac{6 - x}{x^2 - 2x - 24}$
29. $\frac{(a - 3)^2}{9 - a^2}$
30. $\frac{2n^2 - 72}{6n + 36}$
31. $\frac{6 + x - x^2}{x^2 - 9}$
32. $\frac{10 + 3x - x^2}{x^2 - 4}$
33. $\frac{2w^2 - w - 6}{2w - 4}$
34. $\frac{3x^2 - 6x}{6x^2 - 7x - 10}$
35. $\frac{2n^2 + 5n - 3}{4n^2 + 8n - 5}$
36. $\frac{2y^2 - 7y + 3}{6y - 2y^2}$
37. $\frac{3y^2 - 5y + 2}{6y^2 - y - 2}$
38. $\frac{3x^2 - 15x}{3x^2 - 16x + 5}$

Mixed Review Exercises

Simplify. Assume that no denominator equals zero. **2. $6m - 4n$**

1. $10(\frac{1}{2}u + \frac{1}{5}v)$ **$5u + 2v$**
 2. $(-36m + 24n)(-\frac{1}{6})$
 3. $\frac{20a^6b^5}{35a^2b^3} \cdot \frac{4a^4b^2}{7}$
 4. $\frac{(-2y)^4}{(y^2)^4} \cdot \frac{16}{y^4}$
 5. $\frac{2x^4 + 6x^3 + 10x^2}{2x^2}$
 6. $(-10)(-6)(-2)(-5)$ **600**
- $$x^2 + 3x + 5$$

Solve.

7. $3(x + 1) + 1 = 25$ {7}
8. $8y - (5y + 4) = 11$ {5}
9. $(2n - 3) - (5 - 2n) = 16$ {6}

6-2 Multiplying Fractions

Objective: To multiply algebraic fractions.

Multiplication Rule for Fractions To multiply fractions, you multiply their numerators and multiply their denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{For example, } \frac{3}{4} \cdot \frac{5}{8} = \frac{3 \cdot 5}{4 \cdot 8} = \frac{15}{32}$$

Example 1 Multiply: $\frac{5}{6} \cdot \frac{9}{10}$

Solution 1 $\frac{5}{6} \cdot \frac{9}{10} = \frac{5 \cdot 9}{6 \cdot 10} = \frac{45}{60} = \frac{3}{4}$ You can multiply first and then simplify.

Solution 2 $\frac{1\cancel{3}}{2\cancel{6}} \cdot \frac{\cancel{3}^3}{\cancel{10}_2} = \frac{3}{4}$ You can simplify first and then multiply.

Multiply. Express each product in simplest form.

1. $\frac{3}{7} \cdot \frac{35}{9} \cdot \frac{5}{3}$
2. $\frac{5}{16} \cdot \frac{4}{15} \cdot \frac{1}{12}$
3. $\frac{12}{7} \cdot \frac{14}{9} \cdot \frac{8}{3}$
4. $-\frac{5}{2} \cdot \frac{16}{25} - \frac{8}{5}$
5. $\frac{2}{5} \cdot \frac{5}{9} \cdot \frac{9}{10} \cdot \frac{1}{5}$
6. $\frac{8}{5} \cdot \frac{3}{4} \cdot \frac{15}{16} \cdot \frac{9}{8}$
7. $\left(-\frac{3}{2}\right)^2 \cdot \frac{8}{9} \cdot 2$
8. $(-3)^2 \cdot \frac{25}{12} \cdot \frac{75}{4}$

Example 2 a. $\frac{9x}{y^2} \cdot \frac{y^3}{24}$ b. $\frac{x^2 + x - 12}{x^2 + 5x} \cdot \frac{x^2 - 25}{x - 3}$

Solution a. $\frac{9x}{y^2} \cdot \frac{y^3}{24} = \frac{\cancel{3} \cdot 3x}{\cancel{y}^2} \cdot \frac{\cancel{y}^3 \cdot y}{\cancel{3} \cdot 8} = \frac{3xy}{8}$ (y ≠ 0)
 b. $\frac{x^2 + x - 12}{x^2 + 5x} \cdot \frac{x^2 - 25}{x - 3} = \frac{(x+4)(x-3)}{x(x+5)} \cdot \frac{(x+5)(x-5)}{(x-3)}$

$$= \frac{(x+4)(x-5)}{x} \quad (x \neq 0, x \neq -5, x \neq 3)$$

You can leave the answer in factored form.

CAUTION From now on, assume that no denominator equals zero. You won't need to show the excluded values, but know what they are.

Multiply. Express each product in simplest form.

9. $\frac{8}{x^2} \cdot \frac{x^3}{4} \cdot 2x$
10. $\frac{7y}{5} \cdot \frac{10}{21y} \cdot \frac{2}{3}$
11. $\frac{a}{c} \cdot \frac{c}{d} \cdot \frac{d}{e} \cdot \frac{e}{a}$
12. $\frac{6}{x^2} \cdot \frac{5x}{12} \cdot \frac{5}{2x}$
13. $\frac{6w}{v} \cdot \frac{v^3}{3w^2} \cdot \frac{2v^2}{w}$
14. $\frac{8a}{11b^3} \cdot \frac{33b}{4a^2} \cdot \frac{6}{ab^2}$
15. $\frac{2de^2}{5e^2f} \cdot \frac{f^2}{4d} \cdot \frac{f}{10}$
16. $\frac{3rs^2}{4t} \cdot \frac{8t}{9rs} \cdot \frac{2st}{3}$

6-2 Multiplying Fractions (continued)

Multiply. Express each product in simplest form.

17. $\frac{x-3}{x^2} \cdot \frac{2x}{x^2-9} \cdot \frac{2}{x(x+3)}$
18. $\frac{x^2-4}{8x} \cdot \frac{4x^2}{5x+10} \cdot \frac{x(x-2)}{10}$
19. $\frac{a^2-b^2}{a^2} \cdot \frac{a}{2(b-a)} - \frac{a+b}{2a}$
20. $\frac{m}{3(n-m)} \cdot \frac{m^2-n^2}{m^2} - \frac{m+n}{3m}$

Rule of Exponents for a Power of a Quotient

For every positive integer m, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$. For example, $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$.

Example 3 Simplify: a. $\left(\frac{x}{2}\right)^4$ b. $\left(-\frac{x}{3}\right)^2 \cdot \frac{9}{5x}$

Solution a. $\left(\frac{x}{2}\right)^4 = \frac{x^4}{2^4} = \frac{x^4}{16}$
 b. $\left(-\frac{x}{3}\right)^2 \cdot \frac{9}{5x} = \frac{x^2}{9} \cdot \frac{9}{5x} = \frac{x \cdot \cancel{9}}{\cancel{9}} \cdot \frac{\cancel{9}}{5 \cdot x} = \frac{x}{5}$

Multiply. Express each product in simplest form.

21. $\left(\frac{a}{4}\right)^2 \cdot \frac{a^2}{16}$
22. $\left(\frac{c}{3}\right)^3 \cdot \frac{c^3}{27}$
23. $\left(\frac{m}{5}\right)^3 \cdot \frac{m^3}{125}$
24. $\left(\frac{y}{4}\right)^3 \cdot \frac{y^3}{64}$
25. $\left(\frac{2n}{5}\right)^2 \cdot \frac{4n^2}{25}$
26. $\left(\frac{3x}{4}\right)^2 \cdot \frac{9x^2}{16}$
27. $\left(\frac{3w}{7}\right)^2 \cdot \frac{9w^2}{49}$
28. $\left(\frac{7b}{2}\right)^2 \cdot \frac{49b^2}{4}$
29. $\left(\frac{2a}{3b^3}\right)^2 \cdot \frac{4a^2}{9b^6}$
30. $\left(\frac{4m}{5n^2}\right)^2 \cdot \frac{16m^2}{25n^4}$
31. $\left(-\frac{x}{3y}\right)^2 \cdot \frac{x^2}{9y^2}$
32. $-\left(\frac{4b^3}{5}\right)^2 - \frac{16b^6}{25}$
33. $\left(\frac{x}{y}\right)^2 \cdot \frac{y}{x} \cdot \frac{x}{y}$
34. $\left(\frac{2x}{y}\right)^3 \cdot \frac{y^2}{4} \cdot \frac{2x^3}{y}$
35. $\left(-\frac{x}{2y}\right)^2 \left(-\frac{4y}{x}\right) - \frac{x}{y} \cdot 36 \cdot \left(\frac{2z}{y}\right)^3 \cdot \frac{3yz}{8} \cdot \frac{3z^4}{y^2}$

37. Find the area of a square if each side has length $\frac{3x}{4}$ in. $\frac{9x^2}{16}$ in.²

38. Find the volume of a cube if each edge has length $\frac{2n}{3}$ in. $\frac{8n^3}{27}$ in.³

1. $(a+3)(a+8)$
2. $(x-7)(x+2)$
3. $(9x^2+4)(3x+2)(3x-2)$
4. $(2x+1)(x+2)$

Mixed Review Exercises

Factor completely.

5. $(5y+3z)(5y-3z)$
6. $(c+5)^2$
7. $(x+5)(y-4z)$
8. $(3x-1)^2$
1. $a^2 + 11a + 24$
2. $x^2 - 5x - 14$
3. $81x^4 - 16$
4. $2x^2 + 5x + 2$
5. $25y^2 - 9z^2$
6. $c^2 + 10c + 25$
7. $xy + 5y - 4xz - 20z$
8. $9x^2 - 6x + 1$
9. $3x^2 - 11x - 4$
10. $x^4 + 6x^2 - 5x^3$
11. $n^2 + 4n - 12$
12. $y^2 - 5y - 36$
9. $(3x+1)(x-4)$
10. $x^2(x-2)(x-3)$
11. $(n+6)(n-2)$
12. $(y-9)(y+4)$

6-3 Dividing Fractions

Objective: To divide algebraic fractions.

Division Rule for Fractions To divide by a fraction, you multiply by its reciprocal. Remember that the reciprocal of a number n is the number $\frac{1}{n}$ for which $n \cdot \frac{1}{n} = 1$.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad \text{For example, } \frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \cdot \frac{7}{2} = \frac{21}{10}.$$

Example 1 Divide: $\frac{x}{3y} \div \frac{xy}{9}$

Solution $\frac{x}{3y} \div \frac{xy}{9} = \frac{x}{3y} \cdot \frac{9}{xy}$ Multiply by the reciprocal of $\frac{xy}{9}$.

$$= \frac{\cancel{x} \cdot 3}{\cancel{x} \cdot y} \cdot \frac{\cancel{9} \cdot 3}{\cancel{9} \cdot y}$$
 Factor and simplify.

$$= \frac{3}{y^2}$$

Divide. Give your answers in simplest form.

1. $\frac{8}{5} \div \frac{16}{25} \cdot \frac{5}{2}$ 2. $\frac{3}{4} \div \frac{9}{8} \cdot \frac{2}{3}$ 3. $\frac{a}{10} \div \frac{a}{2} \cdot \frac{1}{5}$ 4. $\frac{2x}{5} \div \frac{x}{15} \cdot 6$ 5. $\frac{x^2}{y} \div \frac{x}{y^2}$

6. $\frac{4n^2}{5} \div \frac{8n}{25} \cdot \frac{5n}{2}$ 7. $\frac{ab}{4} \div \frac{a}{b} \cdot \frac{b^2}{4}$ 8. $\frac{c}{3d} \div \frac{c^2}{9d^2} \cdot \frac{3d}{c}$ 9. $\frac{2x^2}{3y} \div \frac{xy}{9} \cdot \frac{6x}{y^2}$ 10. $\frac{3n}{4m^2} \div \frac{1}{12mn}$

11. $\frac{x^2y}{2} \div xy \cdot \frac{x}{2}$ 12. $\frac{8a^2}{3b} \div 4a \cdot \frac{2a}{3b}$ 13. $1 \div \left(\frac{2x}{3}\right)^2 \cdot \frac{9}{4x^2}$ 14. $9 \div \left(\frac{3}{n}\right)^2 \cdot n^2$ 15. $16 \div \left(\frac{2}{a}\right)^3 \cdot 2a^3$

Example 2 Divide: $\frac{15}{x^2 - 16} \div \frac{20}{x - 4}$

Solution $\frac{15}{x^2 - 16} \div \frac{20}{x - 4} = \frac{15}{x^2 - 16} \cdot \frac{x - 4}{20}$ Multiply by the reciprocal.

$$= \frac{\cancel{5} \cdot 3}{(x + 4)(\cancel{x - 4})} \cdot \frac{\cancel{x - 4}}{\cancel{5} \cdot 4}$$
 Factor and simplify.

$$= \frac{3}{4(x + 4)}$$

Divide. Give your answers in simplest form.

16. $\frac{3 + 3b}{6} \div \frac{1 + b}{9} \cdot \frac{9}{2}$ 17. $\frac{4n - 2}{8n} \div \frac{2n - 1}{24} \cdot \frac{6}{n}$ 18. $\frac{x^2 - 4}{3} \div \frac{x + 2}{9} \cdot 3(x - 2)$

19. $\frac{x^2 - 16}{3x} \div \frac{x - 4}{6} \cdot \frac{2(x + 4)}{x}$ 20. $\frac{x^2 - 9}{3} \div \frac{x + 3}{6} \cdot 2(x - 3)$ 21. $\frac{x^2 - 25}{4x} \div \frac{x - 5}{12} \cdot \frac{3(x + 5)}{x}$

6-3 Dividing Fractions (continued)

Divide. Give your answers in simplest form.

22. $\frac{2}{x - 3} \div \frac{2}{3 - x} - 1$ 23. $\frac{4}{6 - 3a} \div \frac{6}{8 - 4a} \cdot \frac{8}{9}$ 24. $\frac{x^2 + 2x}{x^2 - 4} \div \frac{x + 2}{x - 2} \cdot \frac{x}{x + 2}$

25. $\frac{1}{3a - 12} \div \frac{1}{2a - 8} \cdot \frac{2}{3}$ 26. $\frac{3x - 3y}{x} \div \frac{x^2 - y^2}{x^2} \cdot \frac{3x}{x + y}$ 27. $\frac{4}{n^2 - 16} \div \frac{8n - 32}{n + 4}$

Example 3 Divide: $\frac{x^2 - 3x - 10}{2x - 6} \div \frac{x^2 - 4}{x^2 + x - 6}$

Solution $\frac{x^2 - 3x - 10}{2x - 6} \div \frac{x^2 - 4}{x^2 + x - 6} = \frac{x^2 - 3x - 10}{2x - 6} \cdot \frac{x^2 + x - 6}{x^2 - 4}$

$$= \frac{(x - 5)(x + 2)}{2(x - 3)} \cdot \frac{(x + 3)(x - 2)}{(x - 2)(x - 2)}$$

$$= \frac{(x - 5)(x + 3)}{2(x - 3)}$$

Stop; no further simplification is possible.

Divide. Give your answers in simplest form.

28. $\frac{x^2 - 9}{x^2 - 4} \div \frac{x^2 - x - 6}{x^2 + x - 6} \cdot \frac{(x + 3)^2}{(x + 2)^2}$ 29. $\frac{x^2 + x - 20}{5x + 25} \div \frac{x^2 - 4x - 5}{x^2 - 25} \cdot \frac{(x - 4)(x + 5)}{5(x + 1)}$

30. $\frac{x^2 - y^2}{x^2 + y^2} \div (x - y) \cdot \frac{x + y}{x^2 + y^2}$ 31. $\frac{x^2 - x - 6}{x^2 + 2x + 1} \div \frac{x + 2}{x + 1} \cdot \frac{x - 3}{x + 1}$

32. $\frac{x^2 - 3x + 2}{x^2 + 3x + 2} \div \frac{4x - 8}{8x + 8} \cdot \frac{2(x - 1)}{x + 2}$ 33. $\frac{x^2 - 4}{x + 2} \div \frac{x - 2}{x + 1} \cdot x + 1$

34. $\frac{x^2 - 25}{x^2 - 16} \div \frac{4x + 20}{8x - 32} \cdot \frac{2(x - 5)}{x + 4}$ 35. $\frac{4x^2 - y^2}{4y^2 - x^2} \div \frac{2x - y}{2y - x} \cdot \frac{2x + y}{2y + x}$

36. $\frac{x^2 - 3x + 2}{x^2 - 7x + 10} \div \frac{x^2 - 1}{x^2 - 4x - 5} \cdot 1$ 37. $\frac{x^2 - 8x + 15}{x^2 - 9x + 14} \div \frac{x^2 - 9}{x^2 + x - 6} \cdot \frac{x - 5}{x - 7}$

38. $\frac{2x^2 + 7x + 3}{2x^2 + 5x + 2} \div \frac{x^2 - 7x - 30}{x^2 - 6x - 40} \cdot \frac{x + 4}{x + 2}$ 39. $\frac{x^2 + 5x - 6}{x^2 - x - 20} \div \frac{x^2 + 2x - 3}{x^2 - 2x - 15} \cdot \frac{x + 6}{x + 4}$

Mixed Review Exercises

Solve.

1. $3k = 4k - 11$ {11} 2. $5p + 10 = 45$ {7} 3. $(4b - 3) - (3 - 2b) = 30$ {6}

4. $\frac{1}{3}(9k - 6) = 7$ {3} 5. $2n^3 - 32n = 0$ {0, -4, 4} 6. $2x^2 + x = 3$ $\{-\frac{3}{2}, 1\}$

Give the prime factorization of each number.

7. 225 $3^2 \cdot 5^2$ 8. 136 $2^3 \cdot 17$ 9. 140 $2^2 \cdot 5 \cdot 7$ 10. 1250 $2 \cdot 5^4$

6-4 Least Common Denominators

Objective: To express two or more fractions with their least common denominator.

Example 1 Complete: a. $\frac{2}{3} = \frac{?}{15}$ b. $\frac{5}{2a} = \frac{?}{18a^2}$

Solution To write a fraction in a different form, you can multiply the numerator and denominator by the same nonzero number.

$$\begin{aligned} \text{a. } \frac{2}{3} &= \frac{?}{15} \longleftarrow 3 \text{ is multiplied by 5 to get 15.} \\ \frac{2}{3} &= \frac{2 \cdot 5}{3 \cdot 5} = \frac{10}{15} \longleftarrow \text{Therefore, multiply 2 by 5 to get 10.} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{5}{2a} &= \frac{?}{18a^2} \longleftarrow 2a \text{ is multiplied by } 9a \text{ to get } 18a^2. \\ \frac{5}{2a} &= \frac{5 \cdot 9a}{2a \cdot 9a} = \frac{45a}{18a^2} \longleftarrow \text{Therefore, multiply 5 by } 9a \text{ to get } 45a. \end{aligned}$$

Complete. 6. $3(2n - 3)$ 7. $48x$ 8. $15a^2$ 9. $6x^2$ 10. $9mn^2$

$$\begin{aligned} 1. \frac{2}{3} &= \frac{?}{18} & 12. \frac{3}{5} &= \frac{?}{20} & 13. \frac{5}{8} &= \frac{?}{56} & 35 & 4. \frac{2a}{15} &= \frac{?}{45} & 6a & 5. \frac{x-2}{3} &= \frac{?}{12} \\ 6. \frac{2n-3}{5} &= \frac{?}{15} & 7. \frac{8}{15x} &= \frac{?}{90x^2} & 8. \frac{5}{3a} &= \frac{?}{9a^3} & 9. \frac{x}{3y} &= \frac{?}{18xy} & 10. \frac{3n}{4m} &= \frac{?}{12m^2n} \end{aligned}$$

Example 2 Complete: $\frac{2}{x-3} = \frac{?}{(x-3)(x+4)}$

Solution $\frac{2}{x-3} = \frac{?}{(x-3)(x+4)} \longleftarrow (x-3) \text{ is multiplied by } (x+4).$
 $\frac{2}{x-3} = \frac{2(x+4)}{(x-3)(x+4)} \longleftarrow \text{Therefore, multiply 2 by } (x+4).$

Complete.

$$\begin{aligned} 11. \frac{6}{n-1} &= \frac{?}{(n-1)(n+4)} & 6(n+4) & 12. \frac{4}{x+2} &= \frac{?}{(x+2)(x-2)} & 4(x-2) \\ 13. \frac{3}{2x-1} &= \frac{?}{(2x-1)^2} & 3(2x-1) & 14. \frac{5y}{x-7} &= \frac{?}{(x-7)^2} & 5y(x-7) \\ 15. \frac{7}{x-3} &= \frac{?}{4x-12} & 28 & 16. \frac{3}{2x+5} &= \frac{?}{6x+15} & 9 \\ 17. \frac{3}{x+2} &= \frac{?}{x^2-4} & 3(x-2) & 18. \frac{4}{x-1} &= \frac{?}{x^2-1} & 4(x+1) \\ 19. \frac{5}{3-y} &= \frac{?}{3y-y^2} & 5y & 20. \frac{3x}{2+x} &= \frac{?}{2x+x^2} & 3x^2 \end{aligned}$$

6-4 Least Common Denominator (continued)

Example 3 Find the LCD of $\frac{5}{6}$, $\frac{7}{20}$, and $\frac{8}{42}$.

Solution

- Factor each denominator into prime numbers.
 $6 = 2 \cdot 3$ $20 = 2^2 \cdot 5$ $42 = 2 \cdot 3 \cdot 7$
- Greatest power of 2: 2^2
 Greatest power of 3: 3 $2^2 \cdot 3 \cdot 5 \cdot 7 = 420$
 Greatest power of 5: 5
 Greatest power of 7: 7 The LCD is 420.

Find the LCD of each group of fractions.

$$21. \frac{1}{4}, \frac{5}{6} \quad 12 \quad 22. \frac{1}{2}, \frac{3}{8} \quad 8 \quad 23. \frac{3}{2}, \frac{2}{5}, \frac{1}{4} \quad 24. \frac{2}{3}, \frac{5}{9}, \frac{1}{6} \quad 18 \quad 25. \frac{5}{8}, \frac{2}{5}, \frac{4}{3} \quad 26. \frac{2}{3}, \frac{3}{4}, \frac{5}{9} \quad 36$$

Example 4 Find the LCD of $\frac{5}{9x-36}$ and $\frac{4}{5x-20}$.

Solution

- Factor each denominator completely. Factor integers into primes.
 $9x - 36 = 9(x - 4) = 3^2(x - 4)$ $5x - 20 = 5(x - 4)$
- Form the product of the greatest power of each factor.
 $3^2 \cdot 5(x - 4) = 45(x - 4)$
 The LCD is $45(x - 4)$.

Find the LCD of each group of fractions.

$$\begin{aligned} 27. \frac{a+2b}{4}, \frac{2b-a}{6} & 12 & 28. \frac{n-2}{12}, \frac{n+3}{15} & 60 & 29. \frac{n-1}{15}, \frac{n+3}{20} & 60 \\ 30. \frac{2x+3}{12}, \frac{x-4}{8} & 24 & 31. \frac{x+2y}{25}, \frac{2x+y}{20} & 100 & 32. \frac{x^2-x-6}{21}, \frac{x^2-9}{35} & 105 \\ 33. \frac{2}{3t}, \frac{5}{9rt^2} & 9rt^2 & 34. \frac{5}{xy}, \frac{6}{y^2} & xy^2 & 35. \frac{11}{m^2n}, \frac{17}{mn^2} & m^2n^2 \\ 36. \frac{3}{2x+10}, \frac{x}{5x+25} & & 37. \frac{3a}{a+1}, \frac{2}{a-1} & & 38. \frac{3}{a^2-4}, \frac{5}{a+2} & \\ 39. \frac{x}{x^2+3x}, \frac{2x}{x^2-3x} & & 40. \frac{7}{n+3}, \frac{n-1}{n^2+n-6} & & 41. \frac{a+1}{a-2}, \frac{a-5}{a^2-5a+6} & \end{aligned}$$

Mixed Review Exercises 1. $3(n - 3q + 5)$ 3. $(x - 8)(x - 2)$ 4. $(x - 9)(x + 4)$

Factor completely.

$$\begin{aligned} 5. (2x + 1)(x - 3) & 6. (x + 2)(x + 12) & 7. (x - 8)(x + 4) \\ 1. 3n - 9q + 15 & 2. 2x^2 - 8 & 2(x + 2)(x - 2) & 3. x^2 - 10x + 16 \\ 4. x^2 - 5x - 36 & 5. 2x^2 - 5x - 3 & 6. x^2 + 14x + 24 \\ 7. x^2 - 4x - 32 & 8. x^2 + 24x + 144 & (x + 12)^2 & 9. n^2 - 6n & n(n - 6) \end{aligned}$$

6-5 Adding and Subtracting Fractions

Objective: To add and subtract algebraic fractions.

Rules for Fractions

Addition Rule for Fractions

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \quad \text{For example, } \frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$$

Subtraction Rule for Fractions

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c} \quad \text{For example, } \frac{5}{9} - \frac{1}{9} = \frac{5-1}{9} = \frac{4}{9}$$

Example 1 Simplify: a. $\frac{5x}{12} + \frac{x}{12}$ b. $\frac{8x+10}{9} - \frac{2x+1}{9}$

Solution To add or subtract fractions with the same denominator, you add or subtract their numerators and write the result over the common denominator. Then simplify.

$$\begin{aligned} \text{a. } \frac{5x}{12} + \frac{x}{12} &= \frac{5x+x}{12} & \text{b. } \frac{8x+10}{9} - \frac{2x+1}{9} &= \frac{8x+10-(2x+1)}{9} \\ &= \frac{6x}{12} & &= \frac{8x+10-2x-1}{9} \\ &= \frac{\cancel{6}x}{\cancel{6} \cdot 2} & &= \frac{6x+9}{9} \\ &= \frac{x}{2} & &= \frac{\cancel{3}(2x+3)}{\cancel{3} \cdot 3} \\ & & &= \frac{2x+3}{3} \end{aligned}$$

- Simplify.**
- | | | | |
|-----------------------------------|------------------------------------|-------------------------------------|-------------------------------------|
| 5. $\frac{2x+1}{4}$ | 6. $\frac{-2a-5}{9}$ | 7. $\frac{-2y+3}{3}$ | 8. $-3x+2$ |
| 1. $\frac{x}{15} + \frac{4x}{15}$ | 2. $\frac{7x}{12} - \frac{5x}{12}$ | 3. $\frac{3}{x} - \frac{5}{x}$ | 4. $\frac{3}{5x} + \frac{4}{5x}$ |
| 5. $\frac{3x}{4} - \frac{x-1}{4}$ | 6. $\frac{2a}{9} - \frac{4a+5}{9}$ | 7. $\frac{y-2}{3} - \frac{3y-5}{3}$ | 8. $\frac{x+5}{2} - \frac{7x+1}{2}$ |

Example 2 Simplify: a. $\frac{2}{x+3} + \frac{1}{x+3}$ b. $\frac{3}{x-2} + \frac{5}{2-x}$

Solution

a. $\frac{2}{x+3} + \frac{1}{x+3} = \frac{2+1}{x+3} = \frac{3}{x+3}$ Add the numerators.

b. $\frac{3}{x-2} + \frac{5}{2-x} = \frac{3}{x-2} + \frac{5}{-(x-2)} = \frac{3}{x-2} - \frac{5}{x-2}$ Since $2-x = -(x-2)$, the LCD is $x-2$.

$= \frac{3-5}{x-2}$ Subtract the numerators.

$= \frac{-2}{x-2}$, or $-\frac{2}{x-2}$

- 6-5 Adding and Subtracting Fractions (continued)**
9. $\frac{5}{x+4}$ 10. $\frac{2}{y-4}$
- Simplify. 11. $\frac{x+2}{x-1}$ 12. $\frac{-n+1}{n-3}$ 13. $\frac{2}{x-3}$ 14. $\frac{9}{2x-3}$ 15. 3 16. 4
9. $\frac{3}{x+4} + \frac{2}{x+4}$ 10. $\frac{8}{y-4} - \frac{6}{y-4}$ 11. $\frac{x}{x-1} + \frac{2}{x-1}$ 12. $\frac{n}{n-3} - \frac{2n-1}{n-3}$
13. $\frac{4}{x-3} + \frac{2}{3-x}$ 14. $\frac{7}{2x-3} - \frac{2}{3-2x}$ 15. $\frac{3x}{x-y} + \frac{3y}{y-x}$ 16. $\frac{4a}{a-b} + \frac{4b}{b-a}$

Example 3 Simplify: a. $\frac{5}{2m} - \frac{1}{6m^2}$ b. $\frac{a}{6} - \frac{4+3a}{10}$

Solution To add or subtract fractions with different denominators, first rewrite the fractions using their least common denominator.

$$\begin{aligned} \text{a. } \frac{5}{2m} - \frac{1}{6m^2} &= \frac{5 \cdot 3m}{2m \cdot 3m} - \frac{1}{6m^2} & \text{b. } \frac{a}{6} - \frac{4+3a}{10} &= \frac{a \cdot 5}{6 \cdot 5} - \frac{(4+3a) \cdot 3}{10 \cdot 3} \\ &= \frac{15m}{6m^2} - \frac{1}{6m^2} & &= \frac{5a - 3(4+3a)}{30} \\ &= \frac{15m-1}{6m^2} & &= \frac{5a-12-9a}{30} \\ & & &= \frac{-4a-12}{30} \\ & & &= \frac{-4(a+3)}{30} \\ & & &= \frac{-2(a+3)}{15}, \text{ or } -\frac{2(a+3)}{15} \end{aligned}$$

- Simplify.**
- | | | | | |
|--|---|--|--------------------------------------|-------------------------------------|
| 19. $\frac{10x-1}{4x^2}$ | 22. $\frac{28b-7}{40}$ | 25. $\frac{5-3x}{3(x+1)}$ | 27. $\frac{3+4x}{4(x+1)}$ | 28. $\frac{4x+3}{2(x-1)}$ |
| 17. $\frac{3}{n^2} + \frac{2}{n}$ | 3. $\frac{3+2n}{n^2}$ | 18. $\frac{6}{x^2y} - \frac{4}{xy}$ | 6. $\frac{6-4x}{x^2y}$ | 19. $\frac{5}{2x} - \frac{1}{4x^2}$ |
| 21. $\frac{1+4x}{6} + \frac{1-x}{8}$ | 13x+7 | 22. $\frac{4b+1}{8} + \frac{2b-3}{10}$ | 23. $\frac{a-6}{6} - \frac{5-a}{15}$ | 7a-40 |
| 24. $\frac{1+3x}{2} - \frac{6x-1}{6}$ | 4+3x | 25. $\frac{5}{3(x+1)} - \frac{x}{(x+1)}$ | 3x | 1 |
| 27. $\frac{3}{4(x+1)} + \frac{x}{x+1}$ | 28. $\frac{2x}{x-1} + \frac{3}{2(x-1)}$ | 26. $\frac{3x}{x-2} - \frac{1}{2(x-2)}$ | 6x-1 | 1 |
| 30. $\frac{3(a-6)}{10} - \frac{4(a+6)}{-11a}$ | 6-174 | 29. $\frac{4n-3}{15} - \frac{3(n-2)}{10}$ | 12-n | 30 |
| 31. $\frac{3x+4}{4} - \frac{4x}{5} + \frac{x-2}{10}$ | x+16 | 32. $\frac{3n}{4} + \frac{n+2}{3} - \frac{1-n}{6}$ | 5n+2 | 4 |

Mixed Review Exercises

- Simplify.**
- | | | |
|---|-----------------------------|-----------------------------|
| 4. $5y^2 + 6y$ | 5. $6x - 4y$ | 8. $-n^3 + 6n^2 + 4n$ |
| 1. $-4^2 \cdot 3 - 48$ | 2. $(3 \cdot 5 - 10)^2$ | 25 |
| 4. $3y(y-4) + 2y(y+9)$ | 5. $-\frac{1}{5}(-30x+20y)$ | 6. $(4x^2y)(3x^3y^2)(5y^2)$ |
| 7. $\left(\frac{6x^2y^2}{7}\right)\left(\frac{-14xy^2}{3}\right) - 4x^3y^4$ | 8. $n^2(n+6) - (2n^2-4)n$ | 9. $(3-2 \cdot 10)^2$ |
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6-6 Mixed Expressions

Objective: To write mixed expressions as fractions in simplest form.

Vocabulary

Mixed number The sum of an integer and a fraction. For example, $2\frac{1}{3}$.

Mixed expression The sum or difference of a polynomial and a fraction.
For example, $m + \frac{3}{m}$.

Example 1 Write $2\frac{1}{3}$ as a fraction in simplest form.

Solution $2\frac{1}{3} = 2 + \frac{1}{3}$
 $= \frac{2}{1} + \frac{1}{3}$ Write 2 as $\frac{2}{1}$.
 $= \frac{6}{3} + \frac{1}{3}$ LCD = 3
 $= \frac{7}{3}$

Write as a fraction in simplest form.

1. $3\frac{2}{3} \frac{11}{3}$
2. $2\frac{1}{8} \frac{17}{8}$
3. $-3\frac{3}{5} -\frac{18}{5}$
4. $-4\frac{5}{7} -\frac{33}{7}$
5. $5\frac{1}{6} \frac{31}{6}$
6. $6\frac{1}{5} \frac{31}{5}$
7. $-2\frac{3}{4} -\frac{11}{4}$
8. $-1\frac{2}{9} -\frac{11}{9}$

Example 2 Write each expression as a fraction in simplest form.

- a. $x + \frac{2}{x}$ b. $3 - \frac{x-1}{x+2}$

Solution a. $x + \frac{2}{x} = \frac{x}{1} + \frac{2}{x}$ Write x as $\frac{x}{1}$.
 $= \frac{x^2}{x} + \frac{2}{x}$ LCD = x ($\frac{x}{1} = \frac{x \cdot x}{1 \cdot x} = \frac{x^2}{x}$)
 $= \frac{x^2 + 2}{x}$

b. $3 - \frac{x-1}{x+2} = \frac{3}{1} - \frac{x-1}{x+2}$ Write 3 as $\frac{3}{1}$.
 $= \frac{3(x+2)}{x+2} - \frac{x-1}{x+2}$ LCD = $x+2$ ($\frac{3}{1} = \frac{3(x+2)}{x+2}$)
 $= \frac{3x+6-x+1}{x+2}$
 $= \frac{2x+7}{x+2}$

Write each expression as a fraction in simplest form.

9. $6 + \frac{1}{x} \frac{6x+1}{x}$
10. $2 + \frac{5}{a} \frac{2a+5}{a}$
11. $3 - \frac{2}{x} \frac{3x-2}{x}$
12. $5 - \frac{3}{n} \frac{5n-3}{n}$
13. $5a - \frac{2}{a} \frac{5a^2-2}{a}$
14. $6n - \frac{4}{n} \frac{6n^2-4}{n}$
15. $\frac{3}{y} + y \frac{3+y^2}{y}$
16. $4 - \frac{m}{n} \frac{4n-m}{n}$

6-6 Mixed Expressions (continued)

26. $\frac{6a^2 + 4a - 3}{2a + 1}$
27. $\frac{6a^3 + 9a^2 - a - 1}{2a + 3}$
28. $\frac{x^3 + x^2 - 2x - 1}{x + 1}$
17. $2 + \frac{x}{y} \frac{2y+x}{y}$
18. $3 - \frac{2}{x+1} \frac{3x+1}{x+1}$
19. $7 + \frac{y}{y-2} \frac{8y-14}{y-2}$
20. $\frac{x}{x+3} - 4 - \frac{3x+12}{x+3}$
21. $3x + \frac{x}{x-1} \frac{3x^2-2x}{x-1}$
22. $5x - \frac{x}{x+2} \frac{5x^2+9x}{x+2}$
23. $3y + \frac{y}{y-2} \frac{3y^2-5y}{y-2}$
24. $\frac{2n}{2n+3} + 1 \frac{4n+3}{2n+3}$
25. $2x + \frac{x-1}{x+2} \frac{2x^2+5x-1}{x+2}$
26. $3a + \frac{a-3}{2a+1}$
27. $3a^2 - \frac{a+1}{2a+3}$
28. $x^2 - \frac{2x+1}{x+1}$

Example 3 Write as a fraction in simplest form: $x + \frac{2x-3}{x+1} + \frac{5}{x+1}$

Solution $x + \frac{2x-3}{x+1} + \frac{5}{x+1} = \frac{x(x+1)}{x+1} + \frac{2x-3}{x+1} + \frac{5}{x+1}$ The LCD is $x+1$.
 $= \frac{x^2+x+2x-3+5}{x+1}$ Add the numerators.
 $= \frac{x^2+3x+2}{x+1}$
 $= \frac{(x+2)(x+1)}{x+1}$ Factor.
 $= \frac{x+2}{1}$ Simplify.
 $= x+2$

Answers given at the back of

Write each expression as a fraction in simplest form. **this Answer Key.**

29. $\frac{x}{x-1} + \frac{x-1}{x} - 2$
30. $x + \frac{2x+1}{x-2}$
31. $\frac{3}{x+1} + \frac{x}{x+1} - 1$
32. $\frac{x}{x+3} + \frac{x}{x-3} - 3$
33. $x - \frac{4}{x+1} - \frac{3x-1}{x+1}$
34. $\frac{x-1}{x} + \frac{3}{x-1} + 2$
35. $\frac{2a}{a+1} + \frac{3}{a-1} - 1$
36. $2 - \frac{x}{x+3} - \frac{1}{x+3}$
37. $\frac{3x}{x-1} + \frac{2}{x+1} + 1$

Mixed Review Exercises

- Simplify.**
1. $\frac{3a-3b}{3a+12b} \frac{a-b}{a+4b}$
 2. $\frac{a^2-7a+10}{25-a^2} - \frac{a-2}{a+5}$
 3. $\frac{3x^2}{2y^2} + \frac{9xy}{8} \frac{12x^2+9xy^3}{8y^2}$
 4. $\frac{n^2-1}{3} + \frac{n+1}{9}$
 5. $\frac{8}{y^4} \cdot \frac{y^7}{2} 4y^3$
 6. $(-3b^3)^2 9b^6$
 7. $\frac{1}{3xy}, \frac{2}{y^2} 3xy^2$
 8. $\frac{3}{a^2}, \frac{2}{ab} a^2b$
 9. $\frac{1}{x-2}, \frac{3}{x+2}, \frac{5}{x^2-4}$
 $x^2 - 4$

6-7 Polynomial Long Division

Objective: To divide polynomials.

Example 1 Divide $13x - 35 + 12x^2$ by $3x + 7$. Write the answer as a polynomial.

Solution Rewrite $13x - 35 + 12x^2$ in order of decreasing degree of x as $12x^2 + 13x - 35$.

$$\begin{array}{r} 4x - 5 \\ 3x + 7 \overline{)12x^2 + 13x - 35} \\ \underline{12x^2 + 28x} \\ -15x - 35 \\ \underline{-15x - 35} \\ \text{Remainder} \longrightarrow 0 \end{array}$$

Think: $12x^2 \div 3x = ?$
 Multiply $3x + 7$ by $4x$ and subtract.
 Think: $-15x \div 3x = ?$
 Multiply $3x + 7$ by -5 and subtract.

Check: $12x^2 + 13x - 35 \stackrel{?}{=} (4x - 5)(3x + 7) + 0$ { Multiply the divisor by the quotient. Add the remainder.

Therefore $\frac{12x^2 + 13x - 35}{3x + 7} = 4x - 5$.

Since the remainder is 0, both $3x + 7$ and $4x - 5$ are factors of $12x^2 + 13x - 35$.

Divide. Write the answer as a polynomial.

- $\frac{x^2 + 8x + 15}{x + 5} \cdot x + 3$
- $\frac{x^2 - 20 - x}{x - 5} \cdot x + 4$
- $\frac{n^2 - 3n - 18}{n - 6} \cdot n + 3$
- $\frac{-22 + n^2 - 9n}{n + 2} \cdot n - 11$
- $\frac{n^2 + 7n + 10}{n + 5} \cdot n + 2$
- $\frac{3x - 18 + x^2}{x - 3} \cdot x + 6$
- $\frac{-3 - 5x + 2x^2}{2x + 1} \cdot x - 3$
- $\frac{x^2 - 12x + 32}{x - 8} \cdot x - 4$
- $\frac{x^2 - 48 + 8x}{x - 4} \cdot x + 12$
- $\frac{x^2 - 8x - 33}{x - 11} \cdot x + 3$
- $\frac{12 + n^2 - 7n}{n - 3} \cdot n - 4$
- $\frac{2x + x^2 - 15}{x + 5} \cdot x - 3$
- $\frac{6x^2 + x - 40}{3x + 8} \cdot 2x - 5$
- $\frac{3x^2 - 10x - 8}{3x + 2} \cdot x - 4$
- $\frac{3a^2 + 8a + 5}{3a + 5} \cdot a + 1$
- $\frac{-20 + 2x^2 + 3x}{x + 4} \cdot 2x - 5$

Example 2 Divide: $\frac{2a^3 + 5}{a - 2}$. Write the answer as a mixed expression.

Solution

$$\begin{array}{r} 2a^2 + 4a + 8 \\ a - 2 \overline{)2a^3 + 0a^2 + 0a + 5} \\ \underline{2a^3 - 4a^2} \\ 4a^2 + 0a \\ \underline{4a^2 - 8a} \\ 8a + 5 \\ \underline{8a - 16} \\ 21 \longleftarrow \text{Remainder} \end{array}$$

Using zero coefficients, insert missing terms in decreasing degree of a in $2a^3 + 5$. Then divide.

Division ends when the remainder is either 0 or of lesser degree than the divisor.

(Check is on next page.)

6-7 Polynomial Long Division (continued)

Check: $2a^3 + 5 \stackrel{?}{=} (2a^2 + 4a + 8)(a - 2) + 21$

$$2a^3 + 5 \stackrel{?}{=} 2a^3 + 4a^2 + 8a - 4a^2 - 8a - 16 + 21$$

$$2a^3 + 5 \stackrel{?}{=} 2a^3 + (4a^2 - 4a^2) + (8a - 8a) - 16 + 21$$

$$2a^3 + 5 = 2a^3 + 5 \checkmark$$

Therefore $\frac{2a^3 + 5}{a - 2} = 2a^2 + 4a + 8 + \frac{21}{a - 2}$.

Divide. Write the answer as a polynomial or a mixed expression.

- $\frac{a^2 + 3a - 7}{a - 2} \cdot a + 5 + \frac{3}{a - 2}$
- $\frac{k^2 - 7k + 13}{k - 3} \cdot k - 4 + \frac{1}{k - 3}$
- $\frac{6x^2 + x - 6}{3x + 2} \cdot 2x - 1 - \frac{4}{3x + 2}$
- $\frac{x^2 + 9}{x - 3} \cdot x + 3 + \frac{18}{x - 3}$
- $\frac{2a^2 + 5a - 10}{2a - 1} \cdot a + 3 - \frac{7}{2a - 1}$
- $\frac{5 - 2x + x^2}{x - 1} \cdot x - 1 + \frac{4}{x - 1}$
- $\frac{a^3 - 2a^2 - 3a + 4}{a + 1} \cdot a^2 - 3a + \frac{4}{a + 1}$
- $\frac{a^3 + 1}{a + 1} \cdot a^2 - a + 1$
- $\frac{n^3 - 8}{n - 2} \cdot n^2 + 2n + 4$
- $\frac{2x^3 - 7x^2 + 15x - 6}{2x - 1} \cdot x^2 - 3x + 6$
- $\frac{2x^3 + 9x^2 - 27}{x + 3} \cdot 2x^2 + 3x - 9$
- $\frac{x^3 + 4x^2 - 6}{x + 2} \cdot x^2 + 2x - 4 + \frac{2}{x + 2}$
- $\frac{x^3 - 4x^2 + x + 6}{x - 2} \cdot x^2 - 2x - 3$
- $\frac{x^3 + 6x^2 - x - 30}{x - 2} \cdot x^2 + 8x + 15$
- $\frac{2x^3 - x^2 - 5x - 2}{2x + 1} \cdot x^2 - x - 2$
- $\frac{x^3 - 3x^2 + 3x + 4}{x + 2} \cdot x^2 - 5x + 13 - \frac{22}{x + 2}$
- $\frac{4x^3 - 8x^2 + 7x - 2}{2x - 1} \cdot 2x^2 - 3x + 2$
- $\frac{3x^3 - 7x^2 - 22x + 8}{3x - 1} \cdot x^2 - 2x - 8$
- $\frac{8x^3 - 18x^2 + 27x + 8}{4x + 1} \cdot 2x^2 - 5x + 8$
- $\frac{2x^4 + 5x^3 - 4x^2 - 2x + 3}{x + 3} \cdot 2x^3 - x^2 - x + 1$

Mixed Review Exercises

Simplify.

- $\frac{x + 5}{3} + \frac{2x - 3}{3} \cdot \frac{3x + 2}{3}$
- $\frac{a^2}{a + 2} - \frac{4}{a + 2} \cdot a - 2$
- $\frac{3}{x} + \frac{1}{2} \cdot \frac{6 + x}{2x} \cdot \frac{1}{2(x - 4)}$
- $\frac{3c + 1}{4c} + \frac{5}{2c} \cdot \frac{3c + 11}{4c}$
- $\frac{3x + 1}{4} - \frac{2x - 3}{6} \cdot \frac{5x + 9}{12}$
- $\frac{x}{x^2 - 16} - \frac{1}{2x + 8}$
- $x + \frac{3}{x} \cdot \frac{x^2 + 3}{x}$
- $2 + \frac{n}{n - 2} \cdot \frac{3n - 4}{n - 2}$
- $y + 2 + \frac{2y - 1}{y - 1} \cdot \frac{y^2 + 3y - 3}{y - 1}$