10–4 Solving Combined Inequalities

Objective: To find the solution sets of combined inequalities.
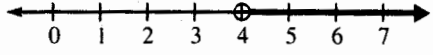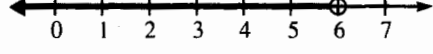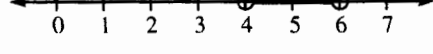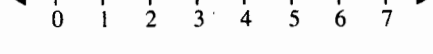
Vocabulary

Conjunction A sentence formed by joining two open sentences by the word *and*. For example, $-1 < x$ *and* $x < 4$, which can also be written as $-1 < x < 4$.

Solve a conjunction To find the values of the variables for which *both* open sentences in the conjunction are true.

Disjunction A sentence formed by joining two open sentences by the word *or*. For example, $y > 1$ *or* $y = 1$.
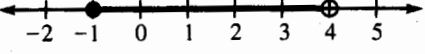
Solve a disjunction To find the values of the variables for which *at least one* of the open sentences in the disjunction is true.

Example 1 Draw the graph of each open sentence.   **Solution**

a. $4 < x$

a.

b. $x < 6$

b.

c. conjunction: $4 < x$ *and* $x < 6$

c.

d. disjunction: $4 < x$ *or* $x < 6$

d.

Draw the graph of each open sentence.

1. $-2 < t$ and $t \le 1$ 2. $r > 2$ or $r \le -1$ 3. $2 \le n$ and $n \le 6$ 4. $x < -1$ or $x \ge 1$

Example 2 Describe the graph of each open sentence.

a. conjunction: $t < 3$ *and* $t \ge 3$ b. disjunction: $t < 3$ *or* $t \ge 3$.

Solution

a. No real number can be less than 3 and also greater than or equal to 3. The solution set is the empty set. It has no graph.

b. Every real number is either less than 3 or greater than or equal to 3. The solution set is {the real numbers}. Its graph is the entire number line.
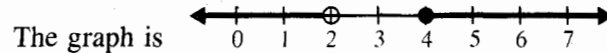

Example 3 Solve the conjunction $-2 \le x - 1 < 3$ and graph its solution set.

Solution 1 Solve the conjunction:

$$
\begin{array}{ccc}
-2 \le x - 1 & \text{and} & x - 1 < 3 \\
-2 + 1 \le x - 1 + 1 & | & x - 1 + 1 < 3 + 1 \\
-1 \le x & \text{and} & x < 4 \\
\end{array}
$$
$$-1 \le x < 4$$

The solution set is $\{-1,$ and all the real numbers *between* -1 and $4\}$.

The graph is

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10–4 Solving Combined Inequalities (continued)

Solution 2
$$-2 \leq \quad x - 1 \quad < 3$$
$$-2 + 1 \leq x - 1 + 1 < 3 + 1 \quad \text{Add 1 to each part of the inequality.}$$
$$-1 \leq \quad x \quad < 4$$

Example 4   Solve the disjunction $2x + 1 < 5$ or $3x \geq x + 8$ and graph its solution set.

Solution

$$
\begin{array}{ccc}
2x + 1 < 5 & \text{or} & 3x \geq x + 8 \\
2x + 1 - 1 < 5 - 1 & & 3x - x \geq x + 8 - x \\
2x < 4 & & 2x \geq 8 \\
x < 2 & \text{or} & x \geq 4
\end{array}
$$

The solution set is $\{4,$ and the real numbers greater than 4 *or* less than 2$\}$.

The graph is

Solve each open sentence. Graph the solution set, if there is one.

5. $-1 < a - 1 < 4$ **6.** $-3 < y + 1 \leq 2$

7. $-2 < -3 + d \leq 1$ **8.** $-4 \leq 2 + r < 2$

9. $-4 \leq 2a + 6 < 10$ **10.** $-3 < 2b + 1 \leq 5$

11. $-8 \leq 3m + 1 < 7$ **12.** $-4 < 3n + 5 \leq 8$

13. $x - 1 < -4$ or $x - 1 > 5$ **14.** $h + 3 \leq -1$ or $h + 3 \geq 1$

15. $2x - 1 \leq -5$ or $2x - 1 > 5$ **16.** $3 + 2y < -5$ or $3 + 2y > 5$

17. $-5x > 20$ or $10 + 5x \geq 0$ **18.** $2d - 3 < -5$ or $5 < 2d - 3$

19. $-3m < 6$ and $18 + 3m < 0$ **20.** $-3 \leq 1 - t$ and $1 - t < 2$

Mixed Review Exercises

Choose a variable and use the variable to write an inequality.

1. The finish line is at least 20 yd away.   **2.** The temperature cannot exceed 25 °C.

3. The weight is at most 105 lb.   **4.** The flight takes at least 2 h.

5. The cost is not more than $75.   **6.** The tolerance is smaller than 1 cm.

7. Ray averages at most 15 points per game.   **8.** Joy won at least 12 tennis matches.

Evaluate each expression if $k = -3$, $m = 9$, and $x = 3$.

9. $|x - k|$ **10.** $|m - k|$ **11.** $|x + k|$

12. $|k - x|$ **13.** $|k - m|$ **14.** $|k + m|$