11-5 Square Roots of Variable Expressions

Objective: To find square roots of variable expressions and to use them to solve equations and problems.

Property

Property of Square Roots of Equal Numbers For any real numbers r and s:
$r^2 = s^2$ if and only if $r = s$ or $r = -s$.

CAUTION When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example, $\sqrt{x^2} = |x|$, not x.

Example 1 Simplify: **a.** $\sqrt{144x^2}$ **b.** $\sqrt{25n^8}$ **c.** $\sqrt{12a^3}$

Solution   **a.** $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$

$\qquad\qquad\qquad = 12|x|$

   b. $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$

$\qquad\qquad\qquad = \sqrt{25} \cdot \sqrt{(n^4)^2}$

$\qquad\qquad\qquad = 5n^4$ (n^4 is always nonnegative)

   c. $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$

$\qquad\qquad\qquad = \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$

$\qquad\qquad\qquad = 2|a|\sqrt{3a}$

Simplify.

1. $\sqrt{81x^2}$

2. $\sqrt{121x^2}$

3. $\sqrt{20x^2}$

4. $\sqrt{45x^4}$

5. $-\sqrt{25x^2}$

6. $-\sqrt{16c^4}$

7. $-\sqrt{64d^8}$

8. $-\sqrt{98n^6}$

9. $\sqrt{225y^4}$

10. $\sqrt{400a^6b^4}$

11. $\sqrt{81m^{12}}$

12. $\sqrt{441n^6}$

13. $\pm\sqrt{75x^2y^3}$

14. $\pm\sqrt{60x^6y^4}$

15. $-\sqrt{121x^2y^2}$

16. $-\sqrt{900a^4b^6}$

17. $\pm\sqrt{\dfrac{81x^8}{100}}$

18. $\pm\sqrt{\dfrac{121}{225x^{10}}}$

19. $\sqrt{\dfrac{x^4y^8}{9z^2}}$

20. $\sqrt{\dfrac{32m^3n^2}{2mn^2}}$

21. $\sqrt{\dfrac{16x^{18}}{3600y^{20}}}$

22. $\sqrt{\dfrac{256x^{40}}{25}}$

23. $\sqrt{2.25x^4}$

24. $-\sqrt{2.56k^2}$

11–5 Square Roots of Variable Expressions (continued)

Example 2   Simplify $\sqrt{m^2 - 8m + 16}$.

Solution $\sqrt{m^2 - 8m + 16} = \sqrt{(m - 4)^2} = |m - 4|$

Simplify.

25. $\sqrt{x^2 + 4x + 4}$

26. $\sqrt{n^2 - 14n + 49}$

27. $\sqrt{x^2 - 6x + 9}$

28. $\sqrt{m^2 - 10m + 25}$

Example 3   Solve $4x^2 = 25$.

Solution 1
$$4x^2 = 25$$
$$4x^2 - 25 = 0$$
$$(2x + 5)(2x - 5) = 0$$
$$2x = -5 \quad \text{or} \quad 2x = 5$$
$$x = -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2}$$

Check: $4\left(\frac{5}{2}\right)^2 \overset{?}{=} 25$ and $4\left(-\frac{5}{2}\right)^2 \overset{?}{=} 25$

$25 = 25 \;\checkmark$ and $25 = 25 \;\checkmark$

The solution set is $\left\{\frac{5}{2}, -\frac{5}{2}\right\}$.

Solution 2
$$4x^2 = 25$$
$$x^2 = \frac{25}{4}$$
$$x = \pm\sqrt{\frac{25}{4}}$$
$$x = \pm\frac{5}{2}$$

Solve.

29. $x^2 = 16$ **30.** $n^2 = 36$ **31.** $x^2 - 9 = 0$ **32.** $d^2 - 25 = 0$

33. $0 = a^2 - 49$ **34.** $0 = m^2 - 64$ **35.** $2m^2 - 18 = 0$ **36.** $40b^2 - 160 = 0$

37. $36y^2 - 16 = 0$ **38.** $4c^2 - 25 = 0$ **39.** $0 = 49z^2 - 9$ **40.** $0 = 45x^2 - 125$

Mixed Review Exercises

Simplify.

1. $\pm\sqrt{80}$ **2.** $-4\sqrt{75}$ **3.** $3\sqrt{256}$

4. $2^{-3} - 3^{-2}$ **5.** $4^3 \cdot 2^{-5}$ **6.** $(3x^2)^3(-2x^4)^2$

Evaluate if $x = 9$, $y = 16$, and $n = 1$.

7. $x^2 + y^2$ **8.** x^2n^2 **9.** $y^2 - x^2$ **10.** $\sqrt{\frac{y}{n}}$ **11.** $\sqrt{\frac{x}{y}}$ **12.** $\left(\sqrt{y}\right)^2$