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7–4 Fractional Equations

Objective: To solve fractional equations.

Vocabulary

Fractional equation An equation with a variable in the denominator of one or more terms. For example, $\frac{3}{x} - \frac{1}{4} = \frac{1}{12}$. To solve a fractional equation, multiply both sides by the LCD to eliminate fractions.

CAUTION Multiplying both sides of an equation by a variable expression sometimes results in an equation that has an extra root. You must check each root of the transformed equation to see if it satisfies the original equation.

Example 1 Solve: $\frac{2}{x} + \frac{1}{4} = \frac{3}{4}$

Solution

$$4x\left(\frac{2}{x} + \frac{1}{4}\right) = 4x\left(\frac{3}{4}\right)$$ Multiply both sides of the equation by the LCD, $4x$.

$$4x\left(\frac{2}{x}\right) + 4x\left(\frac{1}{4}\right) = 3x$$
$$8 + x = 3x$$
$$8 = 2x$$
$$4 = x$$

Notice that x cannot equal 0 because $\frac{2}{0}$ has no meaning.

Check: $\frac{2}{4} + \frac{1}{4} \stackrel{?}{=} \frac{3}{4}$ $\quad \frac{3}{4} = \frac{3}{4}\ \surd$ The solution set is $\{4\}$.

Solve and check. If the equation has no solution, write *No solution*.

1. $\frac{1}{3} + \frac{11}{x} = 4$ **2.** $\frac{16}{x} - \frac{3}{5} = 1$ **3.** $\frac{1}{2} + \frac{3}{x} = 2$

4. $\frac{1}{6} + \frac{2}{x} = \frac{5}{6}$ **5.** $\frac{3}{y} - \frac{1}{4} = \frac{1}{12}$ **6.** $\frac{1}{4} + \frac{2}{x} = \frac{3}{8}$

7. $\frac{5}{x} + \frac{3}{4} = 2$ **8.** $\frac{1}{x} - \frac{1}{2} = -\frac{1}{3}$ **9.** $\frac{7}{2a} - \frac{3}{a} = -\frac{1}{4}$

10. $\frac{3}{n} - \frac{1}{2} = \frac{6}{3n}$ **11.** $\frac{2}{3a} + \frac{5}{6} = \frac{3}{2a}$ **12.** $\frac{2}{a} + \frac{3}{2a} = \frac{7}{6}$

Example 2 Solve: $\frac{6 - x}{4 - x} = \frac{3}{5}$

Solution 1

$$5(4 - x)\left[\frac{6 - x}{4 - x}\right] = 5(4 - x)\left[\frac{3}{5}\right]$$ Multiply both sides by the LCD, $5(4 - x)$.

$$5(6 - x) = (4 - x)(3)$$ Notice that x cannot equal 4.
$$30 - 5x = 12 - 3x$$
$$18 = 2x$$
$$9 = x$$ The solution set is $\{9\}$.

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7–4 Fractional Equations *(continued)*

Solution 2

$$\frac{6-x}{4-x} = \frac{3}{5}$$ Solve as a proportion.

$$5(6-x) = 3(4-x)$$

$$30 - 5x = 12 - 3x$$

$$18 = 2x$$

$$9 = x$$ The solution set is $\{9\}$.

Solve.

13. $\frac{4-x}{6-x} = \frac{5}{6}$

14. $\frac{x+4}{x-1} = 1$

15. $\frac{2}{3} = \frac{x+5}{x+7}$

16. $3 = \frac{x-5}{x-3}$

17. $\frac{x}{x-1} = \frac{6}{5}$

18. $\frac{n}{n-2} = \frac{5}{7}$

19. $\frac{x}{x+3} = \frac{2}{5}$

20. $\frac{x}{x+5} = \frac{3}{2}$

21. $\frac{x-1}{x+3} = \frac{1}{2}$

22. $\frac{5x}{x-1} = 4$

23. $\frac{x+1}{3x-1} = \frac{1}{4}$

24. $\frac{x-1}{x+3} = \frac{3}{5}$

25. $\frac{8}{x+3} = \frac{4}{x}$

26. $\frac{5}{x+2} = \frac{3}{x}$

27. $\frac{2}{x+3} = \frac{3}{x+1}$

28. $\frac{2x-4}{x-2} = 3$

29. $\frac{a+1}{2} = \frac{1}{a}$

30. $\frac{3+x}{2x} = \frac{1}{x}$

31. $\frac{a+2}{6} = \frac{4}{a}$

32. $\frac{1}{x} + \frac{3x}{x-2} = 0$

33. $\frac{4}{x+1} - \frac{1}{x} = 1$

34. $\frac{12}{x+3} = \frac{2}{x-2}$

35. $\frac{2}{x+1} - 1 = \frac{1}{1-x}$

36. $\frac{2}{y+3} - \frac{1}{y-3} = 1$

37. $\frac{2}{x-1} + 3 = \frac{4x}{x-1}$

38. $\frac{3m+5}{6} - \frac{m}{2} = \frac{10}{m}$

39. $\frac{x-3}{x} + \frac{1}{x} = \frac{x+1}{x+4}$

40. $\frac{4}{x+1} - 1 = \frac{1}{x}$

41. $\frac{3}{1-n} + 2 = \frac{5}{1+n}$

42. $\frac{n-2}{n} - \frac{1}{n} = \frac{n-3}{n-6}$

Mixed Review Exercises

Solve.

1. $\frac{3a}{4} + \frac{2a}{5} = 23$

2. $\frac{x}{3} - \frac{x}{2} = 6$

3. $\frac{1}{5}(y-1) + \frac{1}{4}(y+2) = 3$

4. $\frac{8}{3} = \frac{2n}{9}$

5. $\frac{-6}{5t} = \frac{3}{10}$

6. $\frac{3z}{4} = \frac{27}{36}$

Simplify.

7. $(5-3)^3$

8. $3x^2(2x^2 - 5 + 4x)$

9. $4 \cdot 3^2$

10. $(3n^2 + n) + (7 + n^2)$

11. $(4z^2)(3y^2z^2)$

12. $(2p^2q^3)^2$