NAME ______________________________ DATE ______________

12–8 Joint and Combined Variation

Objective: To solve problems involving joint variation and combined variation.

Vocabulary

Joint variation When a variable varies directly as the product of two or more other variables. You can express the relationship in the forms

$$z = kxy, \quad \text{where } k \text{ is a nonzero constant,}$$

and

$$\frac{z_1}{x_1 y_1} = \frac{z_2}{x_2 y_2}.$$

You say that *z varies jointly as x and y.*

Combined variation When a variable varies directly as one variable and inversely as another. You can express the relationship in the forms

$$zy = kx \quad \left(\text{or } z = \frac{kx}{y}\right), \quad \text{where } k \text{ is a nonzero constant,}$$

and

$$\frac{z_1 y_1}{x_1} = \frac{z_2 y_2}{x_2}.$$

You say that *z varies directly as x and inversely as y.*

Example 1 The volume of a right circular cone varies jointly as the height, h, and the square of the radius, r. If $V_1 = 1848\pi$, $h_1 = 9$, $r_1 = 14$, $h_2 = 12$, and $r_2 = 7$, find V_2.

Solution

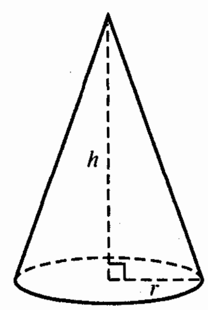
$$\frac{V_1}{h_1 r_1^2} = \frac{V_2}{h_2 r_2^2} \quad \text{Use the form for joint variation.}$$

$$\frac{1848\pi}{9(14)^2} = \frac{V_2}{12(7)^2} \quad \text{Substitute the given values.}$$

$$9(196)V_2 = 12(49)(1848\pi) \quad \text{Solve for } V_2.$$

$$1764V_2 = 1,086,624\pi$$

$$V_2 = 616\pi$$



Solve. Give roots to the nearest tenth. You may wish to use a calculator.

1. d varies jointly as r and t. If $d = 75$ when $r = 30$ and $t = 10$, find d when $r = 18$ and $t = 6$.
2. m varies jointly as v and the square of u. If $m = 60$ when $v = 25$ and $u = 12$, find u when $m = 9$ and $v = 15$.
3. The volume of a pyramid varies jointly as the height and the area of the base. A pyramid 21 cm high has a base area of 25 cm^2 and a volume of 175 cm^3. What is the volume if the height is 15 cm and the area of the base is 36 cm^2?

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12–8 Joint and Combined Variation *(continued)*

Example 2 The power, P, of an electric current varies directly as the square of the voltage, V, and inversely as the resistance, R. If 6 volts applied across a resistance of 3 ohms produces 12 watts of power, how much voltage applied across a resistance of 6 ohms will produce 24 watts of power?

Solution

$\frac{P_1R_1}{V_1^2} = \frac{P_2R_2}{V_2^2}$ Use the form for combined variation.

$\frac{12(3)}{6^2} = \frac{24(6)}{V_2^2}$ Let $P_1 = 12$, $V_1 = 6$, $R_1 = 3$, $P_2 = 24$, and $R_2 = 6$.

$36V_2^2 = 36(24)(6)$ Solve for V_2.

$V_2^2 = 144$

$V_2 = \pm\sqrt{144} = \pm 12$ Discard the negative root since a negative voltage has no meaning.

12 volts will produce the required power.

Solve. Give roots to the nearest tenth. You may wish to use a calculator.

4. c varies directly as a and inversely as b. If $c = 16$ when $a = 96$ and $b = 16$, find c when $a = 60$ and $b = 8$.
5. c varies inversely as the square of h and directly as n. If $c = 1$ when $h = 12$ and $n = 25$, find h when $c = 4.84$ and $n = 4$.
6. w varies directly as u and inversely as v^2. If $w = 8$ when $u = 2$ and $v = 3$, find u when $v = 2$ and $w = 27$.
7. If Q varies directly as the square of t and inversely as the cube of r, and $Q = 108$ when $t = 12$ and $r = 2$, find Q when $t = 15$ and $r = 3$.
8. The power, P, of an electric current varies directly as the square of the voltage, V, and inversely as the resistance, R. If 6 volts applied across a resistance of 3 ohms produces 12 watts of power, how much power will 12 volts applied across a resistance of 9 ohms produce?
9. The volume of a cone varies jointly as its height and the square of its radius. A certain cone has a volume of 154π cm^3, a height of 3 cm, and a radius of 7 cm. Find the radius of another cone that has a height of 7 cm and a volume of 264π cm^3.
10. The distance a car travels from rest varies jointly as its acceleration and the square of the time of motion. A car travels 500 m from rest in 5 s at an acceleration of 40 m/s^2. How many seconds will it take the car to travel 150 m at an acceleration of 3 m/s^2?

Mixed Review Exercises

Solve each system.

1. $2x - y = -1$; $x - y = -2$
2. $x + y = 6$; $x - 2y = 12$
3. $2x - 5y = 2$; $-x + 5y = 4$
4. $3x + 2y = 3$; $x + 2y = 9$