

6-7 Polynomial Long Division

Objective: To divide polynomials.

Example 1 Divide $13x - 35 + 12x^2$ by $3x + 7$. Write the answer as a polynomial.

Solution Rewrite $13x - 35 + 12x^2$ in order of decreasing degree of x as $12x^2 + 13x - 35$.

$$\begin{array}{r}
 4x - 5 \\
 3x + 7 \overline{)12x^2 + 13x - 35} \\
 \underline{12x^2 + 28x} \\
 -15x - 35 \\
 \underline{-15x - 35} \\
 \text{Remainder} \longrightarrow 0
 \end{array}
 \quad \left\{ \begin{array}{l} \text{Think: } 12x^2 \div 3x = ? \\ \text{Multiply } 3x + 7 \text{ by } 4x \text{ and subtract.} \\ \\ \text{Think: } -15x \div 3x = ? \\ \text{Multiply } 3x + 7 \text{ by } -5 \text{ and subtract.} \end{array} \right.$$

$$\begin{array}{l}
 \text{Check: } 12x^2 + 13x - 35 \stackrel{?}{=} (4x - 5)(3x + 7) + 0 \\
 12x^2 + 13x - 35 = 12x^2 + 13x - 35 \quad \checkmark \quad \left\{ \begin{array}{l} \text{Multiply the divisor by the} \\ \text{quotient. Add the remainder.} \end{array} \right.
 \end{array}$$

$$\text{Therefore } \frac{12x^2 + 13x - 35}{3x + 7} = 4x - 5.$$

Since the remainder is 0, both $3x + 7$ and $4x - 5$ are factors of $12x^2 + 13x - 35$.

Divide. Write the answer as a polynomial.

1. $\frac{x^2 + 8x + 15}{x + 5}$

2. $\frac{x^2 - 20 - x}{x - 5}$

3. $\frac{n^2 - 3n - 18}{n - 6}$

4. $\frac{-22 + n^2 - 9n}{n + 2}$

5. $\frac{n^2 + 7n + 10}{n + 5}$

6. $\frac{3x - 18 + x^2}{x - 3}$

7. $\frac{-3 - 5x + 2x^2}{2x + 1}$

8. $\frac{x^2 - 12x + 32}{x - 8}$

9. $\frac{x^2 - 48 + 8x}{x - 4}$

10. $\frac{x^2 - 8x - 33}{x - 11}$

11. $\frac{12 + n^2 - 7n}{n - 3}$

12. $\frac{2x + x^2 - 15}{x + 5}$

13. $\frac{6x^2 + x - 40}{3x + 8}$

14. $\frac{3x^2 - 10x - 8}{3x + 2}$

15. $\frac{3a^2 + 8a + 5}{3a + 5}$

16. $\frac{-20 + 2x^2 + 3x}{x + 4}$

Example 2 Divide: $\frac{2a^3 + 5}{a - 2}$. Write the answer as a mixed expression.

Solution

$$\begin{array}{r}
 2a^2 + 4a + 8 \\
 a - 2 \overline{)2a^3 + 0a^2 + 0a + 5} \\
 \underline{2a^3 - 4a^2} \\
 4a^2 + 0a \\
 \underline{4a^2 - 8a} \\
 8a + 5 \\
 \underline{8a - 16} \\
 21 \quad \longleftarrow \text{Remainder}
 \end{array}
 \quad \left\{ \begin{array}{l} \text{Using zero coefficients, insert} \\ \text{missing terms in decreasing degree} \\ \text{of } a \text{ in } 2a^3 + 5. \text{ Then divide.} \end{array} \right.$$

Division ends when the remainder is either 0 or of lesser degree than the divisor.

(Check is on next page.)

6-7 Polynomial Long Division (continued)

$$\begin{aligned} \text{Check: } 2a^3 + 5 &\stackrel{?}{=} (2a^2 + 4a + 8)(a - 2) + 21 \\ 2a^3 + 5 &\stackrel{?}{=} 2a^3 + 4a^2 + 8a - 4a^2 - 8a - 16 + 21 \\ 2a^3 + 5 &\stackrel{?}{=} 2a^3 + (4a^2 - 4a^2) + (8a - 8a) - 16 + 21 \\ 2a^3 + 5 &= 2a^3 + 5 \checkmark \\ \text{Therefore } \frac{2a^3 + 5}{a - 2} &= 2a^2 + 4a + 8 + \frac{21}{a - 2}. \end{aligned}$$

Divide. Write the answer as a polynomial or a mixed expression.

17. $\frac{a^2 + 3a - 7}{a - 2}$

18. $\frac{k^2 - 7k + 13}{k - 3}$

19. $\frac{6x^2 + x - 6}{3x + 2}$

20. $\frac{x^2 + 9}{x - 3}$

21. $\frac{2a^2 + 5a - 10}{2a - 1}$

22. $\frac{5 - 2x + x^2}{x - 1}$

23. $\frac{a^3 - 2a^2 - 3a + 4}{a + 1}$

24. $\frac{a^3 + 1}{a + 1}$

25. $\frac{n^3 - 8}{n - 2}$

26. $\frac{2x^3 - 7x^2 + 15x - 6}{2x - 1}$

27. $\frac{2x^3 + 9x^2 - 27}{x + 3}$

28. $\frac{x^3 + 4x^2 - 6}{x + 2}$

29. $\frac{x^3 - 4x^2 + x + 6}{x - 2}$

30. $\frac{x^3 + 6x^2 - x - 30}{x - 2}$

31. $\frac{2x^3 - x^2 - 5x - 2}{2x + 1}$

32. $\frac{x^3 - 3x^2 + 3x + 4}{x + 2}$

33. $\frac{4x^3 - 8x^2 + 7x - 2}{2x - 1}$

34. $\frac{3x^3 - 7x^2 - 22x + 8}{3x - 1}$

35. $\frac{8x^3 - 18x^2 + 27x + 8}{4x + 1}$

36. $\frac{2x^4 + 5x^3 - 4x^2 - 2x + 3}{x + 3}$

Mixed Review Exercises

Simplify.

1. $\frac{x + 5}{3} + \frac{2x - 3}{3}$

2. $\frac{a^2}{a + 2} - \frac{4}{a + 2}$

3. $\frac{3}{x} + \frac{1}{2}$

4. $\frac{3c + 1}{4c} + \frac{5}{2c}$

5. $\frac{3x + 1}{4} - \frac{2x - 3}{6}$

6. $\frac{x}{x^2 - 16} - \frac{1}{2x + 8}$

7. $x + \frac{3}{x}$

8. $2 + \frac{n}{n - 2}$

9. $y + 2 + \frac{2y - 1}{y - 1}$