NAME ______________________________ DATE ______________

11-7 Multiplying, Dividing, and Simplifying Radicals

Objective: To simplify products and quotients of radicals.

Vocabulary

Rationalize the denominator The process of eliminating a radical from the denominator of a fraction. Remember that $(\sqrt{a})^2 = a$.

Simplest form of a square-root radical

When all of the following are true:	**Simplest form**	***Not* in simplest form.**
1. No integral radicand has a perfect-square factor other than 1.	$2\sqrt{5}$	$\sqrt{20}$
2. No fractions are under a radical sign.	$\frac{\sqrt{3}}{3}$	$\sqrt{\frac{1}{3}}$
3. No radicals are in a denominator.	$\frac{5\sqrt{2}}{2}$	$\frac{5}{\sqrt{2}}$

Example 1 Simplify $2\sqrt{3} \cdot 3\sqrt{48}$.

Solution

$$\begin{aligned} 2\sqrt{3} \cdot 3\sqrt{48} &= (2 \cdot 3)(\sqrt{3} \cdot \sqrt{48}) \\ &= 6\sqrt{144} \\ &= 6 \cdot 12 \\ &= 72 \end{aligned}$$

Simplify.

1. $6\sqrt{2} \cdot 3\sqrt{2}$
2. $3\sqrt{5} \cdot 2\sqrt{5}$
3. $\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{9}$
4. $\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{16}$
5. $2\sqrt{3} \cdot \sqrt{5}$
6. $4\sqrt{2} \cdot \sqrt{3}$
7. $\sqrt{2} \cdot \sqrt{32}$
8. $\sqrt{3} \cdot \sqrt{27}$
9. $\sqrt{11} \cdot \sqrt{99}$
10. $\sqrt{8} \cdot \sqrt{18}$
11. $4\sqrt{108}$
12. $7\sqrt{80}$

Example 2 Simplify $\sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}}$.

Solution $\sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}} = \sqrt{\frac{7}{6} \cdot \frac{54}{28}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

Simplify.

13. $\sqrt{\frac{7}{10}} \cdot \sqrt{\frac{10}{7}}$
14. $\sqrt{\frac{5}{3}} \cdot \sqrt{\frac{3}{20}}$
15. $\sqrt{\frac{24}{11}} \cdot \sqrt{\frac{33}{2}}$
16. $\sqrt{\frac{7}{5}} \cdot \sqrt{\frac{5}{28}}$
17. $\sqrt{\frac{3}{8}} \cdot \sqrt{\frac{8}{27}}$
18. $\sqrt{\frac{3}{5}} \cdot \sqrt{\frac{125}{3}}$
19. $\sqrt{\frac{7}{3}} \cdot \sqrt{\frac{3}{112}}$
20. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{10}{8}}$

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11–7 Multiplying, Dividing, and Simplifying Radicals *(continued)*

Example 3 Simplify: **a.** $\frac{2}{\sqrt{3}}$ **b.** $\sqrt{\frac{5}{8}}$ **c.** $\frac{5\sqrt{2}}{\sqrt{12}}$ **d.** $\sqrt{4\frac{4}{5}} \cdot \sqrt{3\frac{1}{3}}$

Solution

a. $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{(\sqrt{3})^2} = \frac{2\sqrt{3}}{3}$

b. $\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{5 \cdot 2}}{2(\sqrt{2})^2} = \frac{\sqrt{10}}{4}$

c. $\frac{5\sqrt{2}}{\sqrt{12}} = \frac{5\sqrt{2}}{\sqrt{2^2 \cdot 3}} = \frac{5\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{6}}{2(\sqrt{3})^2} = \frac{5\sqrt{6}}{6}$

d. $\sqrt{4\frac{4}{5}} \cdot \sqrt{3\frac{1}{3}} = \sqrt{\frac{24}{5}} \cdot \sqrt{\frac{10}{3}} = \sqrt{\frac{24}{5} \cdot \frac{10}{3}} = \sqrt{16} = 4$

Simplify.

21. $\frac{3}{\sqrt{5}}$ **22.** $\frac{4}{\sqrt{6}}$ **23.** $\sqrt{\frac{1}{6}}$ **24.** $\sqrt{\frac{3}{8}}$

25. $\frac{6\sqrt{5}}{\sqrt{80}}$ **26.** $\frac{2\sqrt{3}}{\sqrt{48}}$ **27.** $\sqrt{3\frac{3}{4}} \cdot \sqrt{2\frac{2}{3}}$ **28.** $\sqrt{1\frac{1}{6}} \cdot \sqrt{4\frac{2}{3}}$

Example 4 Simplify $\sqrt{3}(\sqrt{3} - 4)$.

Solution $\sqrt{3}(\sqrt{3} - 4) = \sqrt{3} \cdot \sqrt{3} - \sqrt{3} \cdot 4$
$= 3 - 4\sqrt{3}$

Simplify.

29. $\sqrt{2}(\sqrt{2} - 1)$ **30.** $\sqrt{6}(5 - \sqrt{6})$ **31.** $2\sqrt{3}(\sqrt{27} - \sqrt{3})$ **32.** $3\sqrt{5}(2\sqrt{5} - \sqrt{125})$

Mixed Review Exercises

Solve.

1. $x^2 = 121$ **2.** $2x^2 - 128 = 0$ **3.** $16x^2 - 1 = 24$

4. $\frac{1}{c} + \frac{1}{3} = \frac{1}{2}$ **5.** $\frac{3}{5} = \frac{15}{y}$ **6.** $\frac{6b - 1}{3b - 1} = \frac{5}{2}$

Simplify.

7. $15x + 2(3x - 5) + 2$ **8.** $10a + 6 - (6a - 12)$ **9.** $3(2a - 5) - 4(a - 3)$

10. $(-4cd^2)(-3c^2d)$ **11.** $-3m + 2 + 9m - 5$ **12.** $x(x - 1) + (x - 3)(2x - 1)$